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Analytical Modeling of Axial Collapse of Circular Hybrid Tubes

In this paper, considering symmetric collapse mode for hybrid tubes, and using Tsai-Hill fracture criterion to take into account the off-axis strength of an orthotropic lamina, an analytical model is extended and an expression is derived for mean crushing load and fold lengths of circular hybrid tubes. In this model, the influences of the geometrical dimensions including diameter, thickness of metal and composite wall, fiber ply orientation and material properties are studied on the mean crushing load and fold lengths. The validation with the previously published results provides reasonably good agreement.

Keywords: Circular hybrid tubes, Axial collapse, Analytical model, Energy absorption

1 Introduction

Nowadays, increasing speed of vehicles, studying the behavior of energy dissipating devices has gained much more importance to improve crashworthiness characterizations of transportation vehicles such as cars, ships, aircrafts, couches and etc [1]. Circular metal tubes are one of the most widespread shapes of tubular energy dissipating structures that collapse in symmetric or concertina, asymmetric or diamond and mixed modes under quasi-static and dynamic axial loading [2]. In last decades a few methods have been suggested to improve energy absorption capability of these structures like, using filler substances such as metal and polymeric foams [3] and externally reinforcing by composite materials [4-12].

Reinforcing metal tubes by composite layers, hybrid tubes, is one of the most conventional ways to improve their energy absorption and crashworthiness characteristics that was reported by Wang et al. [4] for the first time. Experimental and numerical investigations [5-7] prove that many different parameters such as metal and composite material properties, fiber ply orientation, geometrical dimension of metal tube and composite wall thickness, affect the energy absorption characteristics (specific energy, mean and peak load) of hybrid tubes.

Although, many experimental studies have been carried out in the field of crushing of hybrid tubes under axial loading, analytical models for axial collapse are restricted. Hanefi and

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Wierzbicki [8], based on Alexander's model [9] presented the first analytical model to predict the mean crushing load of circular hybrid tubes that are reinforced with unidirectional fibers in hoop direction. Wang and Lu [10], considering inward and outward folding pattern and using Von Misses criterion instead of Tresca, developed the previous model. Also, they studied the $[\pm\alpha]$ stacking of composite layers, utilizing the stress and strain transformation analysis. Song et al. [5] modified Hanefi's model [8] for dynamic loading by considering strain rate effects for metal material. Shin et al. [11] investigated axial collapse of square hybrid tubes analytically and proposed an expression for mean crushing load. The authors recently have presented a mathematical model to predict mean crushing load and fold lengths of square hybrid tubes with arbitrary stacking sequence [12].

All reported experimental results show that circular hybrid tubes collapse in asymmetric diamond mode, but since the amount of absorbed energy and mean crushing force for diamond and concertina modes are approximately the same, in all presented analytical models, it is assumed that hybrid tubes collapse in concertina mode.

In this paper, based on Alexander's model for collapse of bare metal tubes and using Tsai-Hill criterion to predict fracture of composite layers, a theoretical expression is derived for the mean crushing load and fold lengths of hybrid tubes. In this model, the influences of the geometrical dimensions of metal tube including thickness and diameter, composite wall thickness, fiber ply orientation and material properties on the crushing load have been studied. In this model, equivalent bending moment of hybrid section is determined as bi-material sections [13]. The validation with the previously reported numerical [14] reveals reasonably good agreement.

2 Analytical model

Consider a circular metal tube with average radius of R_0 and thickness of t_m which is externally reinforced with N composite layers. Let t_f is thickness of each composite layer and H be the half fold length. Similar to Hanefi and Wierzbicki [8] and Wang and Lu [10], it's assumed that circular hybrid tubes collapse in symmetric mode based on Alexander's model [9], Fig. 1. The following assumptions are made in this analysis:

1- The folds form outward the tube.

2- The material of metal is regarded as rigid-perfectly plastic and obeys the Von Mises yield criterion.

3- The behavior of composite material is regarded as elastic linear until fracture.

4- Meridian deformation of tube is neglected.

