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Numerical Simulation of Beta Type Stirling Engine Considering Heat and Power Losses

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In this paper, numerical solution of beta-type Stirling engine was presented considering its non-ideal regenerator. To this end, the second-order model including heat and power losses was used. Then, a numerical code was applied for calculating geometrical and physical optimum values of the engine. To confirm the obtained results, the physical and geometrical parameters of the GPU-3 engine were used. According to the obtained results, the values of heat and power losses in the engine were considerable. Based on the results, heat and power losses in the engine led to decreased power and efficiency by 50.1% and 22.7%, respectively. According to the results from the numerical code, the amounts of porosity, frequency, and length of the regenerator were suggested as less than 0.6, 40 to 50 Hz, and 18 to 22 mm, respectively. The results showed a material with high thermal capacity and low conductivity in the optimum physical and geometrical conditions of the engine.

Keywords: Stirling engine, Numerical simulation, Adiabatic model, Non-ideal regenerator, Losses

1 Introduction

Increase of energy cost and environmental pollutions has led to more serious studies on the new power generation engines. Use of non-renewable energy sources of the earth including petroleum, gas, and coal destroys the societies' public wealth and produces one-fourth of the total carbon dioxide of the world. Stirling engine is one of the ideas which have attracted the attention of many interested researchers in recent years.

In physical terms, Stirling engine is an external combustion engine and can use any kinds of external heat sources (industrial waste heat, combustion, and solar energy) for producing mechanical energy.

Many studies have been done on Stirling engines since their invention by Robert Stirling. The first mathematical analysis which was acceptable for analyzing Stirling cycle was presented by Schmidt 50 years after its invention [1]. Schmidt's analysis was done based on the theories of isotherm compression and expansion space.

Using the Schmidt's assumption, the thermodynamic equations were linear and the initial calculations were easily done for measuring the engine's power and efficiency. Today, Schmidt's analysis is widely used for the initial analysis of Stirling engines. Schmidt's cycle

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assumes expansion and compression processes as isotherm processes. However, in practice, this assumption is not true for the engines with 1000 rpm or more engine cycles, since as proved by Rankin, heating or cooling does not occur exactly at constant temperature or constant volume and expansion and compression processes of the cylinders in Stirling engine are closer to the adiabatic values.

Therefore, more appropriate assumptions should be used for thermodynamic modeling in order to get closer to the actual efficiency of the engine by these models. If attempts for modeling Stirling engines are not within the isotherm assumption, the equations will not be clear and can only be solved differentially and using numerical methods. In the adiabatic model, efficiency is turned to a function which depends not only on temperature, but also on swept volume, phase angle, and dead volume. In fact, the power is a function of all the mentioned parameters either in the isotherm or adiabatic cycles.

Finkelstein [2] improved Schmidt's thermodynamic analysis and presented the initial adiabatic analysis. While solving equations in the adiabatic way, compression and expansion spaces are considered adiabatic. Considering the adiabatic assumption, the equations become non-linear and numerical methods should be used for their solving. Since the Finkelstein model was presented, the conducted analysis has been done based on different thermodynamic models (isotherm and adiabatic), use of different heat sources (waste heat, solar, and combustion), and different forms of Stirling engine (gamma-, beta-, and alpha-type engines); the studies done by Urieli and Berchowitz [3] which used the adiabatic thermodynamic model for obtaining the Stirling engine's power and efficiency could be also referred to. Kongtragool and Wongwises [4] conducted the Stirling engine's modeling and optimization by the isotherm model and Tlili et al. [5] as well as Timoumi et al. [6-8] investigated its losses and irreversibility using the Stirling engine's adiabatic modeling. In recent studies done by Tlili et al. [9], Stirling engine was modeled using the solar energy as the heat source. Thombare and Verma [10] gathered the available technologies and obtained achievements with regard to the analysis of Stirling engines and, at the end, presented some suggestions for their applications. Furthermore, Tavakkolpour et al. [11] analyzed the gamma-type Stirling engine by using the Schmidt's theory, solving the equations in the isotherm way, and using flat plate for absorbing solar energy as the hot temperature source. After modeling the Stirling engine, Gostante and Invernizzi [12] investigated the effect of using different gasses on the engines' power and efficiency. Formosa and Despesse [13] conducted the modeling by means of the isotherm model in order to investigate the effects of dead volumes on the engine's power and efficiency. Ziabasharhaqh and Mahmoodi [14] modeled a beta Stirling engine by a non-ideal regenerator and studied the effects of various geometric parameters on it. Hashem et al. [15] conducted numerical modeling by applying the mechanical losses of a gamma engine and the results were compared with the experimental results. In their research, thermal and mechanical losses were included to approximate the experimental results. In the research conducted by Mobarak et al. [16], an unsteady analytical model was developed to calculate the gas leakage mass flow rate by considering an oscillating flow in the annular clearance and to evaluate the power lost in both locations. Modeling by Araoz et al. [17] was based on a second-order Stirling engine model that was previously developed and validated.

Actual engine prototypes present low electrical power outputs and high energy losses. The simulation allowed to evaluate the effect that different design and operational parameters have on the engine performance and, consequently, different performance curves were obtained.

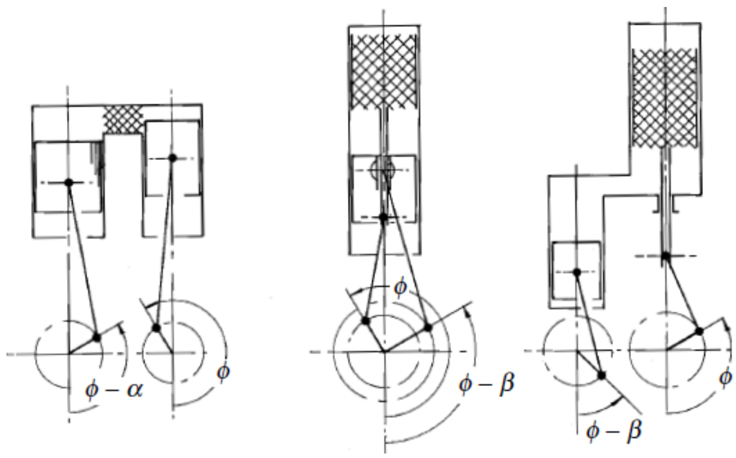
In the present study, the thermodynamic relations of all the engine components were applied. To increase the accuracy of simulation, actual relations were used for the changes of volume of the pistons inside the cylinders and no simplification was used in this regard. At the end, the obtained results were compared with the results presented for the GPU-3, the geometrical and physical specifications of which were presented.

In this study, the simulation of mechanical losses was more accurate than that of previous research and the results closer to the actual results were obtained seen. According to the obtained results, the values of power and heat losses in the engine were considerable and led to the decrease in the engine power and efficiency by 50.1% and 22.7%, respectively. The results proved to be better than those obtained by other analyses and correlated more closely with the experimental data.

