ISME

# A Linear Approach to the Control of Vortex Induced Vibrations of Circular Cylinders with a 2-D Van der Pol Model for Structural Oscillator

**A. Farshidianfar***
Professor

**N. Dolatabadi†**
M.Sc. Graduate

*In the present paper, a new 2-D Van der Polstructural oscillator model is introduced for the vortex induced vibrations of circular cylinders.The main purpose of this task is to control the recently introduced model by means of modern control definitions in state space. In order to control the system, the whole model is linearized about its equilibrium point by deriving state-space matrices. Then, the linear transfer function is obtained and controlled by pole-placement technique which is based on the state variables feedback. Afterwards, this linear controller is applied to the nonlinear system about its equilibrium point by assuming that there is no uncertainty in both physical and mathematical models. Eventually,the results for linear and nonlinear systems are compared.*

*Keywords*: structural oscillator model; VIV; Control; pole-placement.

## 1 Introduction

Vortex induced vibrations (VIV) is a well-known phenomenon to mechanical engineers. When a bluff body is subjected to the fluid flow, the separation of the current introduces the fluctuation of the pressure about the surface of the cylinder. This pressure fluctuation, in turn, results in hydrodynamic forces exerted by the formation of the vortices beyond the body.The environment is rife with potential cases to be subjected to VIV and some of these are: tube bundles in heat exchangers, marine structures such as oil production risers and cables, bridges, power transmission lines, chimneys, tall buildings, aircraft control surfaces, etc. As far as VIV is concerned, the interaction between structures and fluids turns to be significantly important when we are studying infrastructures which are strategically and financially critical.

According to Sarpkaya [1], Gabbai and Benaroya [2], this self-regulated nonlinear phenomena is a six degree of freedom vibration which is reduced to often one degree of freedom in the transverse direction to the flow. There are few theoretical literature even for two dimensional problems [3]. Mostly, to study the dominant equation of structural oscillation, the stiffness and damping coefficients are assumed to be linear [2] while the mass varies due to an added mass introduced by the fluid particles attached to the surface of the cylinder during the oscillation.Farshidianfar and Zanganeh [8] were the first who substituted

*Corresponding author, Professor, Mechanical Engineering Department, Engineering Faculty, Ferdowsi University of Mashhad, PO Box 91775-1111, Mashhad, Iran, farshid@um.ac.ir
† M.Sc. Graduate, Mechanical Engineering Department, Engineering Faculty, Ferdowsi University of Mashhad

the linear damping coefficient of a one dimensional structural oscillator with a Van der Pol model and found more accurate results.

Due to VIV, the structures could experience very large transverse vibrations which yield the failure of them. The increase in reduced velocity from about 4 to 8 results in lock-in and consequently inthe maximum amplitudefor a sinusoidal force on the cylinder which is depicted in Figure 1 as 2P region [1,3,4]. Since it is impossible to avoid the variation of ambient velocity in off-shore circumstances, the control of VIV becomes highlighted.The control of vortex induced vibrations or even the suppression of them becomes significantly important in practical situations, especially in deepwater offshore structures for which fault diagnosis and repair could be expansively difficult and expensive.

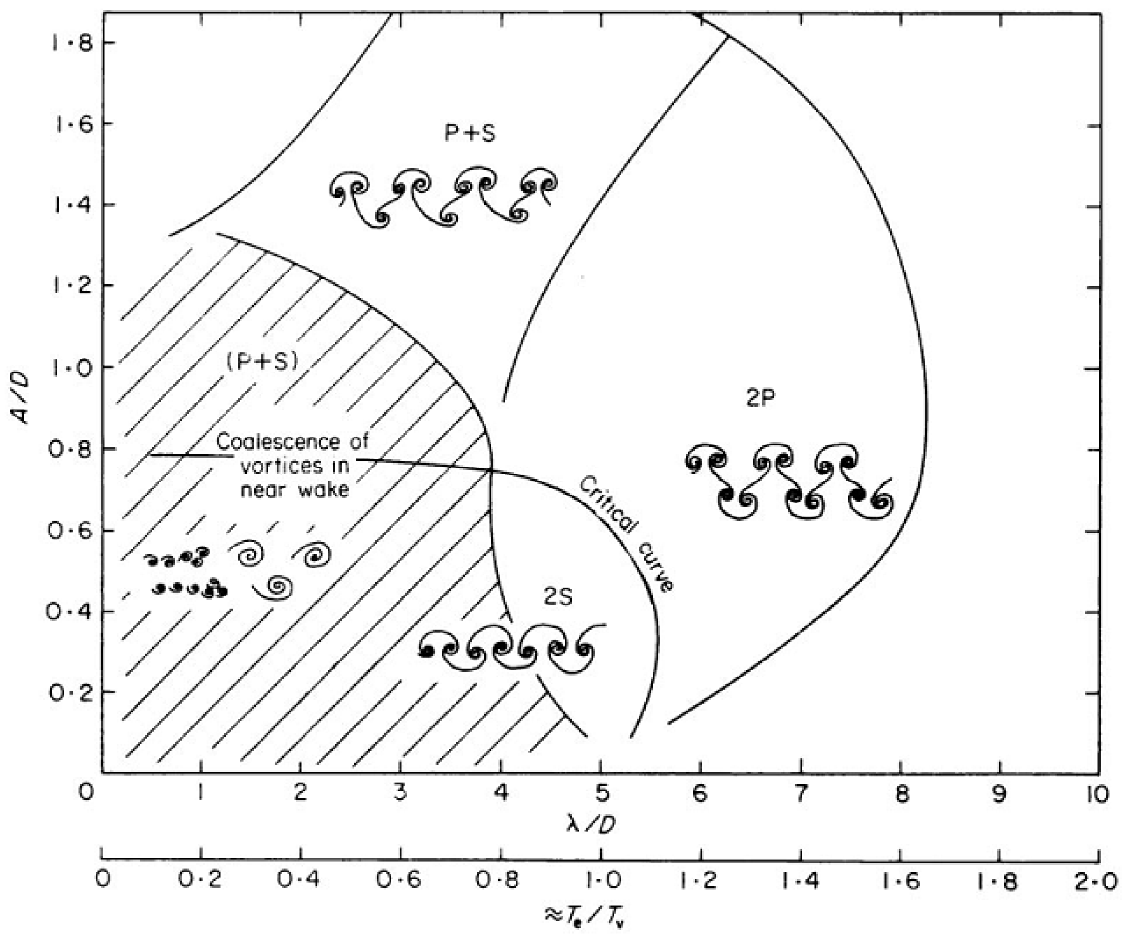


**Figure 1** Map of vortex patterns of a vertical cylinder forced to move along a sinusoidal path in the Re range of 300<Re<1000 [1,3,4]

Generally speaking, the control of such a phenomenon can be divided into two typical categories. The first is passive and the second is active. There are many workable and practical passive solutions to the suppression of VIV, each of which is, to different extent, sufficient. Kumar published an overview on these passive control techniques such as utilization of streamline fairings, helical strakes, perforated surfaces, etcand considered the effectiveness of each method inthe damping of drag and lift forces [3]. Active control, in contrary to passive one, whichis always available and at service, just evinces reactions when the external disturbance is available. In other words the system processes the signals and takes the proper action at a proper time. There are different approaches to active control and the most popular ones are adaptive and robust control methods.

