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Mixed Mode Fracture Analysis of Multiple Interface Cracks

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This paper contains a theoretical formulation of multiple interface cracks in two bonded dissimilar half planes subjected to in-plane traction. The distributed dislocation technique is used to construct integral equations for a dissimilar mediums weakened by several interface cracks. These equations are of Cauchy singular type at the location of dislocation, which are solved numerically to obtain the dislocation density on the faces of the cracks. The dislocation densities are employed to evaluate the modes I and II stress intensity factors for multiple interface cracks. Numerical calculations are presented to show the interaction effects of interface cracks on the stress intensity factors.

Keywords: Multiple interface cracks, Dissimilar mediums, Mixed mode fracture, Stress intensity factor

1 Introduction

In studying the fracture mechanics of bonded materials, in recent past considerable attention has been directed toward studying the mechanical behavior of the interfacial regions where defects usually in the form of voids or cracks often exist. It has been shown that for interface cracks the material properties play an important role in their fracture mechanics behavior. From microelectronics to structural engineering many physical applications require joining of two dissimilar materials. Generally, the main function of the “interface” thus formed is the effective transfer of some flux quantity such as stress, heat or electricity. The interface crack behavior in dissimilar anisotropic composites under mixed mode condition studied by Wang and Choi [1]. Delale and Erdogan [2] investigated interface crack problem between the two homogeneous and non-homogeneous half-plane. In this study, they obtained the integral equations and the asymptotic behavior of the stress state near the crack tip. Two bonded dissimilar homogeneous elastic half-planes under mixed mode loading considered by Delale and Erdogan [3]. They obtained modes I and II stress intensity factors, the energy release rate and the direction of a probable crack growth. Shih and Asaro [4] investigated the near tip elastic-plastic fields of a crack on a bimaterial interface. They found that the oscillations of stresses and the overlap of crack surfaces predicted by the original solutions vanish in the elastic-plastic field in the vicinity of the crack tip in the case where tensile load is dominant.

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Hutchinson and Suo [5] reviewed about the application of the stress intensity factor to the mixed mode fracture of an interface crack. Chen and Erdogan [6] investigated the problem of the interface crack in a non-homogeneous coating bonded to a homogeneous substrate. The symmetric mode I crack problem in an inhomogeneous orthotropic medium, was examined by Ozturk and Erdogan [7]. It has been shown that in the mode I crack problem for an orthotropic inhomogeneous medium, Poisson's ratio has only a negligible influence on the stress intensity factors but the effect of the material inhomogeneity was quite significant. The mixed mode crack problem in-plane elasticity for an inhomogeneous orthotropic medium was treated by Ozturk and Erdogan [8]. It was found that generally the stress intensity factors increase with increasing materials inhomogeneous parameter with decreasing stiffness ratio. Huang and Kardomateas [9] obtained the modes I and II stress intensity factors in a fully anisotropic strip with a central crack. Effects of the materials anisotropy on the mixed mode stress intensity factors of a center crack were studied by Long and Delale [10]. It has been shown that crack length, orientation and the non-homogeneity parameter of the layer have significant effect on the fracture of the FGM layer. Ma et al. [11] analyzed the mixed mode crack problem for a functionally graded orthotropic medium under time-harmonic loading. Mixed-mode fracture analysis of an orthotropic functionally graded material (FGM) coating-bond coat-substrate structure was considered by Dag and IIhan [12].

In this article, the distributed dislocation technique is employed to derive singular integral equations for multiple interface cracks in two bonded dissimilar half plans under in-plane traction. The integral equations are of Cauchy singular types which are solved numerically for the dislocation density on the cracks faces. Finally, numerical calculations have been carried out to show the influences of material properties and crack size upon the modes I and II stress intensity factors.

2 Dislocation solution

We consider a homogeneous half-plane $y > 0$ is bounded to a non-homogeneous half-plane $y < 0$ and dislocation path situated along the positive part of the *x* axis (Figure 1).

Hooke's law in a state of plane stress, linear elastic constitutive relations parameters for the each region, can be expressed in the following form

$$\varepsilon_{xx}(x,y) = \frac{1}{E_0}[\sigma_{xx}(x,y) - \upsilon_0 \sigma_{yy}(x,y)],$$
$$\varepsilon_{yy}(x,y) = \frac{1}{E_0}[\sigma_{yy}(x,y) - \upsilon_0 \sigma_{xx}(x,y)],$$
$$\gamma_{xy}(x,y) = \frac{1}{G}\sigma_{xy}(x,y), \qquad y > 0. \tag{1}$$
$$\varepsilon_{xx}(x,y) = \frac{1}{E(y)}[\sigma_{xx}(x,y) - \upsilon(y) \sigma_{yy}(x,y)],$$
$$\varepsilon_{yy}(x,y) = \frac{1}{E(y)}[\sigma_{yy}(x,y) - \upsilon(y) \sigma_{xx}(x,y)],$$
$$\gamma_{xy}(x,y) = \frac{1}{G(y)}\sigma_{xy}(x,y), \qquad y < 0. \tag{2}$$