2 Thermodynamic analysis

The Stirling engine works in a closed thermodynamic cycle and converts thermal energy into mechanical motion. Considering the physical structure of this engine, it has five main subsystems; each subsystem is considered a control volume in modeling. There are two spaces with variable volumes which are called expansion space and compression space and there are three heat exchanger with the constant volume which are called heater, cooler, and regenerator. Different types of the Stirling engine are identified by the names alpha, beta, and gamma. All of them are similar in terms of thermodynamic cycle; however, there are fundamental differences as far as mechanical mechanisms are concerned. Alpha-type Stirling engine has two pistons in two separate cylinders. A heater is in one cylinder and a cooler is in another. The working gas starts its going and coming movements from the heater and enters the cooler by the regenerator. In the beta-type Stirling engine, there are two pistons inside one cylinder, which are called displacer piston and power piston. The displacer piston moves the working fluid between the cold space and hot space through the heater, cooler, and regenerator and leads to the movement of the power piston. The gamma-type Stirling engine is a combination of alpha and beta types (Figure 1).

In theoretical terms, the efficiency of the Stirling engine is equal to the efficiency of the Carnot cycle. The heat transferred from the working gas to the regenerator during the 4 to 1 process is absorbed by the working gas during the 2 to 3 process. A source for heater and another source for cooler are required to be used during the 1 to 2 and 3 to 4 processes (Figure (2)).



Alpha **Beta** **Gama**

Figure 1 Different configurations of Stirling engines [18]

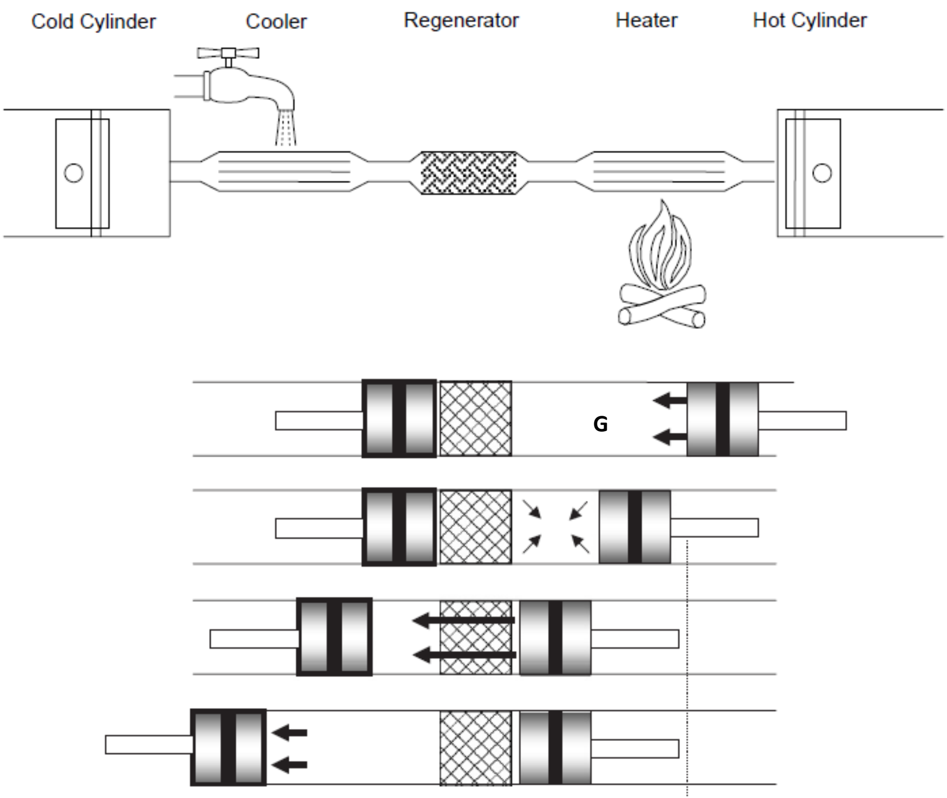


Figure 2 The components of Stirling engine and the operation of their cycle

Regenerators are probably the most impotent components of the Stirling engine which have a fundamental role in increasing its efficiency. As far as the physical structure is concerned, regenerators are made of stainless steel as mesh sheets or stainless steel rods which are stacked. During the half of the engine's work cycle, regenerators act like a sponge and lead to heat absorption from the working cycle. In another half of the cycle, regenerators return heat to the working gas; therefore, there is less amount of heat in the cold area of the engine for dissipation, which leads to the increase of engine efficiency. Thus, the use of regenerators in the Stirling engine decreases the waste heat and, finally, increases the engine efficiency (Figure (3)).

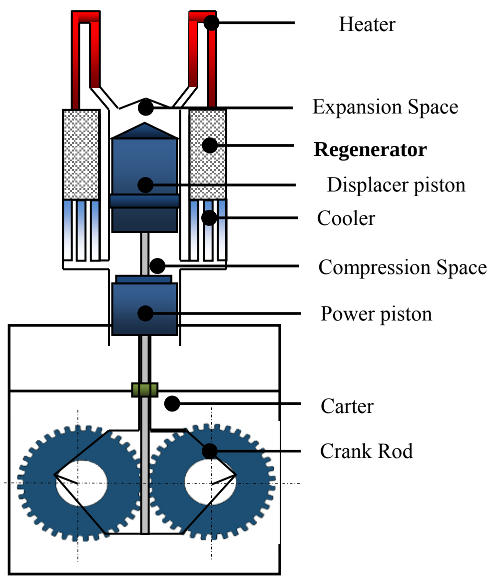


Figure 3 Beta-type Stirling engine Regenerator

The modeling conducted in this paper was for the beta-type engine using the five-volume method. In this method, the components of the Stirling engine were divided into five separate sections and thermodynamic equations were extracted for each unit. Then, appropriate boundary conditions were considered for the movement of the fluid flow inside the components of the engine at the interfaces between the cells. Finally, the numerically obtained equations were solved (Figure (4)).

The engine had a driving mechanism for controlling volume changes during the work cycle and transferring the alternating linear motion of pistons as the angular momentum to the driving shaft (Figure (5)). To test the capability of the performed modeling, reliable input characteristics were required. To this end, precise characteristics of the engine made in General Motors Research Laboratories (GPU-3) were used. In addition to its precise physical and geometrical characteristics, its performance characteristics are also available and have been used by many researchers for validating the implemented modeling.

In Tables (1, 2), physical and geometrical parameters of the engine including precise dimensional characteristics, swept volume by the power piston and displacer piston, clearance values of power piston and displacer piston, number of pipes applied in the heat exchangers, working gas, temperature of hot and cold parts, average pressure of working fluid, working fluid mass, and frequency of the engine are presented. In Table (3), different porosity values with different diameters of wires are given for investigating the best efficiency of the regenerator.

Other input parameters required for the analysis were calculated based on the form and conditions of the engine using the equations available during the numerical solution.