In the upcoming work, we define a new two dimensional Van der Pol damping term and we apply it to a two dimensional structural oscillator. After the linearization of dominant equations, we also design an active controller for the linear form of the nonlinear system. The parameters of the desired controller are designed for the linear model in advance, and then applied to the nonlinear one. The results for both linear system and nonlinear system are considered and compared about the equilibrium point and the uncertainties are neglected. Design and utilization of a linear controller on nonlinear systems can save time, costs and obviate the need for controller.

## 2 VIV model

In this section, to control VIV, initially, the model of structural oscillator will be described. In most cases, for simplicity, the VIV model which could be six degree of freedom is reduced to a one degree of freedom problem [1]. The simplified one degree of freedom equation for the oscillation ofa circular cylinder in linear form is [6]:

$$m\ddot{y}+c\dot{y}+ky=F_1(y,t) \tag{1}$$

According to Ghanem [7], we are able to expand the one dimensional equation of motion to a two dimensional model with the same dominant structural equation in the second direction, but having different amplitudes and time parameters which can be written as it is in equation 2. Figure 2 shows the 2-D structural model of VIV with linear parameters for this model.

$$\begin{cases} m\ddot{y}+c\dot{y}+ky=F_1(y,t)=F_1 \\ m\ddot{x}+c\dot{x}+kx=F_2(x,t)=F_2 \end{cases} \tag{2}$$

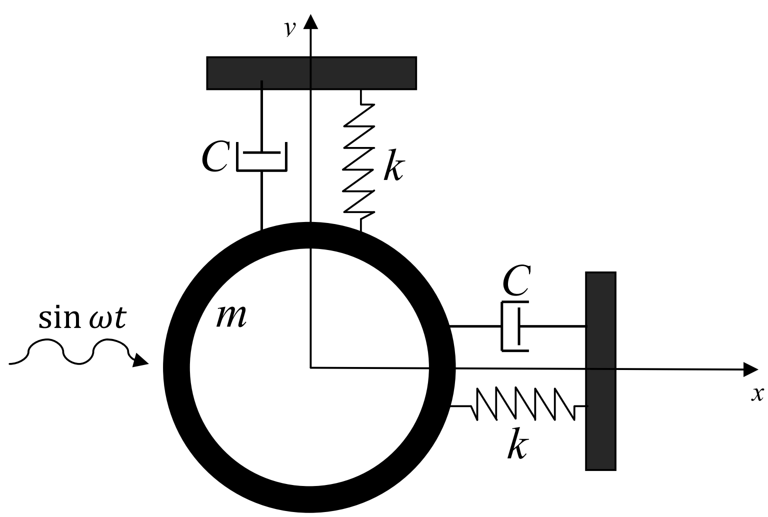


**Figure 2** A 2-D model for the Vortex Induced Vibrations

A Van der Pol oscillator model can be used as the damping term for the structural equation of motion and it is investigated that the new model not only has less discrepancies with respect to the experimental data but also follows it adjacently [8]. Applying this model to the dominant equations, we have:

$$\begin{cases} m\ddot{y}+\varepsilon(y^2-1)\dot{y}+ky=F_1 \\ m\ddot{x}+\varepsilon(x^2-1)\dot{x}+kx=F_2 \end{cases} \tag{3}$$

The mentioned equations above are uncoupled and can be solved separately for each direction. Here, a new two dimensional Van der Pol model is introduced by using the following equation which is introduced by Kaplan [9].

$$\frac{dx}{dt}=\frac{1}{\mu}\left(y-\frac{x^3}{3}+x\right) \tag{4}$$

We can differentiate the equation once and by means of the change of variable it can be rewritten in the form shown in equation 5 [10].

$$\frac{d}{dt}\left[\dot{x}-\frac{1}{\mu}\left(y-\frac{x^3}{3}+x\right)\right]=0 \tag{5}$$

Thus, by expanding the differential equation, it is reshaped as equation 6 where $1/\mu$ can be assumed to be ε.

$$\ddot{x}+\varepsilon\left[x^2-1\right]\dot{x}+\varepsilon\dot{y}=0 \tag{6}$$

The newly introduced Van der Pol term is expressed as equation 7.

$$\varepsilon\left[(x^2-1)\dot{x}+\dot{y}\right] \tag{7}$$

Applying the new Van der Pol model to the dominant equations with the same approach used by Farshidianfar and Zanganeh [8], the new 2D coupled system is obtained in the form of equation 8.

$$\begin{cases} m\ddot{y}+\varepsilon[(y^2-1)\dot{y}+\dot{x}]+ky=F_1 \\ m\ddot{x}+\varepsilon[(x^2-1)\dot{x}+\dot{y}]+kx=F_2 \end{cases} \tag{8}$$

$F_1$and$F_2$ are vortex induced disturbances on the system and are obtained by the solution of the wake oscillator equation.The wake oscillator equation is the dominant oscillatory dynamic model of vortices which is defined by lift and drag coefficients. Generally, the solution to the wake oscillator is assumed to be a harmonic sinusoidal function [11]; thus, we assume $F_1$ and $F_2$ are sinusoidal functions as following.

$$F_1=A_1 \sin \omega_1 t \text{ and } F_2=A_2 \sin \omega_2 t \tag{9}$$

The vortex-excited oscillations reinforce the vortex strength when the amplitude in the transverse direction exceeds a threshold of about 0.1D [12,13]. It is about 0.02D for the in-line oscillations [14,15,16]. Here D is the diameter of the circular cylinder about which the vortex shedding occurs. The ratio of the in-line displacement amplitude to the transverse amplitude is, $A_2/ A_1$= 19% [17].

We assume that the disturbances in both directions are in-phase and running at the same frequencies which means $\omega_1 = \omega_2$ and $\phi = 0$. As a purpose of simplification and normalization, we assume that $A_1 = 1$ which results in $A_2 = 0.2$. Therefore, we have

$$\sin \omega t = U(t) \tag{10}$$

$$F_1=U(t) \text{ and } F_2=0.2U(t) \tag{11}$$

## 2 Linearization

### 2.1 State space matrices

In this section, we first derive the state variables by defining $x = x_1$ and $\dot{x} = \dot{x}_1 = x_2$ in x-direction and $y = x_3$ and $\dot{y} = \dot{x}_3 = x_4$ in y-direction, and then the Jacobian derivativesare applied to state space matrices in order to make equations linear. The state variables are

$$\begin{cases} \dot{x}_1 = x_2 = f_1(t) \\ \dot{x}_2 = \frac{1}{m}\left[0.2U(t) - \varepsilon(x_1^2 - 1)x_2 - \varepsilon x_4 - kx_1\right] = f_2(t) \\ \dot{x}_3 = x_4 = f_3(t) \\ \dot{x}_4 = \frac{1}{m}\left[U(t) - \varepsilon(x_3^2 - 1)x_4 - \varepsilon x_2 - kx_3\right] = f_4(t) \end{cases} \tag{12}$$

The state space equations are in the form of equation 13 for which A, B, C and D matrices should be defined.

$$\begin{cases} \dot{X} = AX + Bu \\ Y = CX \end{cases} \tag{13}$$

We write the Jacobian about the equilibrium point of $X_0$ to obtain matrices $A$ and $B$as following. Without loss of generality, we can assume that the final state is the origin of the state space [18].