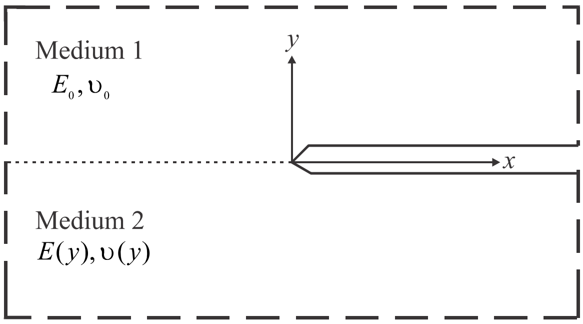

Figure1 Schematic view of the medium

where the E_0 and υ_0 are the elastic constants of the homogeneous half-plane. The equilibrium equations are satisfied by expressing stress components in terms of the Airy stress function $\phi_i(x,y),\ i=1,2.$ as the following form

$$\sigma_{xxi}(x,y)=\frac{\partial^2\phi_i}{\partial y^2},\ \sigma_{yyi}(x,y)=\frac{\partial^2\phi_i}{\partial x^2},\ \sigma_{xyi}(x,y)=-\frac{\partial^2\phi_i}{\partial x\partial y},\ i=1,2 \tag{3}$$

By using the compatibility conditions in terms of the Airy stress function for medium 1 we have

$$\nabla^4\phi_1=0 \tag{4}$$

Young's modulus for medium 2 are assumed to be in the following exponential forms

$$E(y)=E_0e^{\beta y} \tag{5}$$

The stress function for $y<0$ may be shown to satisfy the following differential equation:

$$\nabla^4\phi_2-2\beta\frac{\partial}{\partial y}\nabla^2\phi_2+\beta^2\frac{\partial^2\phi_2}{\partial y^2}-\left(\frac{\partial^2\upsilon}{\partial y^2}-2\beta\frac{\partial\upsilon}{\partial y}+\beta^2\upsilon\right)\frac{\partial^2\phi_2}{\partial x^2}=0. \tag{6}$$

If we now also assume that

$$\upsilon(y)=\upsilon_0e^{\beta y} \tag{7}$$

It is seen that the last term in Eq. (6) vanishes. The solution of Eqs. (4) and (6) is assumed in the form

$$\Phi(s,y)=\int_{-\infty}^{\infty}\phi(x,y)e^{isx}ds \tag{8}$$

The application of Eq.(8) to Eqs.(4) and (6) in conjunction with the property of decaying behavior of the Airy stress function as $|x|\to\infty$ leads to a fourth order ordinary differential equation for $\Phi_i(s,y),\ i=1,2,$ where Φ is the Fourier transform of Airy stress function. The solution to differential equations yields

$$\begin{aligned}\Phi_1(s,y)&=(B_1+B_2y)e^{-|s|y}, \quad y>0\\ \Phi_2(s,y)&=[A_1+A_2y]e^{n_2y}, \quad y<0\end{aligned} \tag{9}$$

where B_1, B_2, A_1 and A_2 are unknown functions and n_2 is the positive root of the characteristic equation obtained from Eq.(6) as

$$(n^2-\beta n-s^2)^2=0. \tag{10}$$

The roots of Eq. (10) are

$$n_1 = n_3 = \frac{\beta}{2} - \sqrt{s^2 + (\frac{\beta}{2})^2},$$
$$n_2 = n_4 = \frac{\beta}{2} + \sqrt{s^2 + (\frac{\beta}{2})^2}, \tag{11}$$

The conditions representing climb and glide edge dislocations, with the Bergers vector b_x and b_y, respectively, are:

$$u_1(x,0^+) - u_2(x,0^-) = b_x H(x)$$
$$v_1(x,0^+) - v_2(x,0^-) = b_y H(x) \tag{12}$$

where $H(.)$ is the Heaviside step function. Moreover, for both types of dislocations the following continuity of stress components along the x-axis should be satisfied. Consequently

$$\sigma_{yy1}(x,0^+) = \sigma_{yy2}(x,0^-)$$
$$\sigma_{xy1}(x,0^+) = \sigma_{xy2}(x,0^-), \quad |x| < \infty \tag{13}$$

By applying Eq. (8) to Eqs. (1) and (2) via Eqs. (3) and (9), displacements in the Fourier domain for the each region may be obtained as

$$U_1(s,y) = \frac{i}{E_0}\left[-2\frac{|s|}{s}B_2 + s(B_1 + B_2 y)(1+\upsilon_0)\right]e^{-|s|y},$$
$$V_1(s,y) = \frac{1}{E_0}[(B_1 + B_2 y)(1+\upsilon_0)|s| + (1-\upsilon_0)B_2]e^{-|s|y}, \quad y > 0,$$
$$U_2(s,y) = \frac{i}{E_0}\left\{\frac{1}{s}[A_1 n_2^2 + A_2(2n_2 + n_2^2 y)]e^{-n_1 y} + \upsilon_0 s[A_1 + A_2 y]e^{n_2 y}\right\},$$
$$V_2(s,y) = \frac{1}{E_0}\left\{s^2[\frac{A_1}{n_1} + A_2(\frac{y}{n_1} + \frac{1}{n_1^2})]e^{-n_1 y} - \upsilon_0[A_1 n_2 + A_2(1 + y n_2)]e^{n_2 y}\right\}, \quad y < 0.$$
$$\tag{14}$$