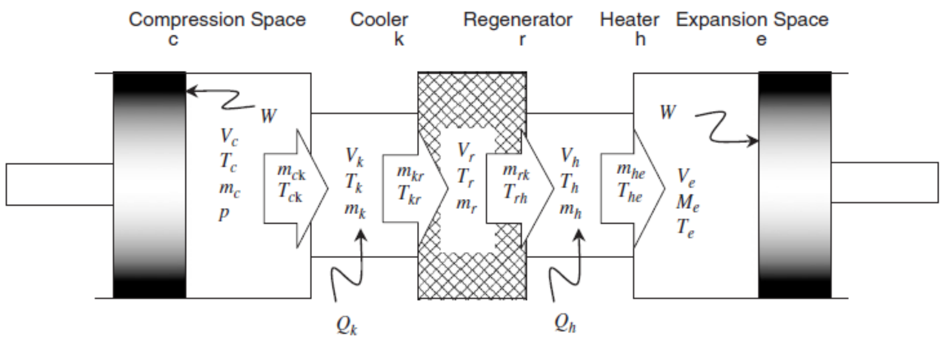


Figure 4 The ideal adiabatic model

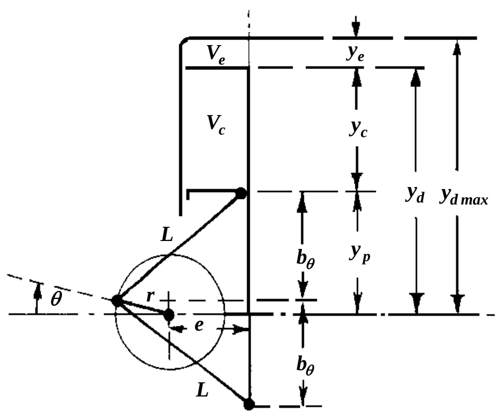


Figure 5 Beta-type Stirling engine volume variations

Table 1 The operational parameters of the GPU-3 engine

Gas	helium
Hot space temperature(Th)	977 K
Cold space temperature(Tk)	288 K
Mean pressure	4130 Kpa
Total mass of gas	1.135 grm
Frequency	41.7 Hz

Table 2 The geometrical parameters of the GPU-3 engine

<table>
<tr><td rowspan="2">Clearance volumes</td><td>Compression space</td><td>28.68 cm3</td></tr>
<tr><td>Expansion space</td><td>30.52 cm3</td></tr>
<tr><td rowspan="2">Swept volumes</td><td>Compression space</td><td>113.14 cm3</td></tr>
<tr><td>Expansion space</td><td>120.82cm3</td></tr>
<tr><td colspan="2">Connecting rod length</td><td>46 mm</td></tr>
<tr><td colspan="2">Eccentricity</td><td>20.8 mm</td></tr>
<tr><td colspan="2">Power piston diameter</td><td>69.9 mm</td></tr>
<tr><td colspan="2">Displacer diameter</td><td>69 mm</td></tr>
<tr><td colspan="2">Displacer length</td><td>37mm</td></tr>
<tr><td colspan="2">Displacer stroke</td><td>31.2 mm</td></tr>
<tr><td colspan="2">Exchanger piston conductivity</td><td>15 W/m.K</td></tr>
<tr><td colspan="2">Exchanger piston stroke</td><td>46 mm</td></tr>
<tr><td rowspan="4">Heater</td><td>Tubes number</td><td>40</td></tr>
<tr><td>Tube inside diameter</td><td>3.02 mm</td></tr>
<tr><td>Tube length</td><td>245.3 mm</td></tr>
<tr><td>Void volume</td><td>70.88 cm3</td></tr>
<tr><td rowspan="5">Cooler</td><td>Tubes number</td><td>312</td></tr>
<tr><td>Interns tube</td><td>1.08mm</td></tr>
<tr><td>Diameter</td><td>46.1 mm</td></tr>
<tr><td>Tube length</td><td>46.1mm</td></tr>
<tr><td>Void volume</td><td>13.8 cm3</td></tr>
<tr><td colspan="3">Regenerator</td></tr>
<tr><td colspan="3">The regenerator body is pipe-like in which metal wires are accumulated on each other</td></tr>
<tr><td colspan="2">Diameter</td><td>22.6 mm</td></tr>
<tr><td colspan="2">Length</td><td>22.6 mm</td></tr>
<tr><td colspan="2">Wire diameter</td><td>40 µm</td></tr>
<tr><td colspan="2">Porosity</td><td>0.697</td></tr>
<tr><td colspan="2">Units numbers/cylinder</td><td>8</td></tr>
<tr><td colspan="2">Thermal conductivity</td><td>15 W/m.K</td></tr>
<tr><td colspan="2">Void volume</td><td>50.55 cm3</td></tr>
</table>

Table 3 Specifications of the regenerator with different wire diameter and porosity

	Porosity	Wire diameter (dm) mm
M1	0.9122	0.0035
M2	0.8359	0.0065
M3	0.7508	0.007
M4	0.7221	0.007
M5	**0.697**	**0.004(GPU-3)**
M6	0.6655	0.008
M7	0.6112	0.008

3 Governing equations

The thermodynamic modeling of the engine is done in three sections. In the first section, the modeling is performed in the isotherm method using the Schmidt's model. The results obtained from this section are used as the initial values for the second section. In the second section, the modeling is done in the ideal adiabatic method and, in the third section, the modeling is done in the non-ideal analysis using thermal and hydraulic losses.

3.1 Isothermal analysis

The aim of isotherm analysis is to obtain a network as a result of the pressure changes and temperature of the working gas using the heat transfer to the inside of the engine. The main attraction of the isotherm analysis is the closed solution method which appears in its equations. The fundamental assumption in this analysis is that the gas in the expansion space and heater and the gas in the compression space and cooler are kept at the heater's and cooler's temperature, respectively. The thermodynamic cycle of isotherm is composed of two isotherm processes and two constant volume processes. In addition, it is assumed that the expansion and compression processes inside the engine are isotherm and the effects of non-ideal regenerator and pressure drop are not considered. The starting point of the analysis is to hold the total mass constant at all the volumes occupied by the gas [10].

3.2 Ideal adiabatic analysis

Adiabatic model is based on the assumption that the heater and cooler are capable of infinite heat exchange and isothermal conditions are established in them. Therefore, the fluid in the heat exchangers is always at the maximum and minimum temperatures of T_{max} and T_{min}, respectively. The temperature of working fluid in the cylinders can be less or more than T_{max} in the expansion space or T_{min} in the compression space during the cycle. To solve this adiabatic state, first, mass is considered constant in the whole system; then, energy equations and perfect gas equation are used to obtain the equations required for measuring the rate of heat transfer to the engine, work done, and, finally, engine efficiency. Considering the system of equations defined for the model, it has been determined that there are 22 variables and 16 differential equations for solving the engine cycle [3].

3.3 Non –ideal analysis

In this section, the regenerator is considered non-ideal; therefore, the equations related to heat and power losses are entered into the numerical code.