$$X_0 = [0\ 0\ 0\ 0]$$

$$J_x = \frac{\partial f_i}{\partial x_j} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{m}[-2\varepsilon x_1 x_2 - k] & \frac{1}{m}[-\varepsilon(x_1^2 - 1)] & 0 & -\frac{\varepsilon}{m} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{\varepsilon}{m} & \frac{1}{m}[-2\varepsilon x_4 - k] & \frac{1}{m}[-\varepsilon(x_3^2 - 1)] \end{bmatrix} \tag{14}$$

Applying $X_0$ to $J_x$ in equation 14, the simplified Jacobian matrix is derived as equation 15 which represents the A matrix in state space about the equilibrium point.

$$J_x[0] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m} & \frac{\varepsilon}{m} & 0 & -\frac{\varepsilon}{m} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{\varepsilon}{m} & \frac{-k}{m} & \frac{\varepsilon}{m} \end{bmatrix} \tag{15}$$

The same process is done to derive $J_u$ which represents B matrix about the equilibrium point.

$$J_u = \frac{\partial f_i}{\partial U} = \begin{bmatrix} 0 \\ \frac{0.2}{m} \\ 0 \\ \frac{1}{m} \end{bmatrix} \tag{16}$$

By applying matrices derived from Jacobian operation to the state space equations, the system will be in the form of differentials which introduces the linear form of it about the equilibrium point [19]:

$$\Delta\dot{x}(t) = J_x(x_0, u_0, t)\Delta x(t) + J_u(x_0, u_0, t)\Delta u(t) \tag{17}$$

$$\Delta\dot{x}(t)=A(t)\Delta x(t)+B(t)\Delta u(t) \tag{18}$$

Since the equilibrium point is located at the origin, it can be easily proven that the Δ's are equal to the main parameters and finally, the state space equation is obtained as it is in equation 19.

$$\Delta x=x-x_0=x-0=x \xrightarrow{yields} \Delta\dot{x}=\dot{x}-0=\dot{x} \text{and} \Delta u=u-0=u$$

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{19}$$

Since we have selected $x = x_1$and $y = x_3$, in the output, the matrix C can be defined asit is in equation 20.

$$y=Cx \xrightarrow{yields} y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \tag{20}$$

Thus, there exists a single input, multi-output (SIMO) model whose state space matrices, in parametric form, are shown by matrices 21.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m} & \frac{\varepsilon}{m} & 0 & -\frac{\varepsilon}{m} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{\varepsilon}{m} & -\frac{k}{m} & \frac{\varepsilon}{m} \end{bmatrix} and B = \begin{bmatrix} 0 \\ \frac{0.2}{m} \\ 0 \\ \frac{1}{m} \end{bmatrix} \tag{21}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

### *2.2 Defining parameters*

For the control of the linear system, we have obtained the required matrices for the state space in terms of dominant parameters. In this part, we have to change the matrices into numerical form. These physical parameters are $\epsilon$, $m$ and $k$ that must be quantified. According to Farshidianfar and Zanganeh[8] and Facchinetti[20], the nonlinearity coefficient ε is rather constant and equal to 0.3. Figure 3 from Zanganeh's paper depicts that the theoretical data with ε = 0.3 complies experimental data with a fairly good approximation.

Stainless steel zeron 100 is assumed to be the preferred material for riser[21]. Based on the stiffness and density as well as the assumed dimensions, it is rational to normalize mass and stiffness by the proportion of mass over stiffness coefficient of

$$m=1.0 \text{ and} k=10$$

By substituting these values into the matrices from equations 21, the final state-space matrices are:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -10 & 0.3 & 0 & -0.3 \\ 0 & 0 & 0 & 1 \\ 0 & -0.3 & -10 & 0.3 \end{bmatrix} \text{and} B=\begin{bmatrix} 0 \\ 0.2 \\ 0 \\ 1 \end{bmatrix}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

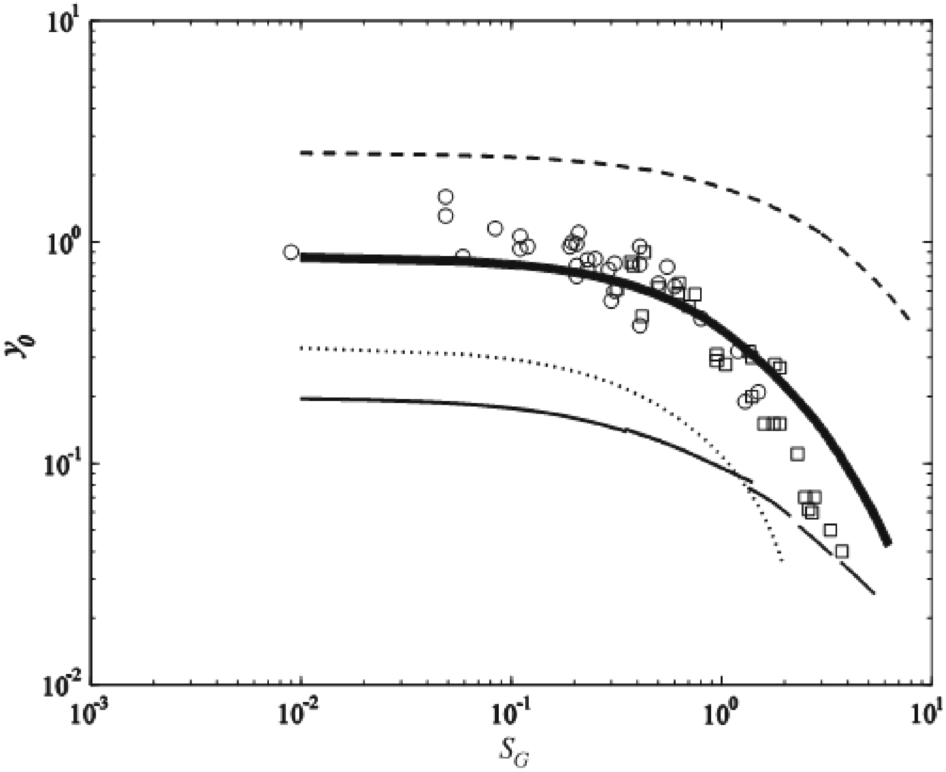


**Figure 3** Structural oscillation amplitude at lock-in as a function of $S_G$. −, Classical wake oscillator model.Results of the modified wake oscillator model: - - -; $\epsilon = 0.1$, , $\epsilon = 0.3$; …, $\epsilon = 0.5$. Empirical data in water: ○, Skop and Balasubramanian [22]. Empirical data in air: □,Skop and Balasubramanian [22].

*2.3 Derivation of the transfer function*

There are two transfer functions between the input and outputs since the system is SIMO with two outputs. Transfer function number one reveals the relation between the input and the output in the x-direction. Transfer function number two connects output in y-direction to the input. These transfer functions are as it is in equation 22.

$$\text{tf output\#1}=\frac{0.2s^2-0.36s+2}{s^4-0.6s^3+20s^2-6s+100} \tag{22}$$

$$\text{tf output\#2}=\frac{s^2-0.36s+10}{s^4-0.6s^3+20s^2-6s+100}$$

Now, it is desirable to study the stability of the system. Therefore, the poles of the plant are calculated and are as following for both transfer functions:

0.0+3.1623i, 0.0-3.1623i, 0.3+3.148i, 0.3-3.148i

There are two poles on the imaginary axis of the S-plane which mea**n**s the system is critically stable. In the upcoming section, the stability of the main system is studied in detail.