By applying Eq. (8) to Eq. (13) and using Eqs. (3) and (9) we find

$$A_1 = B_1, \; A_2 = B_2 - (|s| + n_2)B_1 \tag{15}$$

The conditions (12) may now be used to determine the remaining unknowns B_1 and B_2. To do this, we the first apply Eq.(8) to Eq. (12) to obtain

$$U_1(s,0^+) - U_2(s,0^-) = b_x(\pi\delta(s) + i/s)$$
$$V_1(s,0^+) - V_2(s,0^-) = b_y(\pi\delta(s) + i/s) \tag{16}$$

where $\delta(.)$ is the Dirac delta function. By using Eqs. (16) and (14) it may easily be shown that

$$B_1 = \frac{E_0(\pi\delta(s) + i/s)}{P(s)}\left[-isb_x(n_1^2 - s^2) + 2n_1^2 b_y(n_2 + |s|)\right],$$

$$B_2 = \frac{E_0(\pi\delta(s)+i/s)}{P(s)}\left[isb_x\left((n_1^2+s^2)|s|-s^2(n_1-n_2)\right)+n_1^2 b_y(n_2+|s|)^2\right] \tag{17}$$

where $P(s)=s^4+(3n_1^2-2n_1n_2+n_2^2)s^2+2s^2|s|(n_2-n_1)+4|s|n_1^2n_2+n_1^2n_2^2$.

Substituting Eq.(17) into the first of Eqs. (9) and applying the Fourier transform inversion, the Airy stress function for $y>0$ is expressed as

$$\begin{aligned}\phi_1(x,y)=\frac{E_0}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-|s|y-isx}}{P(s)}\Big\{&\left[n_1^2-s^2-y\left((n_1^2+s^2)|s|-s^2(n_1-n_2)\right)\right]b_x\\ &+\frac{in_1^2(n_2+|s|)}{s}\left[2+y(n_2+|s|)\right]b_y\Big\}ds\end{aligned} \tag{18}$$

The integral in Eq.(18) can be divided into odd and even parts to arrive at

$$\begin{aligned}\phi_1(x,y)=\frac{E_0}{\pi}\int_0^{\infty}\frac{e^{-sy}}{P(s)}\Big\{&\left[n_1^2-s^2-ys\left(n_1^2+s^2-sn_1+sn_2)\right)\right]\cos(sx)b_x\\ &+\frac{n_1^2(n_2+s)}{s}\left[2+y(n_2+s)\right]\sin(sx)b_y\Big\}ds\end{aligned} \tag{19}$$

The stress components in view of Eqs. (19) and (3) for $y>0$ are expressed as

$$\begin{aligned}\sigma_{xx1}(x,y)&=\frac{E_0}{\pi}\int_0^{\infty}\frac{e^{-sy}}{P(s)}\left\{\begin{aligned}&s^2\left[n_1^2-s^2+(2-ys)\left(n_1^2+s^2-sn_1+sn_2)\right)\right]\cos(sx)b_x\\ &+n_1^2(n_2+s)\left[2s+(n_2+s)(-2+sy)\right]\sin(sx)b_y\end{aligned}\right\}ds\\ \sigma_{yy1}(x,y)&=-\frac{E_0}{\pi}\int_0^{\infty}\frac{e^{-sy}}{P(s)}\left\{\begin{aligned}&s^2\left[n_1^2-s^2-ys\left(n_1^2+s^2-sn_1+sn_2)\right)\right]\cos(sx)b_x\\ &+n_1^2(n_2+s)s\left[2+y(n_2+s)\right]\sin(sx)b_y\end{aligned}\right\}ds\\ \sigma_{xy1}(x,y)&=-\frac{E_0}{\pi}\int_0^{\infty}\frac{e^{-sy}}{P(s)}\left\{\begin{aligned}&s^2\left[n_1^2-s^2+(1-ys)\left(n_1^2+s^2-sn_1+sn_2)\right)\right]\sin(sx)b_x\\ &+n_1^2(n_2+s)\left[-2s+(1-ys)(n_2+s)\right]\cos(sx)b_y\end{aligned}\right\}ds\end{aligned} \tag{20}$$

In order to specify the singular behavior of the stress components, the asymptotic behavior of the integrands in Eqs. (20) should be examined. Since the integrands are continuous functions of s and also finite at $s=0$, the singularity must occur as s tends to infinity. We determine the leading terms of Eqs. (20) as $s\to\infty$, employ the following identities:

$$\begin{aligned}&\int_0^{\infty}\sin(sx)e^{-ys}ds=\frac{x}{x^2+y^2},\quad y>0,\\ &\int_0^{\infty}\cos(sx)e^{-ys}ds=\frac{y}{x^2+y^2},\quad y>0,\\ &\int_0^{\infty}s\sin(sx)e^{-ys}ds=\frac{2xy}{(x^2+y^2)^2},\quad y>0,\\ &\int_0^{\infty}s\cos(sx)e^{-ys}ds=\frac{y^2-x^2}{(x^2+y^2)^2},\quad y>0.\end{aligned} \tag{21}$$

