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Stress Intensity Factor of Radial Cracks for Rotating Disks and Cylinders using Average Stress Method

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This article utilizes the average stress method to obtain the stress intensity factor of rotating solid and hollow disks/cylinders containing a radial crack. It is assumed that the cracks are located radially at center, internal or external radius of the geometry. Results are shown for both of the plane stress and plane strain assumptions and are validated against the known data introduced in the literature search.

Keywords: Stress intensity factor, Small crack, Rotating disk and cylinder, Radial crack, Axial crack

1 Introduction

Several researchers have reported stress intensity factors in solid or hollow rotating disks containing radial cracks. Rooke and Tweed [1, 2] expressed stress intensity factor of radial cracks in a finite rotating solid disk in terms of functions of Feredholm equation, then the integral equations are solved using Gauss-Chebyshev quadrature numerically. Owen and Griffiets [3], utilized the FEM to evaluate stress intensity factor in a rotating disk with an edge crack. Their work shows the plot of variation of stress intensity factor versus crack length both for a smooth bore and for a bore containing a keyway. Isida [4] treated stress intensity factor of a rotating solid disk containing an internal crack located at an arbitrary position. Isida used the eigenfunction expansions of the complex stress potentials and presented formulae for stress intensity factor of various crack geometries including special case of central crack. Sukere [5] obtained an approximation for stress intensity factor of edge cracks in rotating disk. This approximation is based on the equivalent stress defined by Williams and Isherwood [6] at crack tip. The same author evaluated stress intensity factor of radial cracks in rotating disks by an elctro-optical technique based on the method of caustics [7]. Gregory [8] obtained a closed form solution for stress intensity factor of a rotating isotropic disk including an edge crack in terms of polynomial series. Schneider and Danzer [9] solved the same problem applying the weight function method. To this aim they used the polynomial fit procedure published by Oliveria and Wu [10] to solve the weight function integral. Gowhari et al. [11] used the finite element method to

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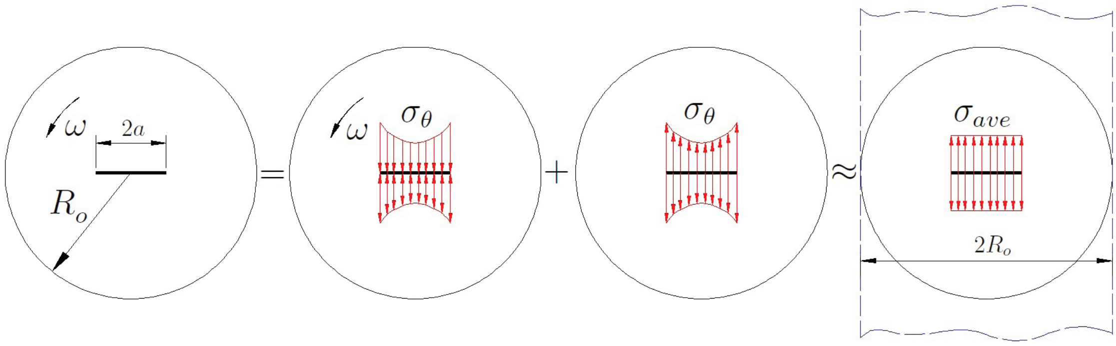

Figure 1 Superposition method applied to approximate stress intensity factor

predict the stress intensity factors for radial single and double edge cracks of annular and solid discs under constant angular velocity. They approximated equivalent prediction equations to evaluate stress intensity factors of these cracked geometries interpolating data obtained from finite element results.

In this paper the stress intensity factor of isotropic solid disks with central or single edge radial crack is approximated utilizing the average hoop stress along the crack face. The proposed simple equations predict the stress intensity factors with engineering accuracy for mentioned geometry. On the other hand, the present work demonstrates formulae for stress intensity factor of radial cracks located at internal or external radius of surface of a hollow disk. These formulae are obtained utilizing the combination of weight function and the average stress method. The results and discussion section of this paper compares presented formulae of this paper with those reported in the literature search whenever they were available. This comparison shows that some of the reported data in the literature search are not compatible with others for the same geometry.

2 Mathematical formulation

Stress intensity factor of rotating disks containing a single radial crack is obtained in this research assuming both the plane stress and plane strain conditions. The crack problems considered in this paper are central crack or radial edge crack for solid disks and radial crack at inside/outside surfaces for hollow disks. The following subsections shows the mathematical procedure to obtain these stress intensity factors.