## 3 Control of the system

Prior to the control studies, we need to investigate the stability of system on its own. For this purpose, it is essential to consider the state controllability and output controllability beforehand to find out whether it can be controlled by simple solutions such as feedback or not.

$$M=[B|AB|A^2B|A^3B]=\begin{bmatrix} 0 & 0.2 & -0.24 & -2.144 \\ 0.2 & -0.24 & -2.144 & 4.7136 \\ 0 & 1 & 0.24 & -9.856 \\ 1 & 0.24 & -9.856 & -4.7136 \end{bmatrix} \quad (23)$$

$$|M|\neq 0$$

And the rank of controllability matrix in equation 23 is 4 which means it is completely state controllable.

$$\text{Rank}(M)=4$$

Output controllability is investigated by the matrix F in equation 24.

$$F=[CB \quad CAB \quad CA^2B \quad CA^3B \quad D] \quad (24)$$

$$\begin{bmatrix} 0 & 0.2 & -0.24 & -2.144 \\ 0 & 1 & 0.24 & -9.856 \end{bmatrix}$$

$$2C_{m\times n}=C_{2\times 4} \xrightarrow{\text{yields}} m=2$$

Rank of the F should be equal to the number of outputs which is interpreted as output controllability.

$$\text{rank}(F)=2$$

### *3.1 Investigations on linearized system*

The behavior of the main nonlinear system, the same as all Van der Pol models, is a limit cycle oscillation. Having linearized the system, it is expected not to have this behavior any more. Figure 4 shows the block diagram of the linearized system. The input of the system which represents the stable condition is zero and a sinusoidal disturbance equal to $F_1$ and $F_2$, as calculated beforehand, are exerted.

The state variables are investigated and their waveforms are depicted in the graph shown in Figure 5. It is clear that state variables are divergent due to the linearization effects, while they are supposed to follow a limit cycle waveform.

Since the system has two outputs, displacementsare depicted in both x and y-directions in Figure 6. It is easily understood that the linear system is unstable and under both linear and nonlinear conditions, the design of a controller is required.