We may write Eqs. (20) as follow

$$\sigma_{xx1}(x,y)=\frac{E_0}{4\pi(x^2+y^2)^2}\left[y(y^2+3x^2)b_x+x(y^2-x^2)b_y\right]+\int_0^\infty [H_{xx1}(x,y,s)-H_{xx1\infty}(x,y,s)]ds,$$
$$\sigma_{yy1}(x,y)=\frac{E_0}{4\pi(x^2+y^2)^2}\left[y(y^2-x^2)b_x-x(x^2+3y^2)b_y\right]+\int_0^\infty [H_{yy1}(x,y,s)-H_{yy1\infty}(x,y,s)]ds,$$
$$\sigma_{xy1}(x,y)=\frac{E_0}{4\pi(x^2+y^2)^2}\left[x(y^2-x^2)b_x+y(y^2-x^2)b_y\right]+\int_0^\infty [H_{xy1}(x,y,s)-H_{xy1\infty}(x,y,s)]ds \tag{22}$$

In Eqs. (22)

$$H_{xx1}(x,y,s)=\frac{E_0e^{-sy}}{\pi P(s)}\left\{\begin{array}{l} s^2\left[n_1^2-s^2+(2-ys)\left(n_1^2+s^2-sn_1+sn_2\right)\right]\cos(sx)b_x \\ +n_1^2(n_2+s)\left[2s+(n_2+s)(-2+sy)\right]\sin(sx)b_y \end{array}\right\},$$
$$H_{yy1}(x,y,s)=-\frac{E_0e^{-sy}}{\pi P(s)}\left\{\begin{array}{l} s^2\left[n_1^2-s^2-ys\left(n_1^2+s^2-sn_1+sn_2\right)\right]\cos(sx)b_x \\ +n_1^2(n_2+s)s\left[2+y(n_2+s)\right]\sin(sx)b_y \end{array}\right\},$$
$$H_{xy1}(x,y,s)=-\frac{E_0e^{-sy}}{\pi P(s)}\left\{\begin{array}{l} s^2\left[n_1^2-s^2+(1-ys)\left(n_1^2+s^2-sn_1+sn_2\right)\right]\sin(sx)b_x \\ +n_1^2(n_2+s)\left[-2s+(1-ys)(n_2+s)\right]\cos(sx)b_y \end{array}\right\},$$
$$H_{1xx\infty}(x,y,s)=\frac{E_0e^{-sy}}{4\pi}\left[(2-ys)\cos(sx)b_x+(-1+ys)\sin(sx)b_y\right],$$
$$H_{1yy\infty}(x,y,s)=\frac{E_0e^{-sy}}{4\pi}\left[ys\cos(sx)b_x-(1+ys)\sin(sx)b_y\right],$$
$$H_{1xy\infty}(x,y,s)=\frac{E_0e^{-sy}}{4\pi}\left[(-1+ys)\sin(sx)b_x+ys\cos(sx)b_y\right]. \tag{23}$$

From Eqs. (22), we may observe that stress components exhibit the familiar Cauchy-type singularity at dislocation location. Moreover, the integrands in Eqs. (22) decay sufficiently rapidly as $s\to\infty$, which makes the integrals susceptible to numerical evaluation.

3 Formulation of multiple cracks

The dislocation solutions obtained in Section 2 is utilized to analyze a dissimilar mediums weakened by N arbitrary straight cracks with smooth face which are located in interface between two dissimilar half planes. Straight cracks configuration may be described in parametric form as,

$$x_i=x_{i0}+sa_i,$$
$$y_i=y_{i0},\qquad i=1,2,...,N,\qquad -1\le s\le 1 \tag{24}$$

where (x_{i0},y_{i0}) and a_i are the center coordinates and half-length of the i*th* crack, respectively. Suppose climb and glide edge dislocations with unknown densities $B_{xk}(t)$ and $B_{yk}(t)$, respectively, are distributed on the segment $a_k dt$ at the surface of *k*th crack, where $-1\le t\le 1$. Employing the principal of superposition, the components of traction vector at a point with coordinates ($x_i(s), y_i(s)$), where parameter $-1\le s\le 1$, on the surface of all cracks yield

$$\sigma_{yy}(x_i(s), y_i(s)) = \sum_{k=1}^{N}\int_{-1}^{1}[k_{yyik}^{11}(s,t)B_{xk}(t) + k_{yyik}^{12}(s,t)B_{yk}(t)]a_k dt,$$

$$\sigma_{xy}(x_i(s), y_i(s)) = \sum_{k=1}^{N}\int_{-1}^{1}[k_{xyik}^{11}(s,t)B_{xk}(t) + k_{xyik}^{12}(s,t)B_{yk}(t)]a_k dt,\ i = 1,2,\ldots,N, -1 \le s \le 1. \tag{25}$$