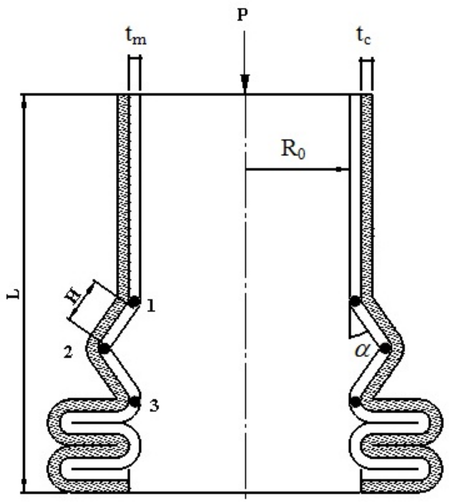

Figure 1 Idealized symmetric collapse mode of axial crushing of a hybrid circular tube

2-1 Dissipated bending energies in plastic hinges

The incremental dissipated bending energy in plastic hinges is given by:

$$dW_b = 4\pi R_0 M_0 d\alpha + 4\pi M_0 (R_0 + H\sin\theta) d\alpha \qquad (1)$$

Then, total dissipated bending energy during fold formation is obtained as:

$$W_b = \int_0^{\frac{\pi}{2}} dW_b = 4\pi M_0 (\pi R_0 + H) \qquad (2)$$

where, M_0 is the fully plastic bending moment of hybrid section of tube that is calculated later.

2-2 Dissipated extensional energy in circumferential deformation

Dissipated extensional energy due to circumferential deformation of hybrid tube includes dissipated energy in metal and composite wall, which are calculated below, respectively:

Incremental of dissipated extensional energy in metal wall, is obtained by:

$$dW_{sm} = 4\pi\sigma_{0m} t_m \cos\alpha d\alpha \int_0^H s ds = 2\pi\sigma_{0m} t_m H^2 \cos\alpha d\alpha \qquad (3)$$

where s is dictance of each point between hinge 1 and 2 from hinge 1 and σ_{0m} is flow stress of metal wall. Thus, total dissipated extensional energy in metal tube due to circumferential extension is given by:

$$W_{sm} = 2\pi\sigma_{0m} t_m H^2 \qquad (4)$$

It's assumed that composite material behaves elastically until fracture. Regarding $\varepsilon_{\varphi i}$ and $\sigma_{\varphi i}$ as circumferential strain and stress of ith layer of composite wall, respectively, dissipated extensional energy in all composite wall is calculated as:

$$W_{sc} = \frac{1}{2}\sum_{i=1}^{N} \sigma_{\varphi i}\varepsilon_{\varphi i} dv \qquad (6)$$

According to linear relation between stress and strain in composite layers, we get :

$$\sigma_{\varphi i} = E_{\varphi i}\varepsilon_{\varphi i} \qquad (7)$$

where:

$$\varepsilon_{\varphi i} = \frac{s}{R_0} \qquad (8)$$

substituting the values of $\sigma_{\varphi i}$ and s from Eq. 7 and Eq. 8 into Eq. 6 leads to:

$$W_{sc} = 2\pi R_0^2 \sum_{i=1}^{N} \int_0^{\varepsilon_{\varphi i}^f} E_{\varphi i}\varepsilon_{\varphi i}^2 t_i d\varepsilon_{\varphi i} \qquad (13)$$

In Eq. 13, $\varepsilon_{\varphi i}^f$ is the fracture strain of ith layer of composite in hoop direction. Considering equal thickness for composite layers , t_f , Eq. 13 can be rewritten as:

$$W_{sc} = \frac{2}{3}\pi R_0^2 t_f \sum_{i=1}^{N} \frac{(\sigma_{\varphi i}^f)^3}{E_{\varphi i}^2} \tag{14}$$

where $\sigma_{\varphi i}^f$, is fracture stress of ith layer in hoop direction and calculated from Tsi-Hill creterion.