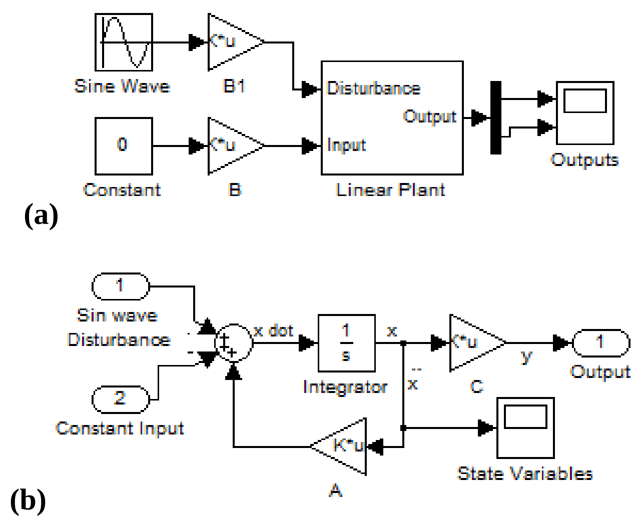


**Figure 4** (a) Block diagram for the main linear system without controller.
(b) Contents of linear plant block in (a)

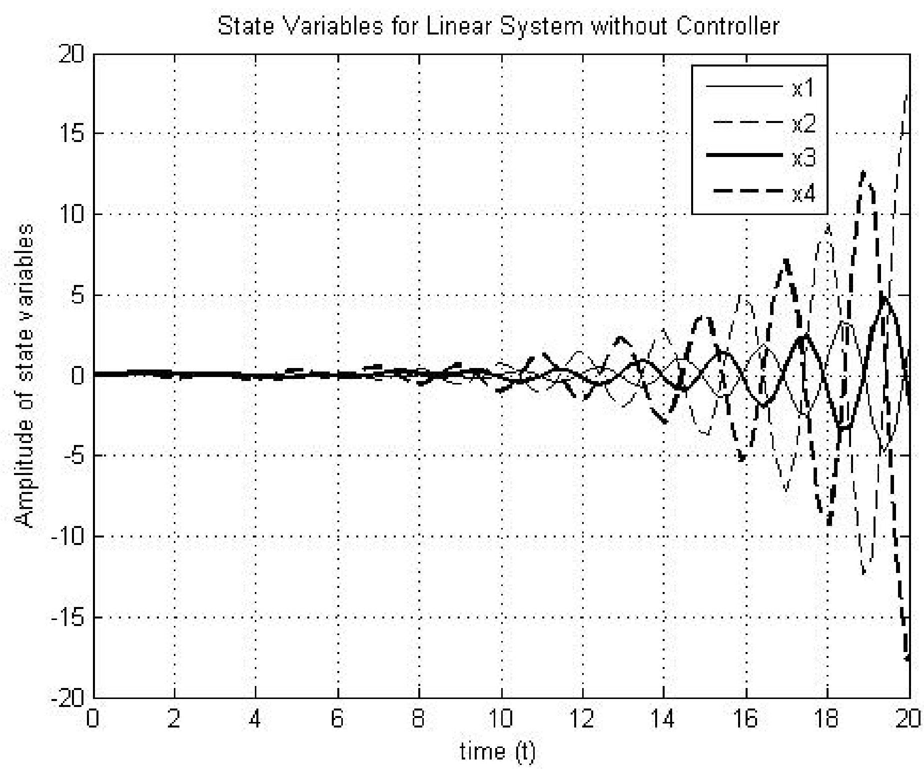


**Figure 5** State variables for the linear system without controller

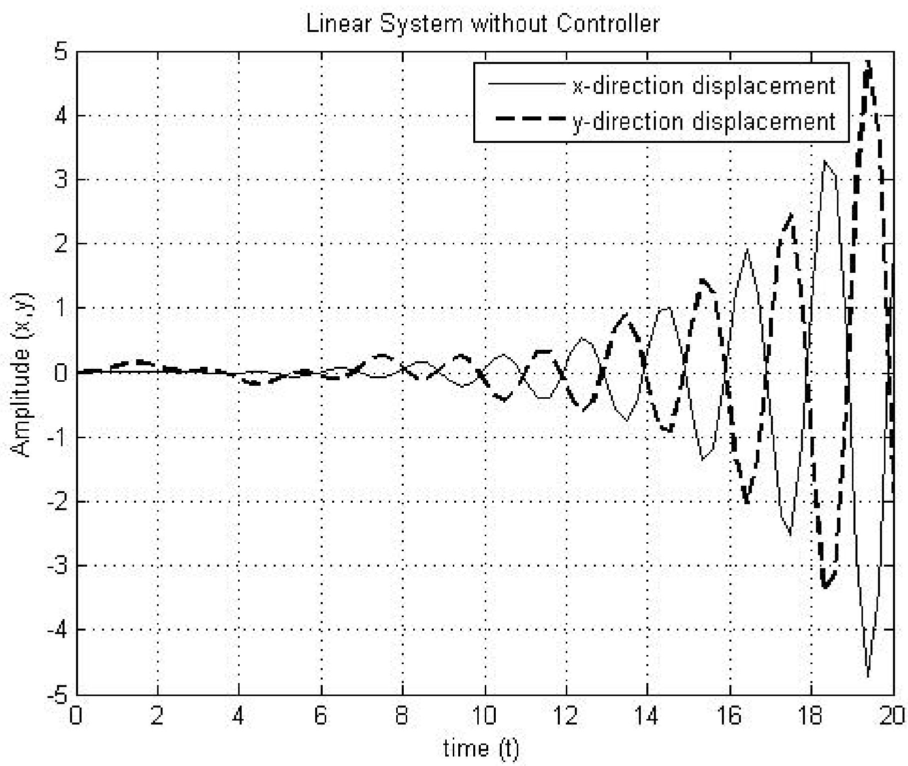


**Figure 6** Out puts in both x and y directions for linear system without controller

### *3.2 Control of linear system applying pole placement technique*

Pole placement technique is the utilization of state feedback on the system state variables. For achieving this goal, desired poles are needed. The desired poles are those selected by designer to stabilize the system by introducing a gain matrix K for feedback controller. These poles, for this case, are picked as shown.

$$P=[-8+4i \quad -8-4i \quad -20 \quad -5]$$

The corresponding characteristic equation for the desired poles is shown in equation 25 where coefficients $\alpha_i$ are used for designing gain matrix *K*.

$$s^4+\alpha_1 s^3+\alpha_2 s^2+\alpha_3 s+\alpha_4=s^4+41s^3+580s^2+3600s+8000 \tag{25}$$

And the characteristic equation for the main system is that of equation 26.

$$|sI-A|=s^4+a_1 s^3+a_2 s^2+a_3 s+a_4=s^4-0.6s^3+20s^2-6s+100 \tag{26}$$

By the use of Ackerman or placement technique [13] to obtain the gain matrix of K, the transformation matrix T is defined in equation 27.

$$T=MW \tag{27}$$

Where M is the controllability matrixin equation 23 and W is the weighting matrix (equation 28) which is defined by coefficients of the characteristic equation,$a_i$'s from equation 26. Using *T* and defined coefficients, we can obtain gain matrix *K* from equation 29.

$$W=\begin{bmatrix} a_1 & a_2 & a_3 & 1 \\ a_2 & a_3 & 1 & 0 \\ a_3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (28)$$

$$K=[\alpha_4\text{-}a_4 \quad \alpha_3\text{-}a_3 \quad \alpha_2\text{-}a_2 \quad \alpha_1\text{-}a_1].T^{-1} \quad (29)$$

$$K=10^4\times[-1.2064 \quad 0.0747 \quad 0.3203 \quad -0.0108]$$

The block diagram of the system for state feedback controller is depicted in Figure7. Figure8 shows the waveform of the input disturbance and outputs of the system simultaneously. Here, it is clear that the output is suitably controlled and the disturbance is damped when the input signal amplitude is 1 and the outputs lay on zero. The state variables are plotted in Figure 9 and those related to y-direction have a larger amplitude than the ones in x-direction as expected. In Figure 10, the output waveformsare considered in detail. The almost straight linesin Figure 8are still in the form of a sinusoidal wave with diminutive amplitude of about $1\times10^{-3}$. This amplitude is smaller in the x-direction as well and is about $0.25\times10^{-3}$. Because this amplitude is trivial compared to the sinusoidal input disturbance, it is rational to neglect these deviations and consider the system fully controlled.

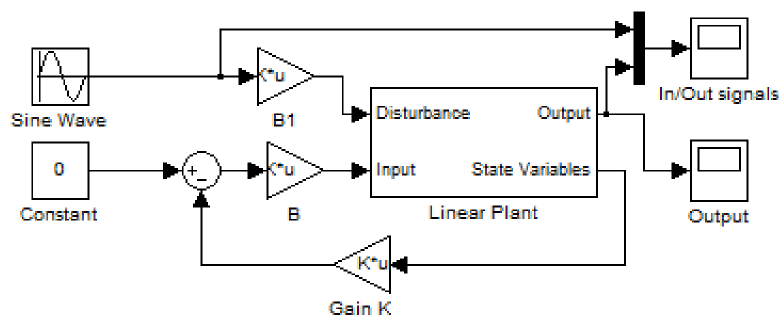


**Figure 7** the block diagram for linear system which is controlled by pole placement technique in the presence of a sinusoidal disturbance and gain matrix K which is defined by Placement or Ackerman technique

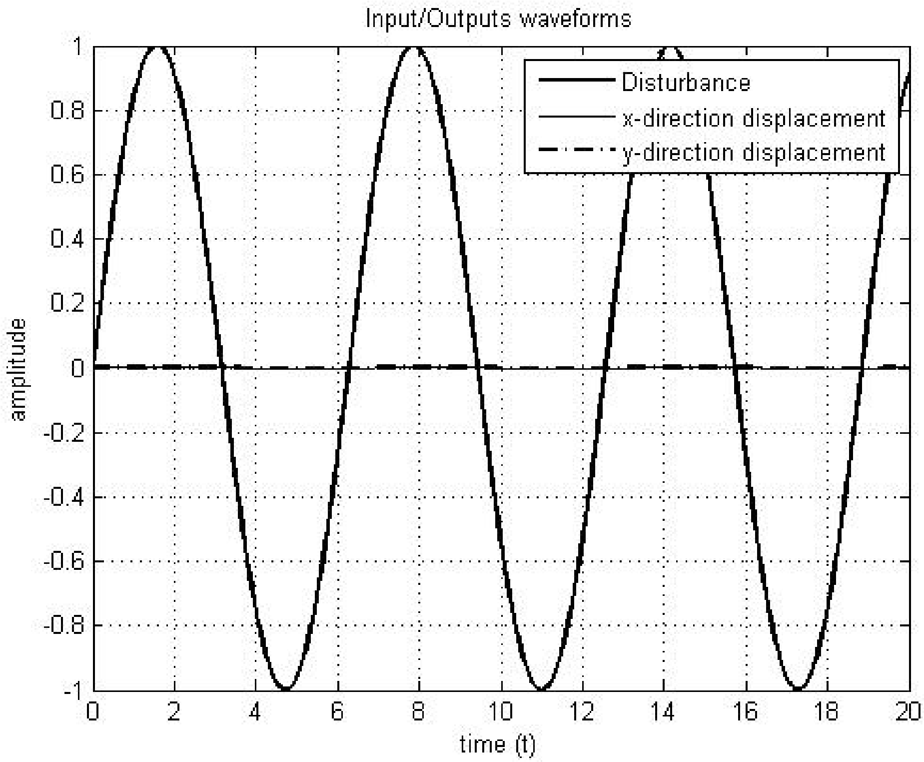


**Figure 8** Common plot of sinusoidal disturbance as an input and outputs in both x and y-directions for a linear model controlled by pole placement technique: the outputs are almost zero and are depicted as straight lines