The kernels k_{yyik}^{11}, k_{yyik}^{12}, k_{xyik}^{11} and k_{xyik}^{12} in integral Eqs. (25) are coefficients of b_x and b_y in stress components σ_{yy} and σ_{xy} in Eqs. (22), respectively. By virtue of Buckner's principle the left side of Eqs. (25), after changing the sign, is the traction caused by external loading on the uncracked medium at the presumed surfaces of cracks. Employing the definition of dislocation density function, the equations for the crack opening displacement across the *i*th crack yield

$$u_{xi}^{+}(s) - u_{xi}^{-}(s) = \int_{-1}^{s} a_i B_{xi}(t)dt,$$

$$u_{yi}^{+}(s) - u_{yi}^{-}(s) = \int_{-1}^{s} a_i B_{yi}(t)dt, \qquad i \in \{1,2,\ldots,N\}. \tag{26}$$

The displacement field is single valued out of an embedded crack surface. Consequently, the dislocation densities are subjected to the following closure requirements:

$$\int_{-1}^{1} B_{xi}(t)dt = 0,$$

$$\int_{-1}^{s} B_{yi}(t)dt = 0, \quad i \in \{1,2,\ldots,N\}. \tag{27}$$

It is worth mentioning that the devised procedure despite its simplicity is capable of handling complicated crack arrangements. To evaluate the dislocation density, the Cauchy singular integral Eqs. (25) and Eqs. (27) ought to be solved simultaneously. The stress fields near the crack tips have the singularity of for the embedded cracks in a medium $1/\sqrt{r}$ where r is the distance from a crack tip. Therefore, the dislocation densities are taken as

$$B_{xi}(t) = \frac{g_{xi}(t)}{\sqrt{1-t^2}}$$

$$B_{yi}(t) = \frac{g_{yi}(t)}{\sqrt{1-t^2}}, \quad -1 < t < 1, \qquad i \in \{1,2,\ldots,N\} \tag{28}$$

Substituting Eqs. (28) into Eqs. (25) and (27) and make use of the numerical solutions of integral equations with Cauchy-type kernel developed by Erdogan et al. [13]. result in $g_{xi}(t)$ and $g_{yi}(t)$. The modes I and II stress intensity factors for embedded cracks derived by Faal and Fariborz [14] are defined as,

$$\begin{Bmatrix} K_{IL} \\ K_{IIL} \end{Bmatrix} = \frac{E_0}{4}\lim_{r_L \to 0}\frac{1}{\sqrt{2r_{Li}}}\begin{Bmatrix} u_{yi}^{+}(s) - u_{yi}^{-}(s) \\ u_{xi}^{+}(s) - u_{xi}^{-}(s) \end{Bmatrix}$$

$$\begin{Bmatrix} K_{IR} \\ K_{IIR} \end{Bmatrix} = \frac{E_0}{4}\lim_{r_R \to 0}\frac{1}{\sqrt{2r_{Ri}}}\begin{Bmatrix} u_{yi}^{+}(s) - u_{yi}^{-}(s) \\ u_{xi}^{+}(s) - u_{xi}^{-}(s) \end{Bmatrix} \tag{29}$$

where the subscripts L and R designate to the left and right tips of crack, respectively, the geometry of crack implies

$$r_{Li} = \left[(x_i(s) - x_i(-1))^2 + (y_i(s) - y_i(-1))^2 \right]^{\frac{1}{2}}$$
$$r_{Ri} = \left[(x_i(s) - x_i(1))^2 + (y_i(s) - y_i(1))^2 \right]^{\frac{1}{2}} \qquad (30)$$

Substituting Eqs. (28) into (26), and results equations into (29) after using the Taylor series expansion of functions $x_i(s)$ and $y_i(s)$ in the vicinity of the points $s = \pm 1$ leads to

$$\begin{Bmatrix} K_{IL} \\ K_{IIL} \end{Bmatrix} = \frac{E_0 \left(\left[x_i'(-1) \right]^2 + \left[y_i'(-1) \right]^2 \right)^{\frac{1}{4}}}{4} \begin{Bmatrix} g_{yi}(-1) \\ g_{xi}(-1) \end{Bmatrix},$$
$$\begin{Bmatrix} K_{IR} \\ K_{IIR} \end{Bmatrix} = -\frac{E_0 \left(\left[x_i'(-1) \right]^2 + \left[y_i'(-1) \right]^2 \right)^{\frac{1}{4}}}{4} \begin{Bmatrix} g_{yi}(1) \\ g_{xi}(1) \end{Bmatrix}. \qquad (31)$$

4 Results and discussion

The method presented above, allows the consideration of a dissimilar mediums with multiple straight cracks subjected to in-plane tractions. The accuracy and validity of the approach is tested by solving some well known problems whose solution has been previously obtained by other authors using different procedures.