2-3 Mean crushing load

Energy method is used to retrieve the mean crushing force, by equating the dissipated energies during forming a fold to external work, and we have:

$$P_m 2H = W_b + W_{sm} + W_{sc} \tag{15}$$

Putting the values of W_b , W_{sm} and W_{sc} from Eq. 2, Eq. 4 and Eq. 14 in Eq. 15, the mean crushing load is defined as:

$$P_m = 2\pi M_0 + \frac{2\pi^2 M_0 R_0}{H} + \pi\sigma_{0m} t_m H + \frac{\pi R_0^2 t_f}{3H} \sum_{i=1}^{N} \frac{(\sigma_{\varphi i}^f)^3}{E_{\varphi i}^2} \tag{16}$$

In Eq. 16, H is unknown and can be determined by minimizing the mean crushing load, thus:

$$\frac{dP_m}{dH} = 0 \Rightarrow H = \sqrt{\frac{2\pi M_0 R_0 + \frac{1}{3} R_0^2 t_f \sum_{i=1}^{N} \frac{(\sigma_{\varphi i}^f)^3}{E_{\varphi i}^2}}{\sigma_{0m} t_m}} \tag{17}$$

2- 4 Equivalent bending moment of hybrid section of tube

Consider hybrid section of tube as Fig. 2 that θ_i ($1 \le i \le N$) is fiber angle of ith composite layer to the axis of tube and σ_{zi}^f is fracture strength of it in the direction of tube axis that is calculated from Tsai-Hill criterion as follows:

$$\sigma_{zi}^f = [\frac{\cos^4\theta}{X_{t,c}^2} + (\frac{1}{S^2} - \frac{1}{X_{t,c}^2})\sin^2\theta\cos^2\theta + \frac{\sin^4\theta}{Y_{t,c}^2}]^{-\frac{1}{2}} \tag{18}$$

where X_t and X_c are tension and compression strength of composite in the direction of fiber and Y_t and Y_c are the same values perpendicular to the fiber directions and S is inplane shear strength.

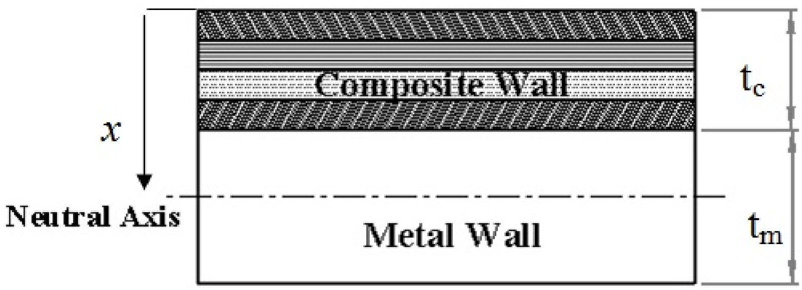

Figure 2 Cross section of circular hybrid tube

Assume that section of hybrid tube is under bending as metal wall has reached full yielding and each composite layer is at the fracture strength. The position of the plastic neutral axis, x, is measured from the end of composite wall and can be defined by force balance on the section of hybrid tube:

$$\sigma_{0m}(t_h - x) = \sigma_{0m}(x - t_c) + \sum_{i=1}^{N} \sigma_{zi} t_i \tag{19}$$

Therefor:

$$x = \frac{t_h + t_C}{2} - \frac{1}{2\sigma_{0m}} \sum_{i=1}^{N} \sigma_{zi} t_i \tag{20}$$

where t_i is the thickness of ith layer of composite and $t_h = t_m + t_c$. Assuming the neutral axis is located at metal section, $x > t_c$, equivalent bending moment of hybrid section is defined to take moment about the neutral axis:

$$M_{0eq} = \sigma_{0m}(t_h - x)\frac{(t_h - x)}{2} + \sigma_{0m}(x - t_c)\frac{(x - t_c)}{2} + \sum_{i=1}^{N} \sigma_{zi} t_i r_i \tag{21}$$

where r_i is distance of center of ith composite layer from the neutral axis and given by:

$$r_i = x - t_c + \sum_{j=1}^{i-1} t_j \tag{22}$$

Now putting the values of x and r_i form Eq. 20 and Eq. 22 in Eq. 21, we have:

$$M_{0eq} = \sigma_{0m}\left[(\frac{t_m}{2})^2 + (\frac{1}{\sigma_{0m}} \sum_{i=1}^{N} \sigma_{zi}^f t_i)^2 \right] + \sum_{i=1}^{N} \left[\sigma_{zi}^f t_i (\frac{t_m}{2} - \frac{1}{2\sigma_{0m}} \sum_{k=1}^{N} \sigma_{zk}^f t_k + \frac{t_i}{2} + \sum_{j=1}^{i-1} t_j) \right] \tag{23}$$