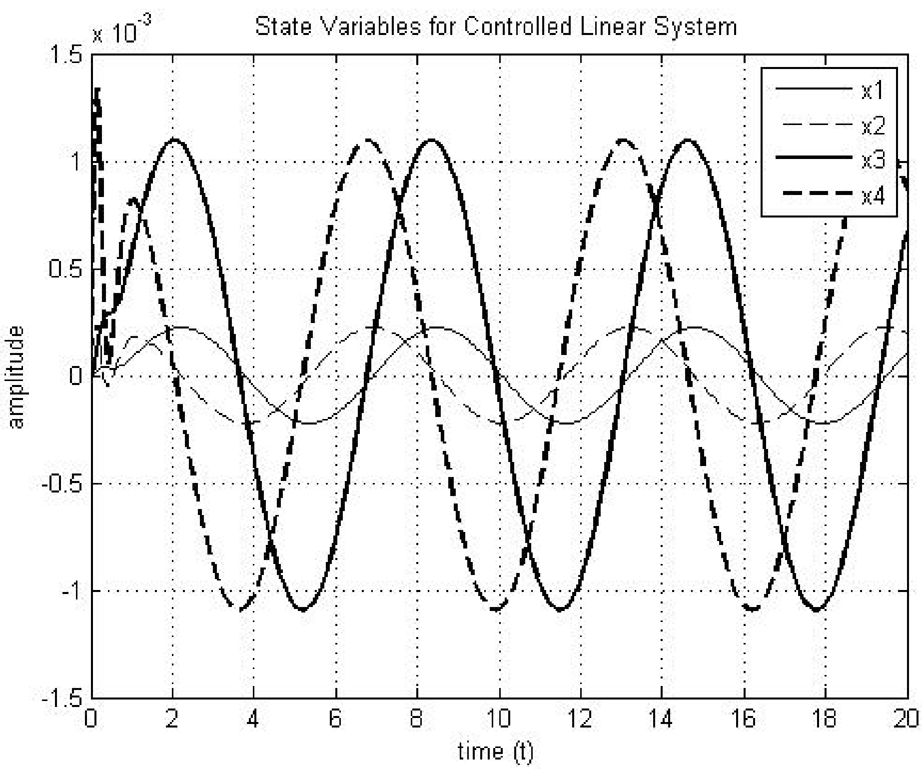


**Figure 9** State variables for linear model controlled by pole placement technique

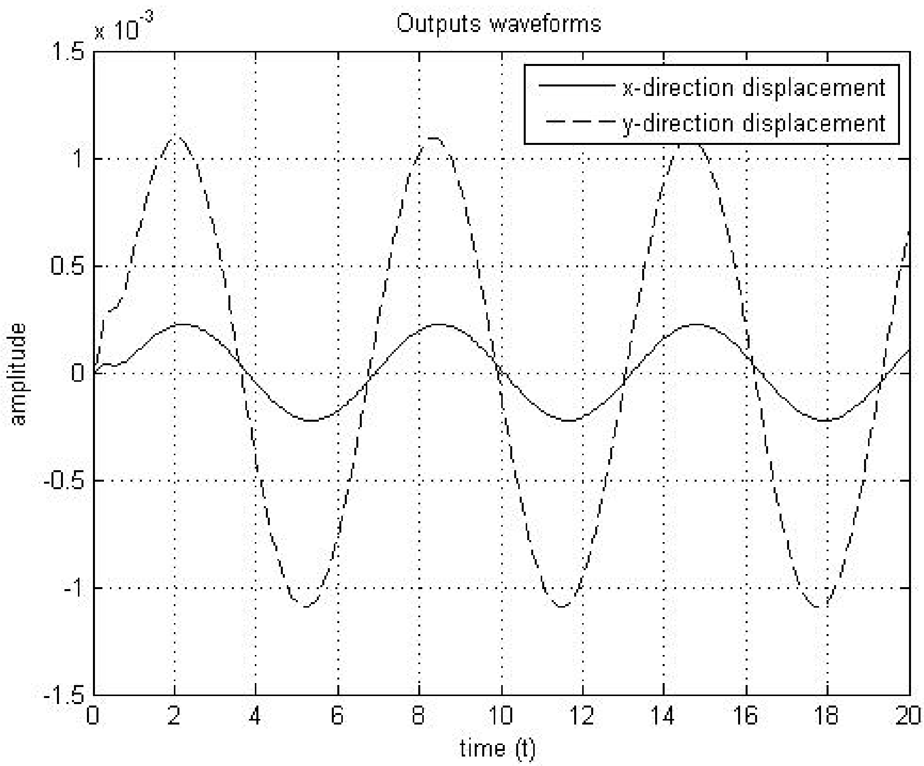


**Figure 10** the outputs in both x and y-directions for linear model controlled by pole placement technique: the maximum amplitude of output signals is about 0.001 which can be assumed as a straight line in comparison with input sinusoidal disturbance

### *3.3 Application of the linear controller on the main nonlinear system*

As most systems have a nonlinear behavior in nature, design and manufacturing of appropriate controllers for nonlinear phenomena could be expensive, time-consuming and intricate; Therefore, Application of linear controllers on nonlinear systems can help, provided they can control the system acceptably. As discussed before, one of common methods is robust control which is based on uncertainties. The approach of this context is the same although the uncertainties are not involved. The linear state feedback controller, which is designed by pole placement technique, is applied to the nonlinear plant as depicted in Figure 11.

State variables are depicted in figure 12. Finally, the output signals in both x and y-directions are plotted in Figure 13. It is clearly understood that the nonlinear system is suitably controlled by the linear controller. To prove the sufficiency of linear controller on the nonlinear system, the so-called error graph is plotted and depicted in figure 14. It is understood that the application of this linear controller on the nonlinear system has been appropriately sufficient, since the deviation of outputs for the nonlinear system from the outputs of the controlled linear system is less that 0.033%.