2.1 Disk with central crack

Consider a rotating disk/cylinder with radius R_o containing a central crack with length $2a$. Applying the superposition method, the stress intensity factor of the rotating cracked disk is equivalent to the stress intensity factor of unrotating cracked disk. Wherein, the stress distribution is equal to the tangential stress of the rotating disk. Now, the first mode SIF of this unrotating disk is approximated with the SIF of a finite strip containing a central crack subjected to uniform stress σ_{ave} along the crack face. Where, σ_{ave} is average of the tangential stress produced by rotation of the uncracked disk over the crack length (See Fig. 1). The tangential stress of a rotating disk for plane stress and plane strain assumptions is:

$$\sigma_\theta = \frac{1+\alpha}{3}\rho\omega^2(R_o^2 - \frac{3\alpha}{1+\alpha}r^2) \tag{1}$$

where α is:

$$\alpha = \frac{1+3\nu}{8} \quad \text{plane stress}$$
$$\alpha = \frac{1+2\nu}{8(1-\nu)} \quad \text{plane strain} \tag{2}$$

The average tangential stress over the crack length is:

$$\sigma_{ave} = \frac{1}{a}\int_0^a \sigma_\theta dr = \frac{1+\alpha}{3}\rho\omega^2 R_o^2[1 - \frac{\alpha}{1+\alpha}(\frac{a}{R_o})^2] \tag{3}$$

For a finite strip of width $2R_o$ with a central crack of length $2a$ subjected to the uniform remote stress σ, K_I is defined by [12]:

$$K_I = \sigma\sqrt{\pi a}\sqrt{\sec\frac{\pi a}{2R_o}} \tag{4}$$

Applying equation (4) to the problem of disk results:

$$K_I = \frac{1+\alpha}{3}\rho\omega^2 R_o^2\sqrt{\sec(\frac{\pi a}{2R_o})}[1 - \frac{\alpha}{1+\alpha}(\frac{a}{R_o})^2]\sqrt{\pi a} \tag{5}$$

2.2 *Disk with edge crack*

The second geometry is a rotating disk/cylinder with an edge crack of the length a. Averaging the tangential stress over the crack length results:

$$\sigma_{ave} = \frac{1}{a}\int_{R_o-a}^{R_o} \sigma_\theta dr = \frac{1+\alpha}{3}\rho\omega^2 R_o^2\{1 - \frac{\alpha}{(1+\alpha)a/R_o}[1 - (1 - \frac{a}{R_o})^3]\} \tag{6}$$

The following equation is proposed for stress intensity factor of a disk with an edge crack subjected to the uniform pressure acting along the crack length a [13].

$$K_I = 1.1215(1 - \frac{a}{2R_o})^{-\frac{3}{2}}\sigma\sqrt{\pi a} \tag{7}$$

Due to the Eq. (7), stress intensity factor of the rotating disk with an edge crack is:

$$K_I = 1.1215(\frac{1+\alpha}{3})(1 - \frac{a}{2R_o})^{-\frac{3}{2}}\rho\omega^2 R_o^2\{1 - \frac{\alpha}{(1+\alpha)a/R_o}[1 - (1 - \frac{a}{R_o})^3]\}\sqrt{\pi a} \tag{8}$$

2.3 *Hollow disk with edge crack at internal face*

Weight function method proposed by Bueckner [14] and Rice [15] is used to compute the first mode stress intensity factor of a rotating hollow disk/cylinder with a radial crack located at its inside surface. Based on this method, if for a cracked body under symmetric loading (called the reference case) the stress intensity factor $K_I^r(a)$ and the displacement field $u_y^r(a)$ are known, then the stress intensity factor for any loading system applied to the same crack may be calculated by the following equation (See Fig. 2):

$$K_I = \int_0^a \sigma(x)\cdot h_I(x,a)dx \tag{9}$$

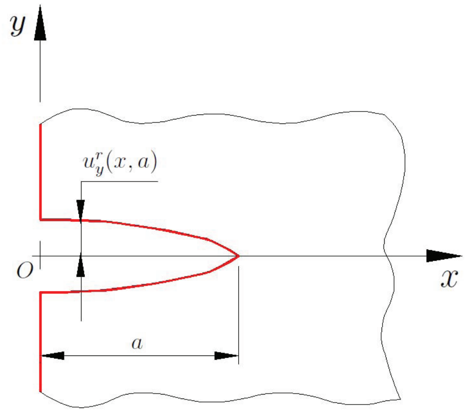

Figure 2 Coordinate system

where $\sigma(x)$ is stress distribution along the crack length in the uncracked body due to the loading for which the stress intensity factor is calculated, and $h(x, a)$ is the Mode I weight function which can be determined from:

$$h_I(x,a) = \frac{E'}{K_I^r(a)} \frac{\partial u_y^r(x,a)}{\partial a} \tag{10}$$

In Eq. (10), $E' = E$, for plane stress, $E = E/(1-\nu^2)$ for plane strain and E is the modulus of elasticity. The solution for crack face displacement field is needed to apply Eq. (9). Displacement field for hollow disks with an internal or external crack is proposed by Pastrama [16, 17] using the FE method. Ma et al. [18] also obtained the weight function $h_I(x, a)$ for the same geometries using the FE method. In this paper, the weight function obtained by Ma et al. for a ring with a radial crack at its inside or outside surface is rearranged in the following form:

$$\begin{aligned}\sqrt{a}h_I(x/a) = & \frac{\sqrt{2}}{\sqrt{\pi}\sqrt{1-x/a}} + A^1(a_0, W)\sqrt{1-x/a} \\ & + A^2(a_0, W)(1-x/a)^{3/2}\end{aligned} \tag{11}$$