Assuming constant thickness for all composite layers and Defining dimensionless ratios $k_i = \frac{\sigma_{zi}}{\sigma_{0m}}$ and $t_r = \frac{t_c}{t_m}$, Eq. 23 can be rewritten as:

$$M_{0eq} = \frac{\sigma_{0m} t_m^2}{4} \left\{ 1 + (\frac{t_r}{N} \sum_{i=1}^{N} k_i)^2 + \frac{t_r}{N} \sum_{i=1}^{N} \left[k_i (2 - 2\frac{t_r}{N} \sum_{j=1}^{N} k_j + \frac{4(i-0.5)}{N} t_r) \right] \right\} \tag{24}$$

On the other hand, according to the Von Mises criterion can be written:

$$M_{0eq} = \frac{2}{\sqrt{3}} \sigma_{0eq} (\frac{t_h}{2})^2 \tag{25}$$

Finally, with substiuting the value of M_{0eq} from Eq. 24 in Eq. 25, the equivalent strength of hybrid section is detemined as:

$$\sigma_{eq} = \frac{\sqrt{3}}{2} \frac{\sigma_{0m}}{(1 + t_r^2)} \left\{ 1 + (\frac{t_r}{N} \sum_{i=1}^{N} k_i)^2 + \frac{t_r}{N} \sum_{i=1}^{N} \left[k_i (2 - 2\frac{t_r}{N} \sum_{j=1}^{N} k_j + \frac{4(i-0.5)}{N} t_r) \right] \right\} \tag{26}$$

Which for particular case N=1, is same as derived expression for bi-material section in [13]:

$$\sigma_{eq} = \frac{\sqrt{3}}{2} \frac{1 + 2kt_r + 2kt_r^2 - k^2 t_r^2}{(1 + t_r)^2} \sigma_{0m} \tag{27}$$

3 Comparison with previously reported results

To verify the presented model, the predicted values for mean crushing load are compared with numerical results reported in [14]. For this goal, the simulated results by software LS_DYNA for circular hybrid tubes with three thicknesses under static crushing are compared with the predicted results by Eq. 16. Two composite materials, including carbon/epoxy and E-glass/epoxy are used to the better validation. The mechanical properties of metal and composite materials were reported in reference [14]. The flow stress of metal is taken into account as $\sigma_{0m} = 0.95\sigma_u$, which σ_u is ultimate strength. The dimensions of metal tube are $R_0 = 29mm$, $t_m = 1mm$ and $t_c = 0.5, 0.8, 1mm$.

Table 1 Comparison of presented model with previously reported results

t_m (mm)	t_c (mm)	Ply pattern	P_m (KN)				Error (%)	
			Presented model		Simulation [14]			
			Carbon/Epoxy	Glass/Epoxy	Carbon/Epoxy	Glass/Epoxy	Carbon/ Epoxy	Glass/ Epoxy
1	0.5	$[\pm 90]_3$	44.97	---	51.70	---	13.01	---
1	0.8	$[\pm 90]_3$	48.64	46.37	57.50	46.70	15.40	0.70
1	1	$[\pm 90]_3$	51.43	---	63.90	---	19.51	---
1	0.8	$[\pm 75]_3$	48.13	42.18	58.40	45.80	17.58	7.90
1	0.8	$[\pm 60]_3$	47.75	46.14	50.20	45.70	4.88	0.90
1	0.8	$[\pm 45]_3$	48.4	46.80	49.10	45.70	1.42	2.40
1	0.8	$[\pm 30]_3$	50.85	48.83	53.50	45.90	4.95	6.30
1	0.8	$[\pm 15]_3$	58.35	54.62	49.00	45.50	19.08	20

Table 1 reveals comparison between the analytical and numerical simulation results. These results reveal that presented model can predict the mean crushing load of multi angle circular hybrid tubes with reasonable accuracy. Of course as it can be seen from Table 1, in some cases the difference between experimental values with the predicted ones by the analytical model is high that is because of two main reasons. The first one corresponds to using the Alexander's model [9] that is a simple and not very precise model on crushing of cylindrical tubes and the other one is because of not considering the deformation and fracture modes in composite layers during crushing phenomenon. It should be noted that the Alexander's model is used because of it's simplicity and to get a closed analytical expression for mean crushing load, similar to [8] and [10]. But the most important advantage of this model is that the possibility of predicting the mean crushing load of circular hybrid tubes reinforced by composite layers in different angles of orientation exist, while the previous models have not got this ability.