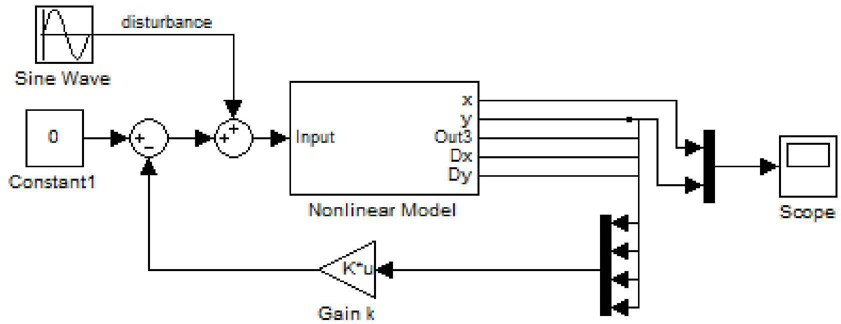


**Figure 11** the block diagram for nonlinear model with linear controller designed by pole placement technique

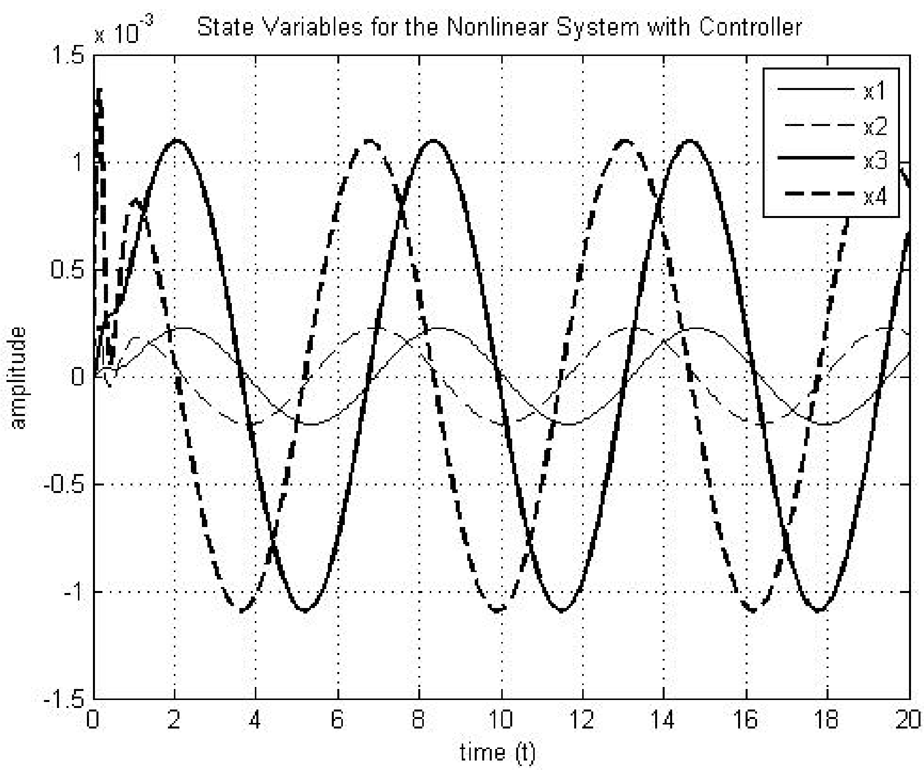


**Figure 12** state variables for nonlinear system with controller obtained from pole placement method

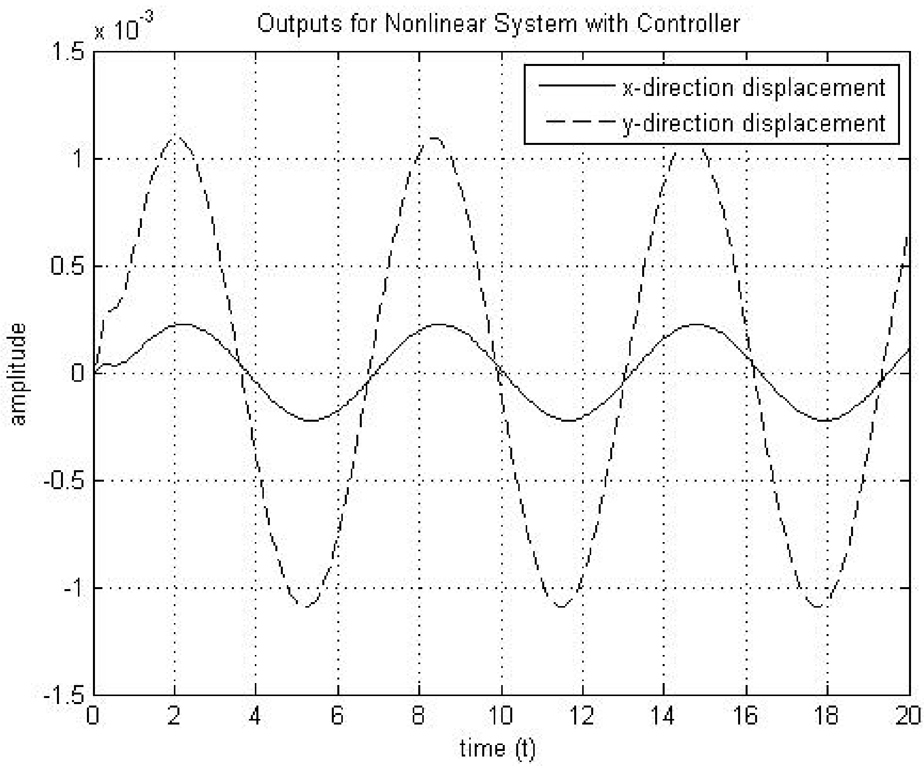


**Figure 13** Outputs of nonlinear model in both x and y-directions controlled by linear controller

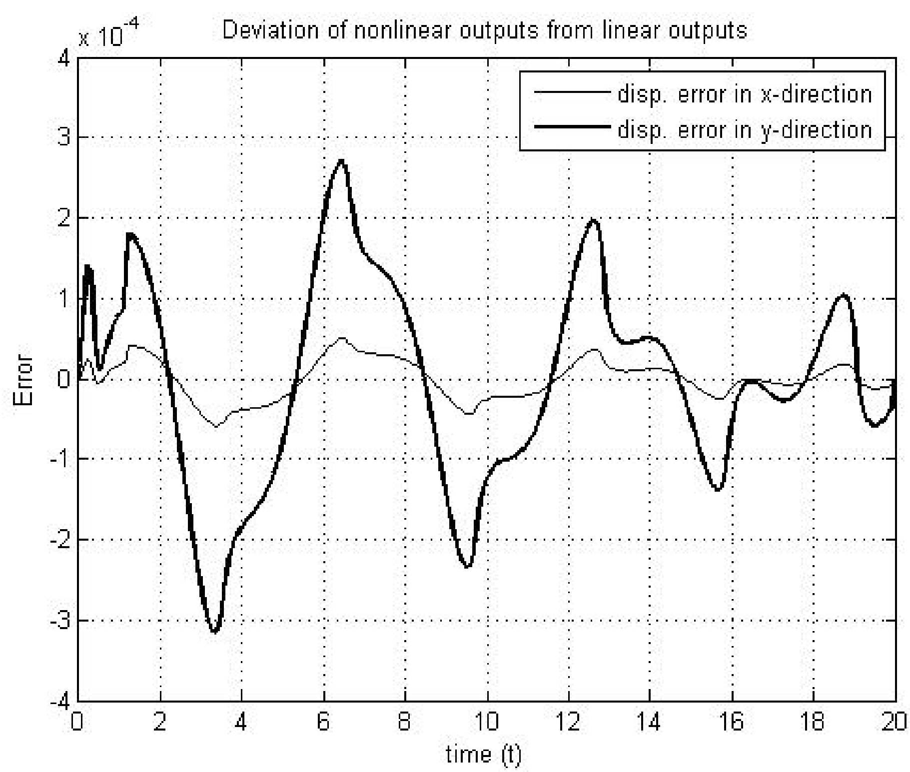


**Figure 14** Deviation of nonlinear system outputs from controlled linear system outputs

## 5 Conclusion

VIV can lead to extreme structural failures if not controlled, especially in harsh environments such as oceans. In this paper a new 2D structural oscillator is introduced which let us model the system with two coupled equations. As the design of nonlinear controllers may bring about extra cost and consume more time, we try to design a linear controller which fits and controls the nonlinear system fairly. Prior to the design of the controller, the linearization process is performed. Then, linear controller, based on the linear system, is designed and considered. Having controlled the linear model, the linear controller is applied to the nonlinear system. As a comparison, it is investigated that both linear and nonlinear systems are toa great extent controlled in which the vibration amplitude converges to zero. It can be concluded that, in the absence of uncertainties, application of linear controllers can reduce the cost, time and complexity and still be functional.