The subsequent polynomials show the coefficients A^1 and A^2 which are functions of normalized crack length $a_0 = a/R_i$ and ratio of inside to outside radius $W = R_i/R_o$:

$$A^i(a_0, W) = \sum_{m=1}^{6,5} \sum_{n=1}^{6} C_{mn}^i \left(\frac{a_0}{1-W}\right)^{m-1} W^{n-1}, i = 1 \text{ or } 2 \tag{12}$$

The values of C_{mn}^1 and C_{mn}^2 for a ring with a radial crack at its inside surface are shown in Tables (1) and (2) respectively. The values of the same parameters for a ring with a radial crack at its outside surface are shown in Tables (3) and (4) respectively. Figure (3) shows comparison between different methods for stress intensity factor of this geometry subjected to uniform pressure along the crack lengths. These results are based on numerical data presented by Andrasik and Praker [19] using weight functions which were generated via the mapping collocation method, numerical results presented by Grandt using the FE method [20] and numerical results produced by closed form solution of elasticity problem presented by Delale [21]. As it may be seen from the figure stress intensity factor used in this article is more compatible to those of Andrasic and Parker and results of Grandt. These results are generated for $R_i/R_o = 0.5$, and

Table 1 Coefficients C^1_{mn} for internal face cracked ring

	n					
m	1	2	3	4	5	6
1	2.5501031E-1	0	0	0	0	0
2	2.9223186E+1	-3.5227005E+2	1.4885594E3	-2.9771731E+3	2.8698853E+3	-1.0740393E+3
3	-4.0255411E+2	4.3836982E+3	-1.8014178E4	3.5722875E+4	-3.4306727E+4	1.2808982E+4
4	1.6754462E+3	-1.7732055E+4	7.2202617E4	-1.4244828E+5	1.3648998E+5	-5.0916840E+4
5	-2.7297581E+3	2.8573816E+4	-1.1586344E5	2.2785452E+5	-2.1786531E+5	8.1192016E+4
6	1.5223427E+3	-1.5816750E+4	6.3981188E4	-1.2551983E+5	1.1974642E+5	-4.4547406E+4

Table 2 Coefficients C^2_{mn} for internal face cracked ring

	n					
m	1	2	3	4	5	6
1	1.3602442E-1	0	0	0	0	0
2	8.5642176E+0	-9.9005569E+1	4.2193509E+2	-8.4729144E+2	8.1781989E+2	-3.0592313E+2
3	-8.6422897E+1	9.9205627E+2	-4.1968433E+3	8.4519102E+3	-8.2090146E+3	3.0980422E+3
4	2.2335414E+2	-2.7056128E+3	1.1601126E+4	-2.3499676E+4	2.2938346E+4	-8.7259707E+3
5	-2.1869618E+2	2.8572034E+3	-1.2418219E+4	2.5240855E+4	-2.4621031E+4	9.3981289E+3
6	6.3620838E+1	-9.8720313E+2	4.3472964E+3	-8.8032422E+3	8.4773438E+3	-3.2020862E+3

Table 3 Coefficients C^1_{mn} for external face cracked ring

	n					
m	1	2	3	4	5	6
1	2.5501031E-1	0	0	0	0	0
2	5.4080143E+1	-5.6152875E+2	2.2443203E+3	-4.3442256E+3	4.0749150E+3	-1.4852133E+3
3	-5.0961920E+2	5.3548418E+3	-2.1458539E+4	4.1624289E+4	-3.9122293E+4	1.4290093E+4
4	1.2670717E+3	-1.3267471E+4	5.3177668E+4	-1.0316833E+5	9.7056742E+4	-3.5495023E+4
5	-9.2447144E+2	9.6765908E+3	-3.8768777E+4	7.5214820E+4	-7.0813688E+4	2.5934543E+4

Table 4 Coefficients C^2_{mn} for external face cracked ring

	n					
m	1	2	3	4	5	6
1	1.3602442E-1	0	0	0	0	0
2	-6.2564392e+001	6.5881152e+002	-2.6154922e+003	5.0134692e+003	-4.6529854e+003	1.6780341e+003
3	6.0184979e+002	-6.2566255e+003	2.4853787e+004	-4.7702477e+004	4.4335598e+004	-1.6012451e+004
4	-1.4538805e+003	1.5061571e+004	-5.9776449e+004	1.1467289e+005	-1.0651307e+005	3.8437199e+004
5	1.0249813e+003	-1.0583187e+004	4.1929215e+004	-8.0310359e+004	7.4472859e+004	-2.6815914e+004