These losses include energy loss caused by pressure drop in heat exchanger, energy loss due to internal conduction between the hot parts and the cold ones in regenerator, energy losses due to external conduction in regenerator, shuttle effect, and residual adiabatic, as well as frictional losses.

3.3.1 Heat losses

In this section, heat losses which must be added to the basic heat input are defined. These are heat losses due to internal conduction, heat losses due to external conduction in regenerator, and energy lost by shuttle effect.

3.3.1.1 Heat losses due to internal conduction between the hot parts and the cold ones in regenerator

In physical terms, regenerators are located between heaters and coolers. Temperature difference of these two heat exchangers leads to the unwanted loss of a considerable amount of heat, the amount of which depends on the thermal conductivity of regenerators, effective area for conduction, and length of regenerators. To obtain the amount of energy loss caused by internal conduction, the following relation can be used [7]:

$$Q_{wrloss} = k \times \frac{A}{L}(T_{wh} - T_{wk}) \tag{1}$$

where A is the effective area for conductivity (m^2), k is regenerator wall conductivity (W/(m.K)), and L is the length of regenerator (m).

3.3.1.2 Heat losses due to external conduction in regenerator

The pressure drop caused by external conduction is created by the non-ideal regenerator of the working gas during its transition through the engine regenerator. ε, as the regenerator effectiveness, explains the rate of heat transferred in the regenerator by the working gas during its transition toward the compression space and received heat in the regenerator by the working gas during its transition toward the expansion space [7]. Therefore, the stored energy by the regenerator at the time of working gas transfer from expansion space to compression space does not completely return to the working gas while being returned.

In ideal cases, regenerator effectiveness is equal to 1 (ε=1). Energy loss by external conduction is equal to [7]:

$$Q_{rloss} = (1-\varepsilon) \times (Q_{r\,max} - Q_{r\,min}) \tag{2}$$

Regenerator effectiveness is obtained by the following equation:

$$\varepsilon = \frac{NTU}{(NTU+1)} \tag{3}$$

where NTU (number of transfer units) is defined in the following way:

$$NTU = \frac{St \times A_{wg}}{2A} \tag{4}$$

where Awg is wetted area of the regenerator surface and St is Stanton number and considered in the following way according to the results by Kays and London [19]:

$$St = \frac{0.46 \times Re^{-0.4}}{Pr} \tag{5}$$

where Pr is Prandtl number and, according to the results by Kays and London [19], it is equal to 0.7.

3.3.1.3. Heat lost by shuttle effect

Moving the displacer between hot and cold parts causes another loss for transferring heat from a hot to a cold space. The displacer absorbs heat from the hot source and restores it to the cold source. Energy loss by shuttle effect can be calculated by the following relation [20].

$$Q_{shuttle} = \frac{0.4Z^2 \times K_g \times D_d}{J \times L_d}(T_e - T_c) \tag{6}$$

where J is the annular gap between the displacer and the cylinder (m), Kg is gas thermal conductivity (W/m.K), Dd is displacer diameter (m), Ld is displacer length (m), Z is displacer stroke (m), and Te and Tc are, respectively, the temperature in the expansion space and in the compression space (K).

3.3.2 Power losses

This section identifies a number of power losses and presents the published equations describing them. Power losses fall under two headings: flow friction and mechanical friction. The adiabatic residual losses are so important.

3.3.2.1 Power dissipation by pressure drop in heat exchanger

In the Stirling engines, there is a relationship between the fluid friction and gas flow movement via heater, cooler, and regenerator. Regenerator porosity causes pressure drop of the working fluid. Decrease in the power of the engine is the result of the pressure drop through heat exchanger. In addition, the movement of working fluid through the heat exchanger leads to pressure drop through heat exchanger and, finally, decreases the work done and the heat required for the engine; where Dp, the frictional drag force in the heat exchanger, can be calculated by the following relation:

$$DP_{r,h,k} = \frac{2 \times f_{r,h,k} \times \mu \times V_{r,h,k} \times G \times l_{r,h,k}}{m_{r,h,k} d_{r,h,k}^{\,2}} \tag{7}$$

where G is the working gas mass flow (kg/m^2.s) in heat exchanger, V is heat exchanger volume (m3), l is heat exchanger length (m), μ is dynamic viscosity of gas (kg/m.s), m is mass of working gas inside heat exchanger (kg), d is the hydraulic diameter of the heat exchanger (m), and f is Reynolds friction factor inside heat exchanger, which is calculated for the regenerator based on the values estimated by Kays and London [19] and for heater and cooler based on Organ relation [18] as shown below:

$$f_r = 54 + 1.43 \times Re^{0.78} \tag{8}$$

$$f_{h,k} = 0.0791 \times Re^{0.75} \tag{9}$$

By inserting pressure drop in the following relation, the amount of heat dissipation and power loss in the heat exchanger can be obtained:

$$Q_{r,h,kdiss} = \frac{Dp \times m_{r,h,k}}{\rho} \tag{10}$$

$$dW_{work} = Dp \times dV \tag{11}$$

3.3.2.2 Mechanical losses

Mechanical friction due to the seals and the bearings is hard to compute reliably and must be essentially measured. However, if the engine itself were used, the losses due to mechanical friction would be combined with power required or delivered by the engine. If indicated and brake power were determined, then mechanical friction losses would be the difference.

The friction losses should be measured directly by having the engine operate at the design average pressure with very large dead volume so that very little engine action is possible. The engine needs not to be heated, but the seals and bearing need to be at design temperature. The amount of wasted work caused by the mechanical losses defined by Martini [21] in Figure (6). Edward [22] derived correlation for mechanical friction loss for the piston in terms of the friction mean effective pressure (f_{mep}). Since it approximates a linear relationship between f_{mep} and engine speed, then the following correlations are used.

$$f_{mep} = 6.8948 \times 10^{-2} \times (0.002N + 1) \tag{12}$$

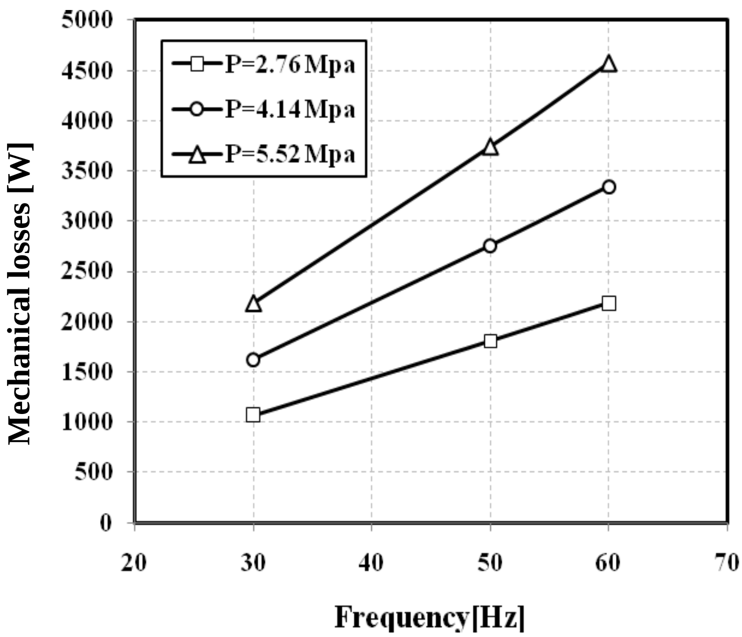


Figure 6 Adiabatic residual power losses versus frequency (Martini, 1983)

where N (RPM) and fmep (bar). Then, the power loss due to this friction effect, based on the mean effective pressure definitions, is:

$$W_{Mechanicalloss} = 2 \times 14.6706 \times (0.002N + 1) \times (V_H + V_C) \quad (13)$$