## References

[1] Sarpkaya, T., "A Critical Review of the Intrinsic Nature of Vortex-induced Vibrations", Journal of Fluids and Structures, Vol. 19, pp. 389–447, (2004).

[2] Gabbai, R. D., and Benaroya, H., “An Overview of Modeling and Experiments of Vortex-induced Vibrations of Circular Cylinders”, Journal of Sound and Vibration, Vol. 282, pp. 575-616, (2005).

[3] Williamson, C.H.K., and Govardhan, R., "A Brief Review of Recent Results in Vortex-induced Vibrations", Journal of Wind Engineering and Industrial Aerodynamics, Vol. 96, pp. 713-735, (2008).

[4] Williamson, C.H.K., and Govardhan, R., "Vortex-induced Vibration", Annu. Rev. Fluid. Mech, Vol. 36, pp. 413-455, (2004).

[5] Kumar, R. A., Chan-Hyun S., and Gowda B. H.L., "Passive Control of Vortex-induced Vibrations: An Overview", Recent Patents on Mechanical Engineering, Vol. 1, pp. 1–11, (2008).

[6] Carbonell, P., Xiaodong, W., and Zhong-ping, J., "On the Suppression of Flow-induced Vibration with a Simple Control Algorithm", Communications in Nonlinear Science and Numerical Simulation, Vol. 8, pp. 49-64, (2003).

[7] Ghanem, R., and Gattulli, V., "Adaptive Control of Flow-induced Oscillations Including Vortex Effects", International Journal of Non-linear Mechanics, Vol. 34, pp. 853-868, (1999).

[8] Farshidianfar, A., and Zanganeh, H., "A Modified Wake Oscillator Model for Vortex-induced Vibration of Circular Cylinders for a Wide Range of Mass-damping Ratio", Journal of Fluids and Structures, Vol. 26, pp. 430-441, (2010).

[9] Kaplan, D., and Glass, L., "*Understanding Nonlinear Dynamics*", Springer-Verlag New York, Inc, ISBN 0-387-94440-0, pp. 240–244, (1995).

[10] Nayfe, A.H., "*Introduction to Perturbation Techniques*", John Wiley and Sons, Inc, pp. 147–159, New York, (1993).

[11] Fossen, T.I., "*Guidance and Control of Ocean Vehicles*", John Wiley and Sons, pp.73–76, New York, (1994).

[12] Davies, M.E., “A Comparison of the Wake Structure of a Stationary and Oscillating Bluff Body using a Conditional Averaging Technique”, Journal of Fluid Mechanics, Vol. 75, pp. 209-231, (1976).

[13] Atsavapranee, P., Benaroya, H., and Wei, T., “Vortex Dynamics in the Near Wake of a Freely-oscillating Cylinder”, ASME Fluids Engineering Division, Summer Meeting, Washington, DC, pp.1-6, (1998)

[14] Mandini, R.V., “Kinematics of Vortices in the Early Wake of Vibrating Circular Cylinders”, Master’s Thesis, University of California, Berkeley, CA, USA, (1961).

[15] Sainsbury, R.N., and King, R., “The Flow Induced Oscillation of Marine Structures”, Proceedings of the Institution of Civil Engineers, Vol. 49, pp. 269-302, (1971).

[16] Sugimoto, T., Saito, S., Matsuda, K., Okajima, A., Kiwata, T., and Kosugi, T., “Water Tunnel Experiments on In-line Oscillation of a Circular Cylinder with a Finite Span Length”, Conference on Bluff Body Wakes and Vortex-induced Vibrations, (BBVIV3), Port Douglas, Australia, (2002).

[17] Williamson, C.H.K., and Jauvtis, N., "The Effects of Two Degrees of Freedom on Vortex-induced Vibration at Low Mass and Damping", J. Fluid. Mech., Vol. 509, pp. 23-62, (2004).

[18] Ogata, K., "*Modern Control Engineering*", Prentice-Hall, Inc., 3rd Edition, pp. 786–896, (1997).

[19] Khaki-Sedigh, A., "*Modern Control Systems*", University of Tehran Press 2235,ISBN 9-640-34774-4, pp. 52–57, (2006).

[20] Facchinetti, M.L., de-Langre, E., and Biolley, F., "Coupling of Structure and Wake Oscillators in Vortex-induced Vibrations", Journal of Fluids and Structures, Vol. 19, pp. 123-140, (2004).

[21] WILCO (Oilfield services PTE LTD), Zeron®100 (UNS S32760) SlicklineData Sheet, www.weir.co.uk.

[22] Skop, R.A., and Griffin, O.M., “A New Twist on an Old Model for Vortex-excited Vibrations”, Journal of Fluids and Structures, Vol. 11, pp. 395-412, (1997).

## Nomenclature

$a_i$ : coefficients of characteristic equation
$A$ : coefficient matrix for state variables in state space
$A_i$ : amplitude of force term onstructural oscillator
$B$ : coefficient matrix for input and disturbance signals
$c$ : damping term in linear oscillator model
$C$ : state variables' coefficient in output
$D$ : diameter of the cylinder
$F$ : output controllability matrix
$F_i$ : variable force term on structural oscillator
$J$ : Jacobian matrix
$k$ : stiffness term in linear oscillator model
$K$ : gain matrix for state feedback
$m$ : mass parameter
$M$ : controllability matrix
$P$ : desired poles matrix for pole placement
$t$ : dimensionless time
$tf$ : transfer function
$T$ : transformation matrix
$U$ : common term of force on the structure
$y$ : dimensionless displacement of cylinder in y direction
$W$ : weight matrix for transformation matrix
$x$ : dimensionless displacement of cylinder in x direction
$x_i$ : state variables
$X_0$ : equilibrium point matrix for state variables
$\alpha_i$ : coefficients of characteristic equation for desired poles
$\omega$ : frequency of force term
$\omega_i$ : directional frequency of directional force term
$\varepsilon$ : parameter of van der Pol equation
$\mu$ : inverse of Van der Pol parameter
$\phi$ : phase between lift and drag forces

**چکیده**

در این مقاله، ارتعاشات القایی ناشی از گردابه ها بطور مختصر معرفی می گردد. در ادامه مطلب، مدل جدید و غیرخطی که معادله سازه تحت ارتعاش را بصورت دوبعدی تعریف می نماید بدست آورده شده است. از آنجا که بررسی و طراحی یک کنترلر غیرخطی مناسب که بتواند سیستم های غیرخطی را کنترل نماید زمان و هزینه بیشتری نیاز دارد، کاربرد کنترلرهای خطی مناسبی که بتواند سیستم های غیرخطی را کنترل نماید می تواند بعنوان یک امتیاز محسوب شود. لذا مدل غیرخطی معرفی شده را ابتدا خطی سازی می کنیم و کنترلر خطی مربوط به آنرا با استفاده از فیدبک بر روی متغیرهای حالت طراحی می نمائیم. نهایتا، کنترلر خطی طراحی شده را بر روی سیستم غیرخطی اعمال می نمائیم و کارآیی آنرا مورد بررسی قرار می دهیم.