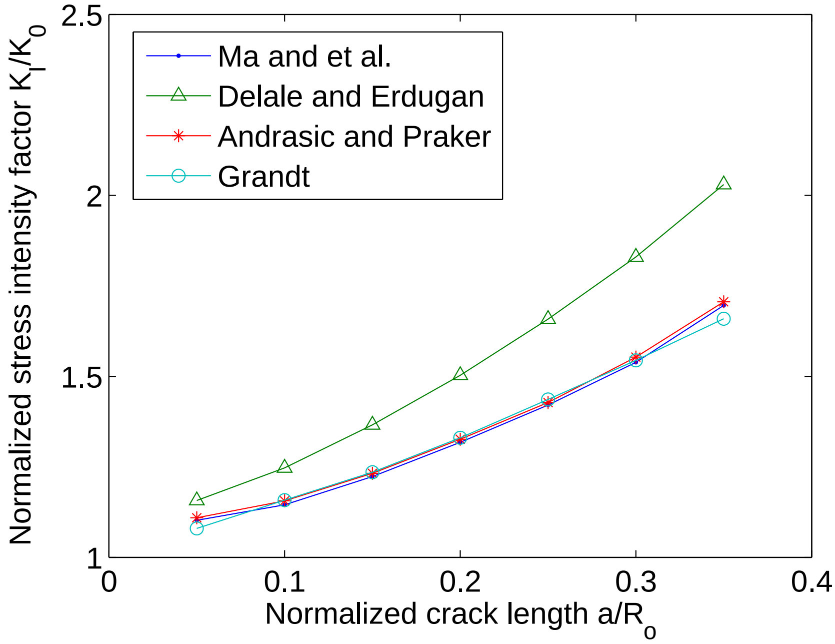

Figure 3 Stress intensity factor of an edge crack at inner surface of a disk or cylinder subjected to the uniform pressure along the crack length with $\nu = 0.3$ and $R_i/R_o = 0.5$

$\nu = 0.3$. Maximum difference between the results of Eq. (13) and results of Andriask and Parkder is less than 1% while the maximum difference between Eq. (13) and results of Delale and Erdugan is 16.5%.

Substituting Eq. (11) into Eq. (9) results to the stress intensity factor of a ring with a radial crack located at its internal or external surface subjected to a uniform stress distribution σ acting along the crack length.

$$K_I = (\frac{2\sqrt{2}}{\pi} + \frac{2}{3}A^1(a_0, W) + \frac{2}{5}A^2(a_0, W))\sigma\sqrt{\pi a} \tag{13}$$

Putting the average hoop stress along the crack length corresponding to an uncracked rotating hollow disk in Eq. (13), results to the stress intensity factor of this geometry. The hoop stress is:

$$\sigma_\theta = \frac{1+\alpha}{3}\rho\omega^2(R_i^2 + R_o^2 + \frac{R_i^2 R_o^2}{r^2} - \frac{3\alpha}{1+\alpha}r^2) \tag{14}$$

The average hoop stress along the crack length is:

$$\begin{aligned}\sigma_{ave} &= \frac{1}{a}\int_{R_i}^{R_i+a} \sigma_\theta dr \\ &= \frac{1+\alpha}{3}\rho\omega^2[1 + W^2 - \frac{W^2}{a_0}(\frac{1}{W+a_0} - \frac{1}{W}) - (\frac{\alpha}{1+\alpha})\frac{(W+a_0)^3 - W^3}{a_0}]\end{aligned} \tag{15}$$

Substitution σ_{ave} obtained in Eq. (15) instead of σ in Eq. (13), results to first mode of stress intensity factor of the ring with a radial crack at inside surface. That is:

$$\begin{aligned}K_I = &\frac{1+\alpha}{3}\rho\omega^2[1 + W^2 - \frac{W^2}{a_0}(\frac{1}{W+a_0} - \frac{1}{W}) - (\frac{\alpha}{1+\alpha})\frac{(W+a_0)^3 - W^3}{a_0}]\times \\ &[\frac{2\sqrt{2}}{\pi} + \frac{2}{3}A^1(a_0, W) + \frac{2}{5}A^2(a_0, W)]\sqrt{\pi a}\end{aligned} \tag{16}$$

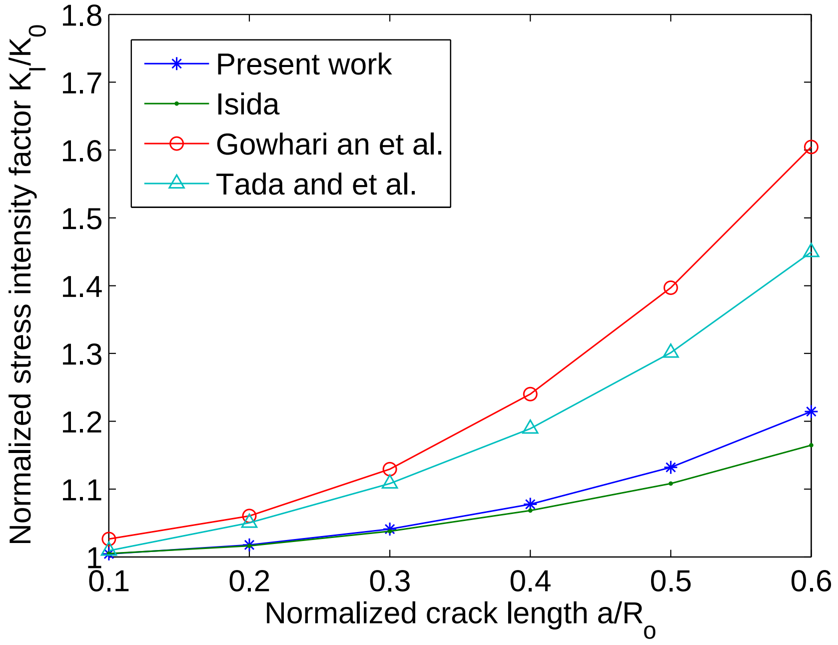

Figure 4 Stress intensity factor of central crack in rotating solid disk under plane stress assumption for $\nu = 0.3$