Finally, the actual work of Stirling engine is obtained by calculating the amount of wasted work caused by the pressure drop of the working gas:

$$actW_{work} = W_{work} - dW_{work} - W_{Mechanical\ Loss} \quad (14)$$

To obtain the amount of heat required by obtaining the amounts of thermal loss in the regenerator and its produced heat caused by pressure drop, the actual heat required for the engine and, finally, thermal efficiency of the Stirling engine can be calculated using Equations (14, 15):

$$actQ_{Hpower} = Q_{Hpower} + Q_{r\,loss} + Q_{wrloss} + Q_{shuttle} - Q_{rdiss} - Q_{hdiss} - Q_{kdiss} \quad (15)$$

$$\eta = \frac{actW_{work}}{actQ_{H_{Power}}} \quad (16)$$

4 Solution methods

For the numerical solution of the equations obtained from the adiabatic model, the equations of pressure and mass changes should be simultaneously solved in the compression space along with the energy equations. The best method for the numerical solution is to use the initial values method. In this method, the initial values of all the variables are clear at the zero point at the beginning and the final values of the equations are solved using these initial values so that their obtained functions include all the available variables along with the functions related to the volume changes of the engine at different angles of the crank. To solve it, y vector is defined as a function of available variables. For example, yp shows the gas pressure of the system, y_{mc} indicates the mass of working fluid in the compression space, and so on. If the initial values of the variables are available, y vector is defined as $y(t_0)=y0$. Accordingly, the value of y(t) is determined from the differential equations of Dy=F(t,y) and the determined values satisfy differential and initial equations simultaneously. In fact, in this numerical method, first, the initial values are calculated at time t0 and, afterwards, new values are calculated at time $t_1=t_0+\Delta t$ by a small increment of time. Therefore, there is a wide set of direct circles at different times which obtain the correct values of y(t). In the adiabatic solution method, initial values of mass changes and pressure in the compression space should enter the equations along with other information of the problem such as volume changes. These values are obtained from the first section, i.e. solving the equations in the isotherm model. By inserting the operational and geometrical specifications of the GPU-3 engine and the initial values obtained from isotherm solution, the heat values are obtained for the heater, cooler, work, and engine efficiency. The numerical code is able to enter the considered changes in different parts in terms of physical and geometrical aspects and show the value changes of the power and engine efficiency by applying the new conditions.

5 Numerical results and discussions

To validate the obtained results, the geometrical and physical parameters of the engine manufactured by General Motors Research Laboratories (GPU-3) were used Table (1).
The comparison of the obtained results with the published ones [3] in ideal adiabatic analysis indicated the accuracy of the performed modeling (Figure (7) and Table (4)). By solving Equations 1 to 11 and applying 360 degree changes to the engine crank angle according to Figure (5), Qk, Qh, Qr, and W are obtained for each crank angle (0 to 360º) based on Equation (10). Qh is the amount of heat transferred to the heater, Qk is the amount of heat transferred by the cooler, Qr is the amount of heat regenerated by regenerator, and W is the net power from the engine during the cycle. Effects of non-deal regenerator on the efficiency and output power of the engine are shown in Table (5).
Using non-ideal regenerator, the mean effective temperature becomes differences for the heater and cooler first resulting this changing is reduction in power output from 8008 W to 7122 W.

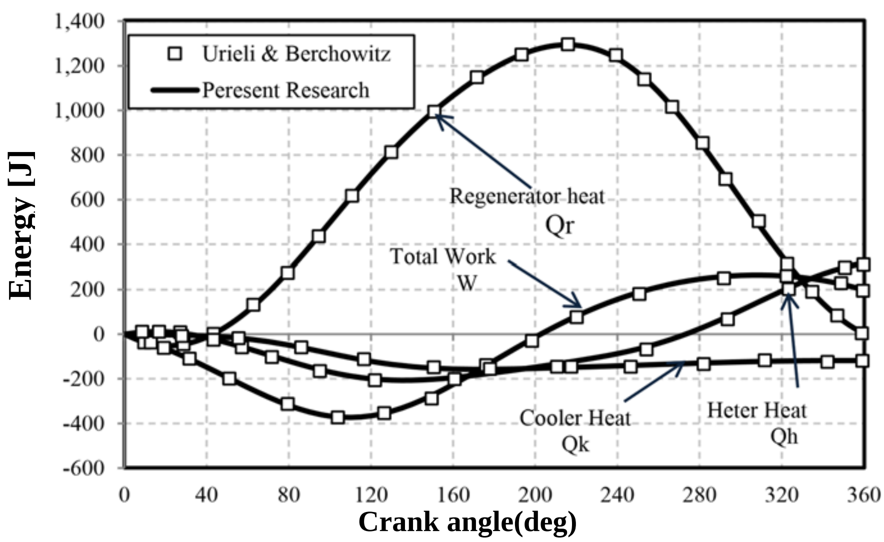


Figure7 Validation of the computational method [3]

Table 4 Result of the ideal adiabatic model (Ideal regenerator ε=1)

	P[W]	η
Urieli & Berchowitz, (1984)	8300	62.5%
Timoumi et al., (2008)	8462	62.06%
Present work	8008	62.6%
Experiment result (Martini,1983)	3960	35%

Table 5 Result of the ideal adiabatic model (Non-ideal regenerator ε=0.986)

	P[W]	η
Urieli & Berchowitz, (1984)	7414	59.5%
Present work	7122	57.6%

Table 6 Result of the non-ideal analysis model

Energy dissipation in regenerator by pressure drop	Q_{rdiss}	[W]	114.6
Energy dissipation in heater by Pressure drop	Q_{hdiss}	[W]	17.3
Energy dissipation in cooler by pressure drop	Q_{kdiss}	[W]	2.9
Energy losses due to internal conduction	Q_{wrloss}	[W]	544.1
Energy losses due to external conduction	Q_{rloss}	[W]	699.2
Energy lost by Shuttle effect	$Q_{shuttle}$	[W]	32
Wasted work caused by the pressure drop	dW_{work}	[W]	769.5
Wasted work caused by adiabatic residual	dW_{work}	[W]	2100
Actual work	$actW_{work}$	[W]	4158
Actual heat required for the engine	$actQ_h$	[W]	10421
efficiency	η	[%]	39.9

Now, considering the obtained results from ideal adiabatic analysis and by applying thermal and hydraulic losses, the operational values of the engine can be obtained (Table (6)). The obtained results showed the amount of internal and external thermal losses, dissipation heat caused by pressure drop, heat transferred to the engine, as well as power and efficiency of the engine. The comparison of the results obtained from Tables (5) and (6) indicated the decrease of engine power as a result of pressure drop and adiabatic residual loss by 50.1% and decrease of engine efficiency by 22.7%. Considering the obtained results, the amount of hydraulic and thermal lossess in the Stirling engine was very high and the optimum geometrical design and determination of optimum physical characteristics of the engine could have a significant role in obtaining the highest output power and efficiency in the engine.