The analysis, developed in the preceding section, allows the consideration of a dissimilar mediums with multiple straight cracks subjected to in-plane tractions. The validation of the formulation is accomplished by comparing our results with those obtained by Delale and Erdogan [2], wherein the crack is under constant normal traction. In this case, excellent agreement is observed for modes I and II stress intensity factors. The calculated stress intensity factors are given in Table (1).

As expected, in the nonhomogeneous medium $\beta > 0$, with increasing values of β, stiffness of the plane decreasing and consequently the stress intensity factors increase with increasing the FG constant. Also note that due to the lack of symmetry with respect to $y = 0$ plane, the stress intensity factors are one of mixed mode.

Next example, we consider two equal-length collinear cracks are located at the interface of two bonded dissimilar half-planes. Figures (1) and (2). The center of cracks remain fixed while the crack length are changing with the same rate. The dimensionless modes I and II stress intensity factors versus crack length for two different non-homogeneity constant and $2b = 2.5cm$ are depicted in Figures (1) and (2). The variation of modes I and II stress intensity factors of R tips are much smaller than that of L.

This attributes to weak interaction of cracks and large variation of stress fields around L tips. The next example, consider the variation of dimensionless modes I and II stress intensity factors, versus the dimensionless b/a for two different FG constant which are placed at the interface between two dissimilar mediums (Figures (4) and (5)). The crack length remain fixed while the center of cracks changing with the same rate. The problem is symmetric with respect to the y-axis.

We observe that with increasing the center of cracks modes I and II stress intensity factors decreasing because the interaction between cracks is weak.

Table 1 Comparisons of the normalized mixed-mode stress intensity factors for a crack subjected to uniform normal stress

	βa	0	0.1	0.25	0.5	0.75	1.0	1.25	1.5	2.0	2.5	3.0	5.0
$\frac{K_I}{K_0}$	Delale and Erdogan [2]	1.0	1.032	1.081	1.164	1.249	1.336	1.425	1.517	1.706	1.903	2.107	2.998
	Present Work	1.0	1.0319	1.0807	1.1636	1.2487	1.3359	1.4251	1.5165	1.7053	1.9022	2.1066	2.9957
$\frac{K_{II}}{K_0}$	Delale and Erdogan [2]	0.0	0.013	0.033	0.069	0.109	0.150	0.194	0.241	0.341	0.449	0.564	1.097
	Present Work	0.0	0.0127	0.0330	0.0693	0.1084	0.1502	0.1942	0.2406	0.3398	0.4472	0.5623	1.0892

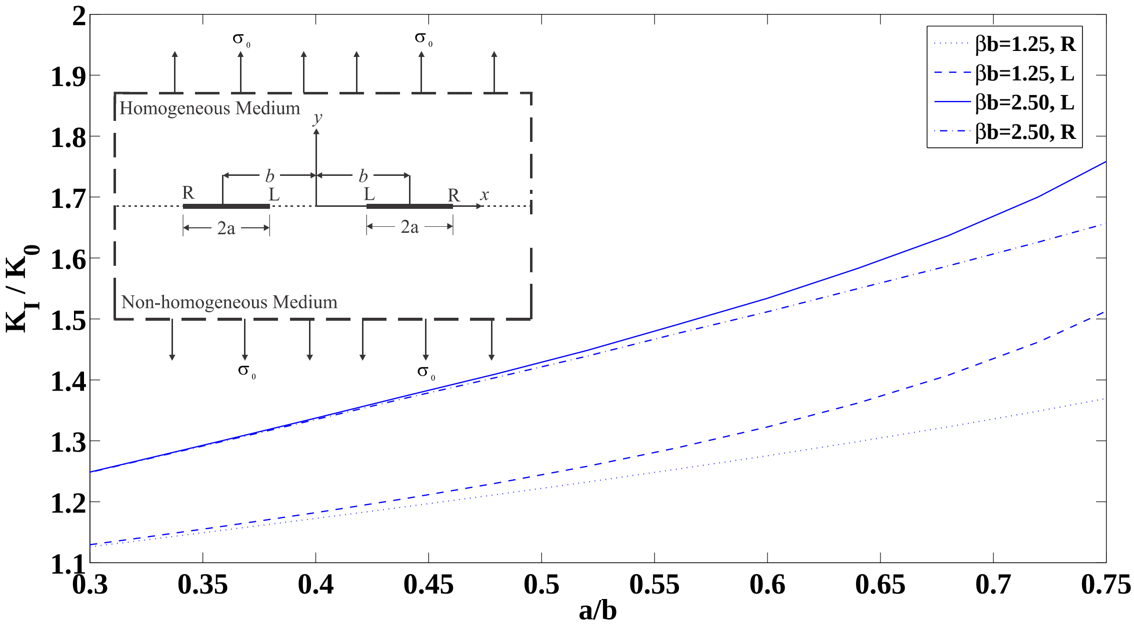

Figure 2 Variation of dimensionless mode I stress intensity factors for two collinear cracks with changing a/b