2.4 Hollow disk with edge crack at external face

Averaging the hoop stress given by Eq. (14) of a rotating hollow disk/cylinder, along the crack length placed radially at external surface, results to:

$$\begin{aligned}\sigma_{ave} &= \frac{1}{a}\int_{R_o-a}^{R_o} \sigma_\theta dr \\ &= \frac{1+\alpha}{3}\rho\omega^2[1+W^2-\frac{W^2}{a_0}(1-\frac{1}{1-a_0})-(\frac{3\alpha}{1+\alpha})\frac{1-(1-a_0)^3}{a_0}] \qquad (17)\end{aligned}$$

Substitution the average stress (17) in Eq. (13), the stress intensity factor for a rotating ring with radial crack at its external face is obtained:

$$\begin{aligned}K_I = \frac{1+\alpha}{3}\rho\omega^2[1+W^2-\frac{W^2}{a_0}(1-\frac{1}{1-a_0})-(\frac{\alpha}{1+\alpha})\frac{1-(1-a_0)^3}{a_0}]\times \\ [\frac{2\sqrt{2}}{\pi}+\frac{2}{3}A^1(a_0,W)+\frac{2}{5}A^2(a_0,W)]\sqrt{\pi a} \qquad (18)\end{aligned}$$

3 Results and discussion

To check the accuracy of the method, this section calculates and compares the values of stress intensity factors obtained in previous sections with results given in the literature search.

First, the problem of rotating solid disk with a central radial crack under plane stress assumption is considered. Figure (4), shows the comparison of normalized stress intensity factor K_I/K_0 versus normalized crack length a/R_o obtained in present work to exact solution of Isida [4], FE results of Gowhari [11] and the formula proposed by Tada [22] for $\nu = 0.3$. Where:

$$K_0 = \frac{1+\alpha}{3}\rho\omega^2 R_o^2\sqrt{\pi a} \qquad (19)$$

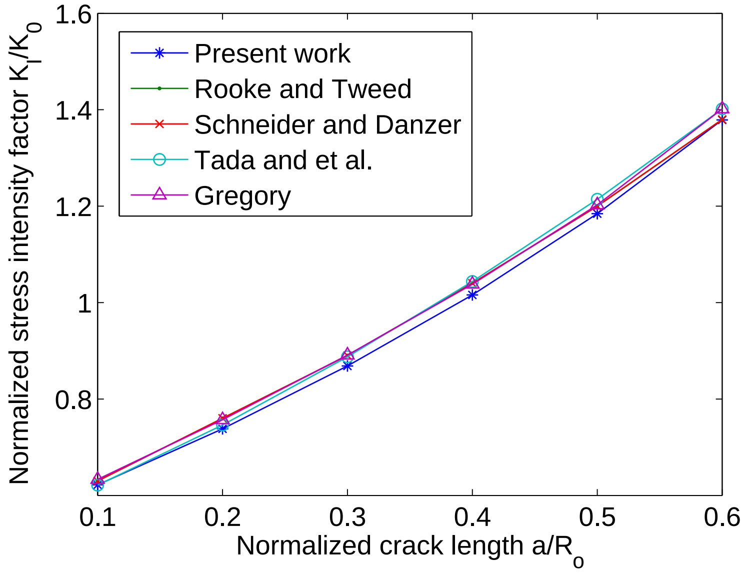

Figure 5 Stress intensity factor of an edge crack in rotating solid disk under plane strain assumption for $\nu = 0.2$

The figure shows that the proposed formula in this work is close to the exact solution of Isida even for long cracks, comparing to other methods. The maximum difference of presented formula is less than 4.5% for $a/R_o \leq 0.6$ and less than 1% for $a/R_o \leq 0.4$.

In Fig. (5), the normalized stress intensity factor K_I/K_0 of a rotating solid disk containing a radial edge crack for plane stress condition and $\nu = 0.2$ is presented and is compared with the numerical solution proposed by Rooke and Tweed[1] [2], weight function polynomials proposed by Schneider and Danzer [9], asymptotic interpolation of Tada [22] and closed form solution presented by Gregory [2] [8]. As it may be seen, a close solution to other methods is obtained using the proposed formula in this work. The maximum difference between results of present work and Rooke is less than 3%.

Figure (6) shows comparison of stress intensity factor of the same previous problem under plane stress assumption. The stress intensity factor proposed in current article is compared with the FE results of Gowhari et al. [11], asymptotic interpolation of Tada [22], and exact closed form solution of Gregory [8]. The maximum difference between the present work in Fig. (6) with those obtained by Gregory is less than 2.5%.