The thermal and hydraulic losses are mainly located in the regenerator. The reduction of these losses improves the engine performance. To investigate the influence of these losses on the engine performance, we changed one important parameter in the model each time and kept others unchanged.

5.1 Effect of regenerator porosity

Regenerator porosity has an important role in the amount of hydraulic and thermal losses, efficiency, and output power of the engine Stirling engine. Hydraulic diameter of the regenerator, gas speed, regenerator heat transfer, and regenerator effectiveness all depends on the regenerator porosity. Thermal losses by internal and external conduction and energy dissipation by pressure drop through heat exchanger for different values of regenerator porosity (according to Table (3)) are given in Table (7) and Figure (8). The working fluid moves from the expansion space and first enters the heater and then goes through the regenerator. The motion of working fluid from the inside of the porous environment of the regenerator and conflict with the regenerator matrix transfer the heat from the working gas to the regenerator matrix and generate pressure drop caused by the motion of the working gas inside it. Then, the working fluid enters the cooler and compression spaces. After this stage, the working fluid moves toward the expansion space again and causes pressure drop inside the regenerator due to absorbing previously left heat in the regenerator matrix. The results indicate the increase of thermal loss and decrease of pressure drop in the regenerator by increasing regenerator porosity.

Table7 Effect of regenerator porosity on engine performance

	Regenerator porosity[mm]				
	0.912	0.8359	0.722	**0.697**	0.612
Q_{rloss} [W]	3760	3451.3	1717	**699.2**	1125
Q_{wrloss} [W]	544.1	544.1	544.1	**544.1**	544.1
Q_{rdiss} [W]	8.3	10.9	34.6	**114.6**	80
Q_{hdiss} [W]	17.6	17.6	17.4	**17.3**	17.4
Q_{kdiss} [W]	3.1	3.1	3	**2.9**	3
$Q_{shuttle}$ [W]	31.8	31.8	31.9	**32**	31.9
dW_{work} [W]	169	185.3	323.3	**769.5**	584.1
$actW_{work}$ [W]	4214	4338.9	4551	**4158**	4532
$actQ_h$ [W]	12946	12839	11427	**10421**	11127
η[%]	32.6	33.8	39.8	**39.9**	40.7

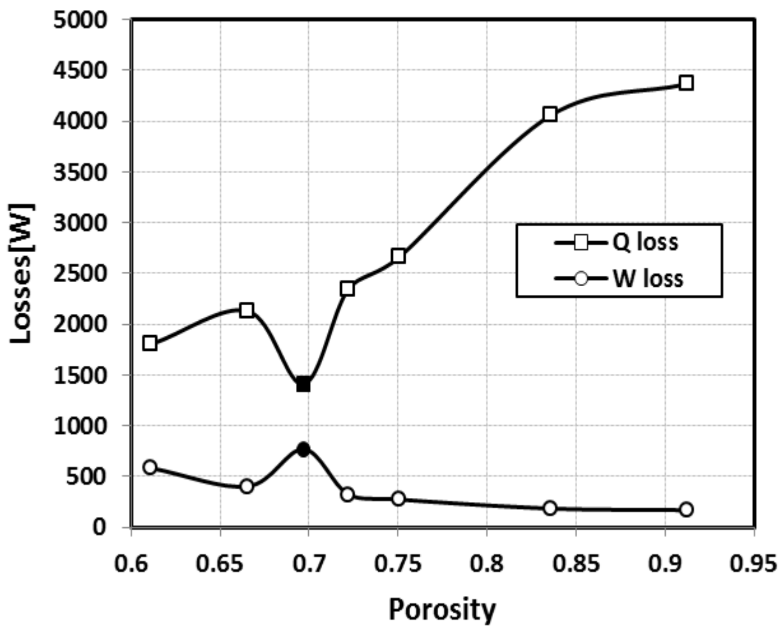


Figure 8 Effect of regenerator porosity on engine losses

As regenerator porosity is increased, the efficiency and output power of engine are decreased (Figure (9)), which is caused by the increase of thermal losses in the regenerator. According to the results, to increase the engine power output, the values 0.6112 for regenerator porosity are proposed.

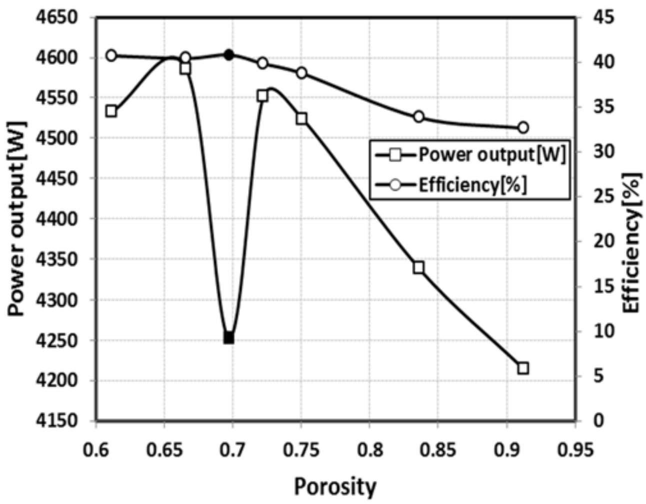


Figure 9 Effect of regenerator porosity on the efficiency and output power of engine

5.2 Effect of engine frequency

Effects of operational frequency of the engine on the thermal and hydraulic losses, efficiency, and output power of the engine are shown in Table (8) and Figures (10,11), which demonstrates increase of thermal losses and pressure drop loss due to increase in operational frequency and increase of engine power output with the increase of functional frequency of the engine. According to the results, within the frequency of 40 to 50 Hz, the engine has the most optimum efficiency and output power.