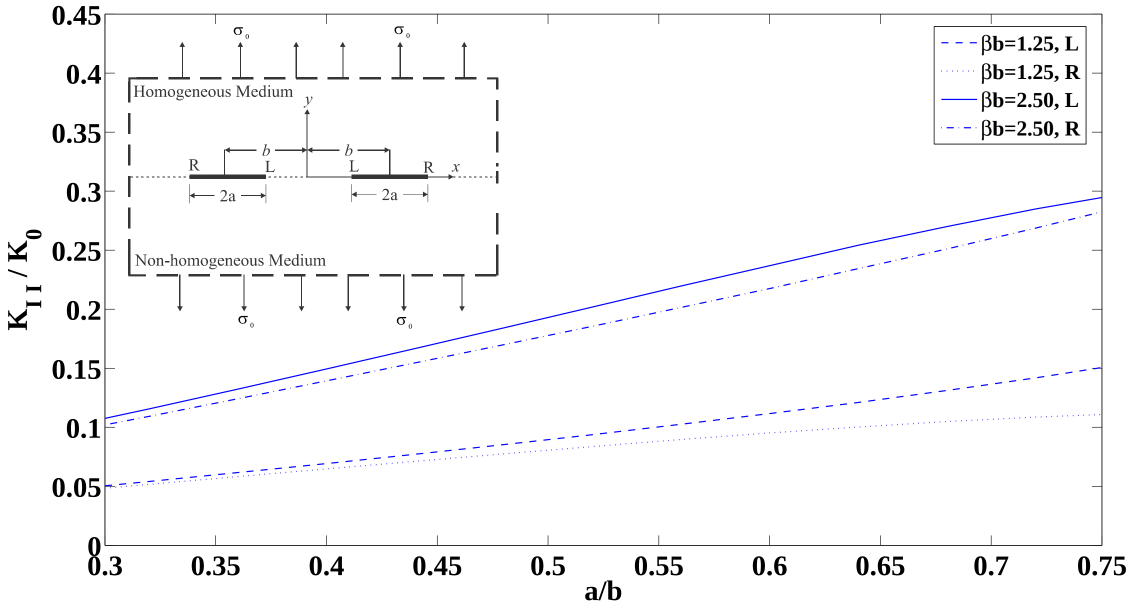

Figure 3 Variation of dimensionless mode II stress intensity factors for two collinear cracks with changing a/b

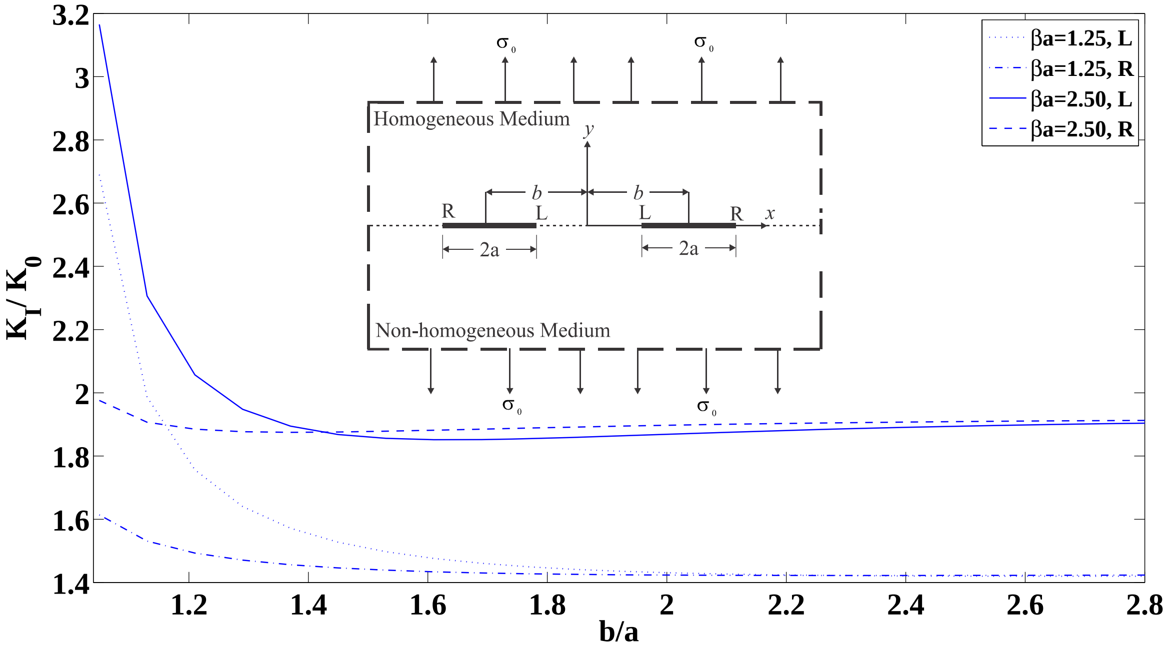

Figure 4 Variation of dimensionless mode I stress intensity factors for two collinear cracks with changing b/a