Figure (7) shows the comparison between Eq. (16) and results presented by Delale and Erdogan [21] obtained by truncating the series up to the 140 terms and using a curve fitting procedure for stress intensity factor of a rotating disk containing a radial crack at its internal surface for plane stress condition and with $\nu = 0.3$ and $R_i/R_o = 0.5$. The difference between presented formula in this work and results presented by Delale and Erdogan is 16.2% which is around the same difference shown in Fig. (3) for the uniform pressure loading along the crack length[3]. It should be mentioned that presented data of Delale and Erdogan for a pressurized

[1]Results of Rooke and Tweed originally are normalized to $K_0 = \frac{\rho\omega^2(3-2\nu)}{8(1-\nu)}\sqrt{\pi a}$. In Fig. (5) their results are normalized to K_0 given by Eq. (19).

[2]Resutls of Gregory originally are normalized to $K_0 = 1/3\rho\omega^2\sqrt{\pi a}$. In Figs. (5) and (6), results of Gregory are normalized to K_0 given by equation (19).

[3]Gowhari and et al.[11], proposed $K_I/K_0 = (0.2+\nu)^{-0.01}[8.04 + (\frac{R_i}{R_o})^{0.025}(\frac{a}{R_o})^{-0.013}]$ for this case. Since this formula predicts a very large stress intensity factor and differs around 3.5 times to results of Delale and et al., a mistake might be happened in this formula and so, it is not plotted in Fig. (7).

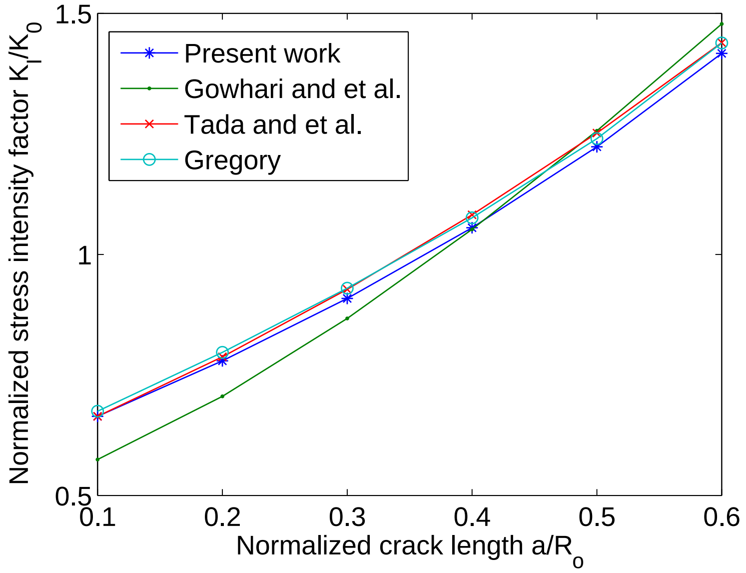

Figure 6 Stress intensity factor of an edge crack in rotating solid disk under plane stress assumption and $\nu = 0.2$

thick cylinder containing a radial edge crack at its inside surface differs with results of Ma et al. [18], Grandte [20] and Andrasic and Parker [19] in the same order of magnitude of the uniform pressure loading along the crack length given in Fig. (3). Tabular data of normalized stress intensity factors based on the formula presented in this work for internal face cracked edge rotating hollow disks and cylinders are shown in Tables (5) and (6) respectively. The values of K_I/K_0 in tables are shown for various $a/(R_o - R_i)$, $R_i/(R_o - R_i)$ and $\nu = 0.3$.

Figure (8) shows comparison of stress intensity factor of a rotating hollow disk obtained by present work with those presented by truncated series solution of Delale and Erdogran [21] and FE results of Gowhari et al. [11] for $\nu = 0.3$. The maximum difference between results of present work and results of Delale and Erdogan is less than 2%. It should be mentioned that a good agreement is obtained for stress intensity factor presented by Delale and Erdogan [21], results of Ma and et al. [18], Grandte [20] and Andrasic and Parker [19] for a pressurized disk containing and external edge crack. The normalized stress intensity factor for plane stress and plane strain conditions are shown in Tables (7) and (8) respectively. The results of K_I/K_0 in these tables are presented for different values of $R_i/(R_o - R_i)$ and $a/(R_o - R_i)$.

4 Conclusion

In this paper average stress method is used to evaluate formulae for stress intensity factor of cracked rotating solid/hollow disks for both of the plane stress and plane strain conditions. With a good accuracy, the simple equation presented in this work for rotating disks including a central crack, was the closest one to the exact solution in comparison with results obtained by other methods, including the FEM. The simple formula presented for rotating disk with an edge crack shows close agreement to the numerical values obtained by exact solution of the elasticity problem and other methods. Formulas in the form of polynomials presented for the stress intensity factor of rotating hollow disks with a crack at outside surface shows a good agreement with numerical results obtained by truncated series.and