Table 8 Effect of operational frequency on engine performance

	Frequency[Hz]				
	16.7	33.3	**41.7**	50	58.3
Q_{rloss} [W]	199.1	514.9	**699.2**	895.6	1104
Q_{wrloss} [W]	544.1	544.1	**544.1**	544.1	544.1
Q_{rdiss} [W]	15.1	69.1	**114.6**	173.3	247.3
Q_{hdiss} [W]	1.4	9.3	**17.3**	28.5	43.7
Q_{kdiss} [W]	0.2	1.6	**2.9**	4.9	7.5
$Q_{shuttle}$ [W]	32.2	32	**32**	31.9	31.9
dW_{work} [W]	92.1	453	**769.5**	1188	1725
$actW_{work}$ [W]	736.5	3184	**4158**	5194	6010
$actQ_h$ [W]	4496	8443	**10421**	12374	14323
η[%]	16.4	37.7	**39.9**	42	42

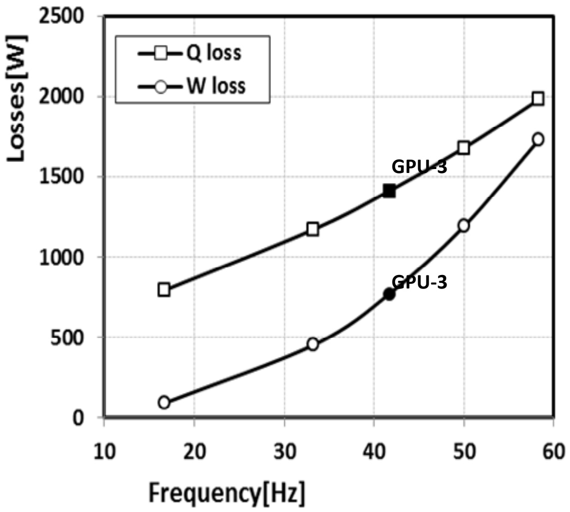


Figure 10 Effect of operational frequency on engine losses

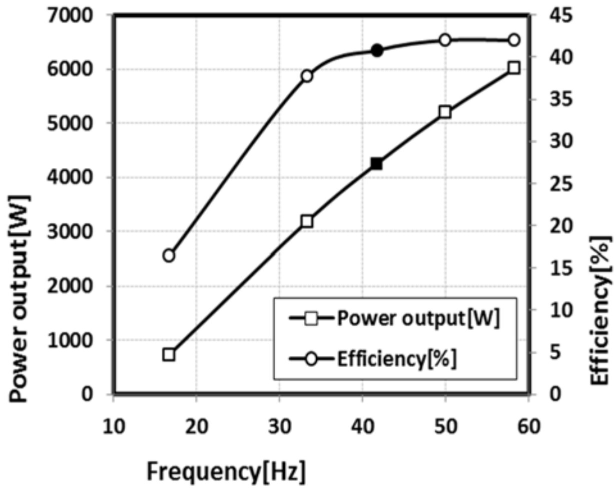


Figure 11 Effect of operational frequency on the efficiency and output power of engine

5.3 effect of regenerator length

Effect of regenerator length on the thermal and hydraulic losses, efficiency, and output power of engine is shown in Table (9) and Figures (12, 13). Considering the obtained results, thermal losses are decreased with the increase of regenerator length, while the level of work loss caused by pressure drop has an incremental trend with the increase of regenerator length. Output power of the engine has a detrimental trend with the increase of regenerator length. The location between 18 and 22 mm in Figure (13) is the optimum regenerator length due to having appropriate efficiency and power for the engine.

Table 9 Effect of o regenerator length on engine performance

	Regenerator length[mm]				
	5	15	**22.6**	35	55
Q_{rloss} [W]	3022	1055	**699.2**	446.3	277.3
Q_{wrloss} [W]	2459	819.8	**544.1**	351.3	223.6
Q_{rdiss} [W]	27.3	77.7	**114.6**	173.3	266
Q_{hdiss} [W]	17.9	17.4	**17.3**	17.1	16.9
Q_{kdiss} [W]	3.4	3	**2.9**	2.8	2.7
$Q_{shuttle}$ [W]	31.7	31.9	**32**	32.1	32.2
dW_{work} [W]	268.3	553.3	**769.5**	1122	1695
$actW_{work}$ [W]	5383	4820.9	**4158**	3344	2000
$actQ_h$ [W]	16582	11810	**10421**	8941	7349
η[%]	32.5	40.8	**39.9**	37.4	27.2

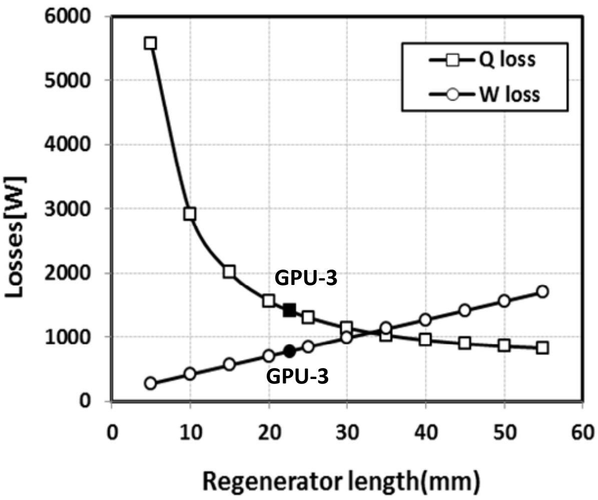


Figure 12 Effect of regenerator length on engine losses

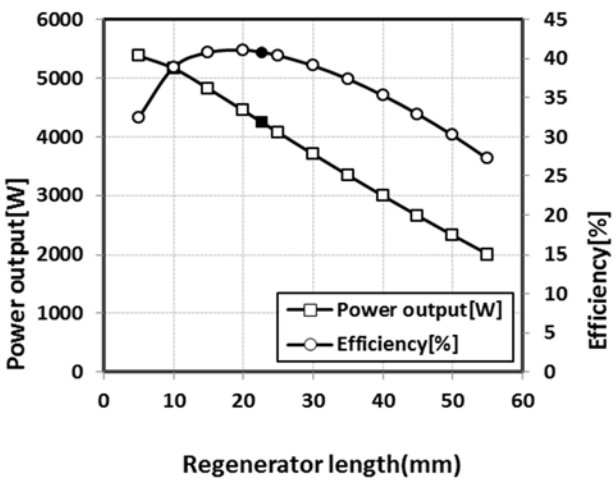


Figure 13 Effect of regenerator length on the efficiency and output power of engine

5.4 Effect of regenerator matrix conductivity

The performance of the Stirling engine depends on the thermal conductivity of materials constituting the regenerator matrix. Effect of regenerator matrix conductivity on the thermal and hydraulic losses, efficiency, and output power of engine is shown in Table (10) and Figures (14, 15). The matrix of the regenerator can be made of several materials. To increase heat exchange of the regenerator and to reduce the internal losses by conductivity, a material with low conductivity must be chosen. Figure (14) shows that, with an increase of the matrix, regenerator thermal conductivity leads to a reduction of the work loss due to an increase of pressure drop internal heat exchanger. Thermal losses by internal and external conduction and energy dissipation by pressure drop through heat exchanger for different regenerator matrix conductivity are given in Figure (14).