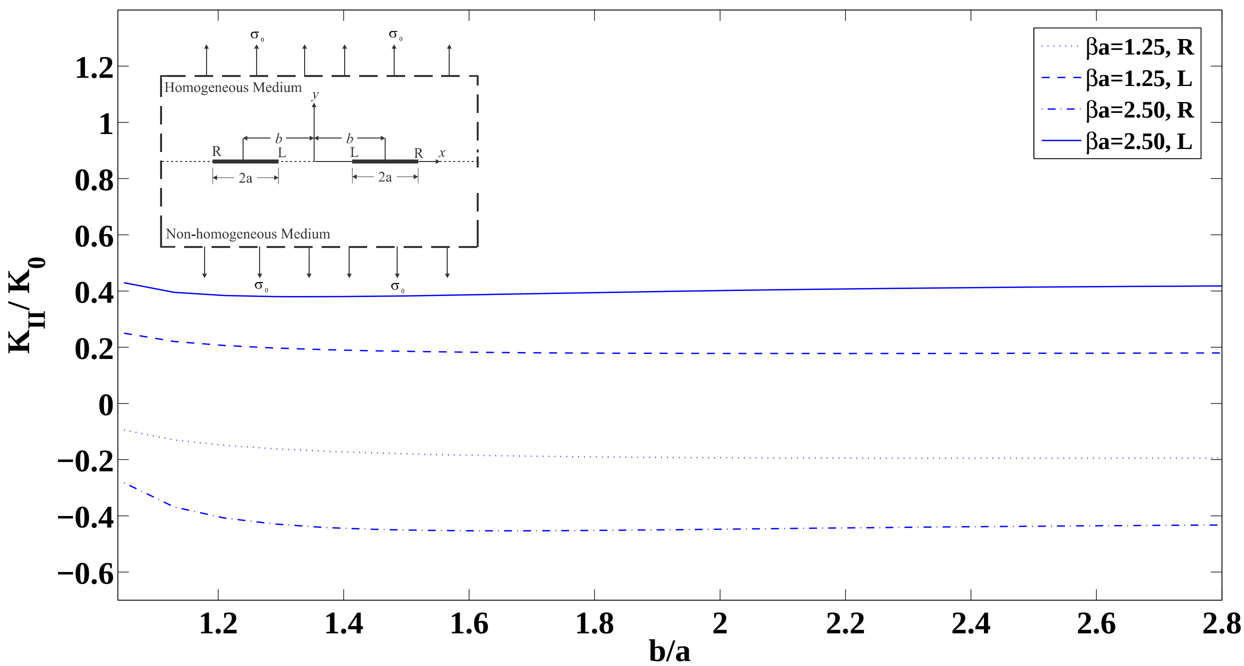

Figure 5 Variation of dimensionless mode II stress intensity factors for two collinear cracks with changing b/a

5 Conclusions

This paper presents efficient analytical method for the evaluation of modes I and II stress intensity factors for multiple interface cracks problem in dissimilar mediums. A solution for the stress field caused by the Volterra types climb and glide edge dislocations in a dissimilar mediums is first obtained. The solutions are in accordance with the well-known results in literature. The stress components are used as the Green's function to derive integral equations for the analysis of multiple interface cracks. To show the applicability of the procedure more examples are solved where in the interaction between cracks is investigated.

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Nomenclature

a	Half lengths of crack
A_1, A_2, B_1, B_2	Unknowns coefficients
b_x, b_y	Dislocation densities
E_0	Elastic modulus
G	Shear elastic modulus
$g_{xi}(t), g_{yi}(t)$	Regular terms of dislocation densities
$k_{yyik}(s,t), k_{xyik}(s,t)$	Kernel of integral equations
$H(x)$	Heaviside step function
$K_{IR}, K_{IIR}, K_{IL}, K_{IIL},$	Stress intensity factors of left and right side of crack
K_0	Stress intensity factor of a crack in infinite plane
N	Number of cracks
u, v	Displacement components
x, y	Coordinates
r_{Li}, r_{Ri}	Distance from right and left crack tips
$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$	Stress components
σ_0	Applied tractions at infinity

Greek Symbols

β	Nonhomogeneous constant
υ_0	Poisson's ratio
$\delta(s)$	Dirac delta function
$\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$	Strain components
ϕ	Airy stress function

چکیده

این مقاله حاوی یک فرمولاسیون تئوری از چندین ترک واقع در فصل مشترک دو نیم صفحه‌ی غیر مشابه، نسبت به بارگذاری درون صفحه‌ای می‌باشد. روش توزیع نابجایی برای بدست آوردن معادلات انتگرالی در دو محیط غیر مشابه تضعیف شده توسط چندین ترک واقع در فصل مشترک دو محیط استفاده شده است.
این معادلات در محل نابجایی دارای تکینگی از نوع کوشی می‌باشند که بصورت عددی برای بدست آوردن دانسیته نابجایی بر روی سطوح ترک‌ها حل شده‌اند. دانسیته‌های نابجایی برای بدست آوردن ضرایب شدت تنش مود ترکیبی I و II برای ترک‌های واقع در فصل مشترک دو محیط غیر مشابه بکار می‌روند.
محاسبات عددی برای نشان دادن اثرات اندرکنش ترک‌های واقع در فصل مشترک بر روی ضرایب شدت تنش ارائه شده است.