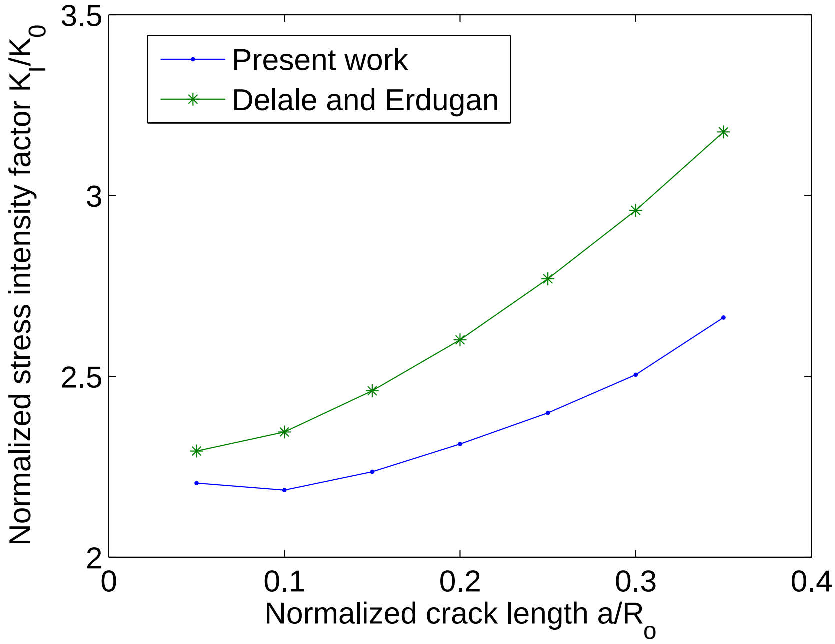

Figure 7 Stress intensity factor of an edge crack in rotating solid disk under plane stress condition with $R_i/R_o = 0.5$ and $\nu = 0.3$

Table 5 Normalized stress intensity factor for internal face crack in rotating hollow disks $\nu = 0.3$

	$R_i/(R_o-R_i)$				
$a/(R_o-R_i)$	1/3	1/2	1	2	3
0.01	2.2159	2.2518	2.3430	2.4479	2.5101
0.02	2.1568	2.2052	2.3197	2.4368	2.5046
0.03	2.1015	2.1623	2.2986	2.4281	2.5014
0.04	2.0498	2.1230	2.2797	2.4216	2.5004
0.05	2.0016	2.0869	2.2628	2.4173	2.5015
0.06	1.9567	2.0538	2.2478	2.4151	2.5046
0.07	1.9148	2.0234	2.2346	2.4149	2.5098
0.08	1.8758	1.9955	2.2230	2.4165	2.5168
0.09	1.8396	1.9700	2.2131	2.4200	2.5258
0.1	1.8059	1.9466	2.2048	2.4252	2.5367
0.2	1.5820	1.7991	2.1852	2.5551	2.7360
0.3	1.4920	1.7388	2.2359	2.7842	3.0718
0.4	1.4610	1.7146	2.3128	3.0681	3.5109
0.5	1.4479	1.7145	2.3991	3.3824	4.0284
0.6	1.4506	1.7494	2.5042	3.7183	4.6049
0.7	1.5134	1.8423	2.6628	4.0808	5.2251
0.8	1.7335	2.0202	2.9340	4.4858	5.8752

Table 6 Normalized stress intensity factor for internal face crack in rotating hollow cylinders $\nu = 0.3$

	$R_i/(R_o - R_i)$				
$a/(R_o - R_i)$	1/3	1/2	1	2	3
0.01	2.2094	2.2403	2.3174	2.4024	2.4525
0.02	2.1501	2.1936	2.2939	2.3911	2.4468
0.03	2.0947	2.1507	2.2727	2.3822	2.4434
0.04	2.0430	2.1112	2.2535	2.3755	2.4420
0.05	1.9947	2.0750	2.2364	2.3709	2.4428
0.06	1.9496	2.0417	2.2212	2.3684	2.4456
0.07	1.9076	2.0111	2.2077	2.3678	2.4503
0.08	1.8685	1.9831	2.1960	2.3690	2.4569
0.09	1.8321	1.9574	2.1858	2.3721	2.4653
0.1	1.7982	1.9339	2.1771	2.3768	2.4756
0.2	1.5725	1.7838	2.1536	2.5002	2.6666
0.3	1.4798	1.7203	2.1989	2.7196	2.9898
0.4	1.4453	1.6920	2.2692	2.9917	3.4124
0.5	1.4280	1.6868	2.3479	3.2919	3.9095
0.6	1.4256	1.7154	2.4440	3.6116	4.4623
0.7	1.4812	1.7996	2.5909	3.9555	5.0552
0.8	1.6885	1.9649	2.8455	4.3384	5.6748