Table10 Effect of Regenerator thermal conductivity on engine performance

	Regenerator thermal conductivity[W/Mk]				
	5	**15**	25	35	40
Q_{rloss} [W]	699.2	**699.2**	699.2	699.2	699.2
Q_{wrloss} [W]	181.4	**544.1**	906.9	1269	1451
Q_{rdiss} [W]	114.6	**114.6**	114.6	114.6	114.6
Q_{hdiss} [W]	17.3	**17.3**	17.3	17.3	17.3
Q_{kdiss} [W]	2.9	**2.9**	2.9	2.9	2.9
$Q_{shuttle}$ [W]	32	**32**	32	32	32
dW_{work} [W]	769.5	**769.5**	769.5	769.5	769.5
$actW_{work}$ [W]	4253	**4158**	4253	4253	4253
$actQ_h$ [W]	10058	**10421**	10783	11146	11328
η[%]	42.3	**39.9**	39.4	38.2	37.5

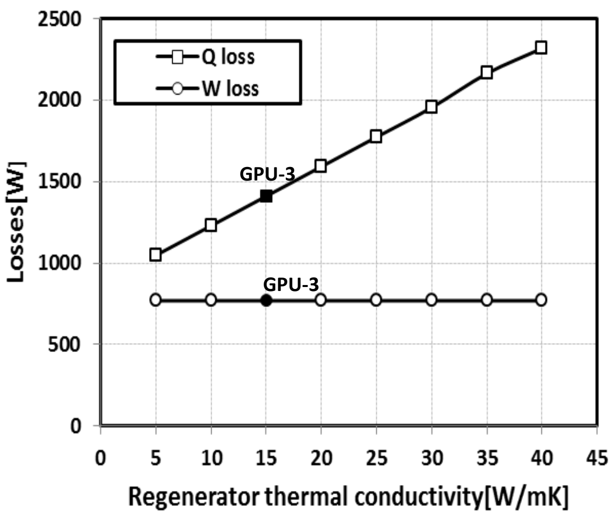


Figure 14 Effect of regenerator matrix conductivity on engine losses

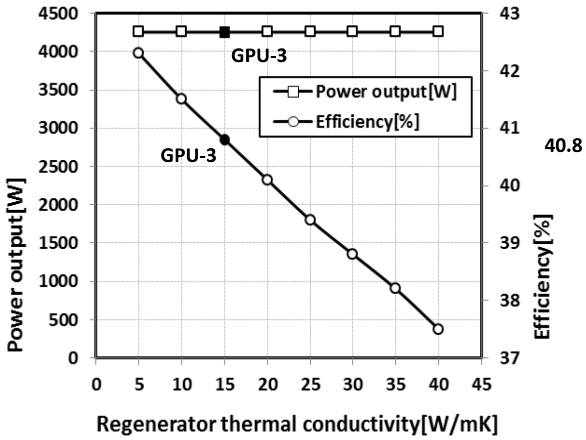


Figure 15 Effect of regenerator matrix conductivity on the efficiency and output power of engine

6 Conclusion

In this study, numerical modeling of beta-type Stirling engine was done considering power and heat losses. First, main parameters were calculated using the ideal adiabatic model and compared with the published values. Then, the effects of power and heat losses were inserted in the numerical code and the performance of the engine was investigated using different geometrical and physical parameters of the regenerator, the results of which are shown below:

- As equations related to power and heat losses were imposed, power and efficiency of the engine were decreased by 50.1% and 22.7%, respectively.

- Decrease in regenerator matrix porosity led to the increase of power and output efficiency of the Stirling engine. Pressure drop in regenerator decreased with the increase of regenerator porosity. In fact, as the porosity of regenerator increased, resistance of regenerator matrix against the movement of operating gas flow decreased; as a result, its pressure drop decreased.

- Efficiency and output power of the engine depended on the intensity of its operational frequency. Power and efficiency increased with the increase of frequency. The most optimum amount for the highest power and efficiency was within the range of 40 to 50 Hz.

- By observing the results with regard to the effect of increase in regenerator length on the efficiency and output power of the engine, the best length value of 18 to 22 mm was recommended for the most optimum value of efficiency and output power of the engine, The obtained results demonstrated the increase of work loss caused by increasing pressure drop for the higher values of regenerator length. Also, the values of thermal losses caused by internal and external conductivity increased with the increase in the regenerator length.

- Regenerator matrix can be made of different materials. The performance of the engine depended on the materials applied in the regenerator matrix. To increase the temperature changes of regenerator and decrease internal losses by thermal conductivity, a material with high thermal capacity and low conductivity should be selected.

- In theory, Stirling engines have high efficiency and power, while constructed samples have low efficiency and power. The main reason of the difference between theoretical and practical results is due to the existence of considerable losses such as external and internal conductivity losses, shuttle effect, residual losses, and pressure drop of the regenerator. These losses depend on the physical and geometrical parameters of the designed sample.

- This study could help determine the influence of geometrical and physical parameters on the performance of GPU-3 Stirling engine and, therefore, identify the optimal design parameters.

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Nomenclatures

A	: Area
C_p	: Specific heat at constant pressure
C_v	: Specific heat at constant volume
D	: Diameter
Dm	: Wire diameter
e	: Eccentricity
L	: Connecting rod length
M	: Mass of working gas in the engine
m	: Mass of gas in different components
P	: Pressure
Q	: Heat
r	: Connecting rod length

R : Gas constant
T : Temperature, K

Greek symbols
γ : Ideal gas specific heat ratio
θ : Crank rotational angle, radians

Subscripts
c : Compression space
ck : Interface between compression space and cooler
cl : Clearance space in engine
e : Expansion space
h : Heater
he : Interface between heater and expansion space
k : Cooler
kr : Interface between cooler and regenerator r regenerator
rh : Interface between regenerator and he

چکیده

در این مقاله حل عددی موتور استرلینگ نوع بتا با درنظر گرفتن بازیاب غیر ایده آل در آن، انجام شده است. برای این منظور از مدل مرتبه دوم، شامل افت‌های حرارتی و توان موتور استفاده شده است.

در ادامه، با استفاده از کد عددی، مقادیر بهینه ابعادی و عملکردی موتور، محاسبه شده است. برای صحه‌گذاری نتایج بدست آمده، از پارامترهای هندسی و عملکردی موتورGPU-3 استفاده شده است. با استناد به نتایج بدست آمده، مقادیر افتهای حرارتی و توان در موتور بسیار قابل توجه می باشد. بر اساس نتایج بدست آمده، افت توان موتور به میزان ۵۰/۱ درصد و راندمان آن به میزان ۲۲/۷ درصد می باشد.

با استفاده از کد عددی و نتایج بدست آمده، مقدار تخلخل کمتر از ۰/۶، فرکانس ۴۰ تا ۵۰ هرتز و طول ۱۸ تا ۲۲ میلمیتر و موادی با ظرفیت حرارتی بالا و ضریب هدایت حرارتی پایین برای بازیاب در شرایط بهینه هندسی و فیزیکی موتور، پیشنهاد می‌شود.