4 Conclusion

In the present paper, an analytical model is developed for axial collapse of circular hybrid tubes based on Alexander's model and an expression is derived for the mean crushing load and fold wave length in terms of geometrical dimensions of metal tube including thickness and diameter, composite wall thickness, fiber orientation of each layer and also metal and composite material properties. In this model, strength of each composite layer in the direction of tube axis is calculated using Tsai-Hill criterion. To validate the model, predicted values are compared with numerical results in literature that show reasonably good agreement. This model has an important advantage over previous models that can predict mean crushing load of a circular tube reinforced by composite layers in different angles of orientation, also the effect of number of composite layers is observed.

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Nomenclature

dv	:	Element of volume
dW_b	:	Incremental absorbed bending energy
dW_{sm}	:	Incremental absorbed extensional energy in metal wall
E_1	:	Elasticity modulus in fiber direction
E_2	:	Elasticity modulus perpendicular to fiber direction
$E_{\varphi i}$	:	Elasticity modulus of ith composite layer in hoop direction
G_{12}	:	In-plane shear modulus
H	:	Half fold length
k_i	:	$=\sigma_{zi}/\sigma_{0m}$
L	:	Length of tube
M_0^{eq}	:	Equivalent bending moment of hybrid section
N	:	Number of composite layers
P_m	:	Mean crushing load
$P_{\max}$	:	Maximum crushing load
r_i	:	Distance of center of ith composite layer to neutral axis
R_0	:	Average radius of metal tube
s	:	Distance
S	:	In-plane shear strength
SAE	:	Specific absorbed energy
t_c	:	Thickness of composite wall
t_f	:	Thickness of each composite layer
t_h	:	Thickness of hybrid tube
t_m	:	Thickness of metal wall
t_r	:	$=t_c/t_m$
W_b	:	Absorbed bending energy
W_{sm}	:	absorbed extensional energy in metal wall
W_{sc}	:	absorbed extensional energy incomposite wall
x	:	Neutral axis
X_c	:	compressive strength in fibre direction
X_t	:	tensile strength in fibre direction
Y_c	:	compressive strength in perpendicular to fibre

Y_t : tensile strength in perpendicular to fibre

Greek symbols

α : Rotation angle of fold
δ_e : Effective crushing length
$\varepsilon_{\varphi i}$: Strain of ith composite layer in hoop direction
$\varepsilon_{\varphi i}^f$: Fracture strain of ith composite layer in hoop direction
η_e : Crush force uniformity factor
η_f : Energy efficiency
θ_i : Angle of fiber orientation angle of ith composite layer to the tube axis
ρ : Density
σ_{0m} : flow stress of the metal tube
σ_{eq} : Equivalent strength of hybrid section
σ_u : Ultimate strength of metal
σ_{zi} : Strength of ith composite layer in the direction of tube axis
σ_{zi}^f : Fracture strength of ith composite layer in the direction of tube axis
$\sigma_{\varphi i}$: Fracture strength of ith composite layer in hoop direction
$\sigma_{\varphi i}^f$: Fracture strength of ith composite layer in hoop direction

چکیده

در این مقاله با درنظر گرفتن شیوه فروریزش چیندار برای لوله‌های استوانه‌ای فلزی- مرکب و استفاده از معیار شکست سای - هیل در محاسبه مقاومت لایه‌های دیواره مرکب، رابطه‌ای تحلیلی جهت محاسبه متوسط نیروی لهیدگی و طول چین لوله‌های استوانه‌ای فلزی مرکب تحت فروریزش محوری شبه‌استاتیک ارائه می گردد. در این مدل، تأثیر پارامترهای هندسی لوله شامل قطر، ضخامت دیواره‌های فلزی و مرکب، جهت الیاف در هر لایه، خواص مکانیکی ماده فلزی و مرکب روی متوسط نیروی لهیدگی و طول چین بطور تحلیلی ارائه می‌شود. مقایسه نتایج تحلیلی مدل حاضر با کارهای منتشر شده قبلی نشان می دهد این مدل علاوه بر توانایی تخمین متوسط نیرویی لهیدگی بر حسب زاویه الیاف در هر لایه، از دقت قابل قبولی نیز برخوردار می باشد.