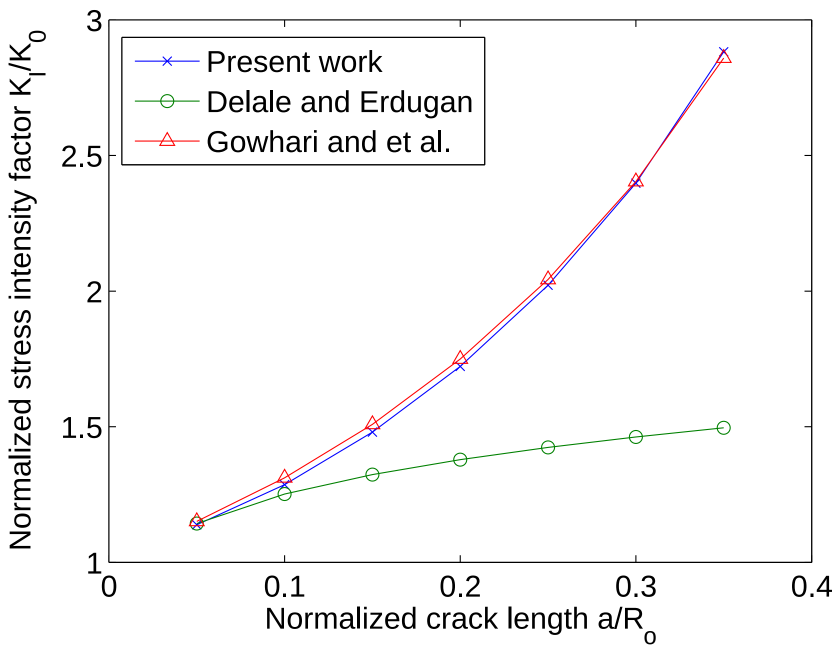

Figure 8 Stress intensity factor of an edge crack in rotating solid disk under plane stress condition with $R_i/R_o = 0.5$ and $\nu = 0.3$

Table 7 Normalized stress intensity factor for external face crack in rotating hollow disks $\nu = 0.3$

	$R_i/(R_o - R_i)$				
$a/(R_o - R_i)$	1/3	1/2	1	2	3
0.01	0.6263	0.7347	1.0469	1.4837	1.7497
0.02	0.6349	0.7426	1.0549	1.4915	1.7580
0.03	0.6437	0.7508	1.0635	1.5002	1.7673
0.04	0.6527	0.7595	1.0726	1.5099	1.7777
0.05	0.6618	0.7684	1.0822	1.5204	1.7891
0.06	0.6710	0.7777	1.0924	1.5319	1.8017
0.07	0.6804	0.7873	1.1031	1.5443	1.8153
0.08	0.6900	0.7972	1.1143	1.5577	1.8300
0.09	0.6997	0.8074	1.1260	1.5719	1.8458
0.1	0.7095	0.8178	1.1382	1.5870	1.8627
0.2	0.8173	0.9375	1.2864	1.7869	2.0954
0.3	0.9445	1.0816	1.4804	2.0752	2.4508
0.4	1.0964	1.2516	1.7223	2.4553	2.9411
0.5	1.2809	1.4578	2.0218	2.9361	3.5805
0.6	1.5096	1.7213	2.3981	3.5333	4.3855
0.7	1.7992	2.0782	2.8821	4.2698	5.3750
0.8	2.1745	2.5835	3.5189	5.1771	6.5706

Table 8 Normalized stress intensity factor for external face crack in rotating hollow cylinders $\nu = 0.3$

	$R_i/(R_o - R_i)$				
$a/(R_o - R_i)$	1/3	1/2	1	2	3
0.01	0.5243	0.6328	0.9449	1.3816	1.6475
0.02	0.5332	0.6410	0.9531	1.3895	1.6557
0.03	0.5422	0.6495	0.9618	1.3982	1.6649
0.04	0.5514	0.6583	0.9710	1.4078	1.6752
0.05	0.5607	0.6675	0.9807	1.4183	1.6864
0.06	0.5701	0.6769	0.9909	1.4296	1.6987
0.07	0.5797	0.6866	1.0015	1.4418	1.7120
0.08	0.5894	0.6966	1.0126	1.4548	1.7263
0.09	0.5993	0.7069	1.0242	1.4687	1.7417
0.1	0.6093	0.7174	1.0363	1.4834	1.7581
0.2	0.7181	0.8366	1.1816	1.6771	1.9827
0.3	0.8456	0.9793	1.3705	1.9551	2.3246
0.4	0.9969	1.1474	1.6059	2.3214	2.7963
0.5	1.1800	1.3509	1.8974	2.7855	3.4119
0.6	1.4064	1.6102	2.2640	3.3628	4.1880
0.7	1.6924	1.9602	2.7358	4.0761	5.1435
0.8	2.0627	2.4550	3.3569	4.9564	6.3003

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Nomenclature

a:	Half of the crack for central crack and crack length for edge crack
a_0:	Normalized crack length
E:	Modulus of elasticity
K_I^r:	Stress intensity factor of reference case
K_I:	Stress intensity factor of mode I
R_i:	Inside radius of disk
R_o:	Outside radius of disk
u_y^r:	Displacement field of reference case
W:	Ratio of inside to outside radius

Greek symbols

ν:	Poisson ratio
ω:	Angular velocity
ρ:	Density
σ_θ:	Hoop stress
σ_{ave}:	Average stress over crack length

چکیده

این مقاله از روش تنش متوسط جهت تعیین ضریب شدت تنش یک دیسک/سیلندر چرخان توپر/ توخالی که شامل یک ترک شعاعی است، بهره می‌برد. ترکها در جهت شعاعی و در لبه خارجی یا لبه داخلی دیسک توخالی و یا در لبه خارجی و مرکز دیسک فرض شده‌اند. نتایج برای هر دو حالت تنش و کرنش صفحه‌ای نشان داده شده‌اند و صحت آنها بوسیله مقایسه با نتایج معرفی شده در سوابق علمی مقاله بررسی شده است.