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Hybrid differential transform-finite difference solution of 2D transient nonlinear annular fin equation

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In the present paper, hybrid differential transform and finite difference method (HDTFD) is applied to solve 2D transient nonlinear straight annular fin equation. For the case of linear heat transfer the results are verified with analytical solution. The effect of different parameters on fin temperature distribution is investigated. Effect of time interval of differential transform on the stability of results has been examined. Results show the excellent capability of HDTFD to solve different engineering problems and also indicate that appropriate selection of differential transform time interval can solve the divergence problem of the method and lead to reduction in computational costs.

Keywords: differential transform, Finite difference, Transient non-linear heat conduction, Time interval of differential transform

1. Introduction

One of the primary ways to enhance heat transfer rate is to use extended surfaces or fins. The heat transfer mechanism of fin is to conduct heat from the heat source to fin surface by its thermal conduction, and then dissipates it to surrounding medium by convection. Most common application areas of fins are in combustion engines, heat exchangers and cooling of electronic equipments.

Early studies about thermal analysis of fins could be found in [1]. Hsiung and Wu [2] used the Laplace transformation and the integral method to study the transient heat transfer in straight fins of various shapes. Cheng and Chen [3] used a hybrid method to study the transient response of annular fins of various shapes subject to constant base temperatures. Arslantruk [4] investigated the optimum dimension of an annular fin with uniform thickness under thermally non-symmetric convective boundary condition. Recently Iborra and Campo [5] studied the temperature distribution and fin efficiency for annular fins of uniform thickness analytically. Lai et al. [6] introduced a discrete

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model for calculating thermal performance of straight annular fin with variable thermal properties.

In the recent years the concept of differential transform method, first introduced by Zhou [7], has received considerable attention as a powerful tool to solve PDEs. This method constructs an analytical solution in the form of a polynomial, with significantly less symbolic computation of the necessary derivatives of data functions, in comparison to Taylor series method. Combination of this method with numerical procedures results in a hybrid differential transform finite difference (HDTFD) method with remarkable computational cost efficiency and excellent solubility of different engineering problems. Bor and Chao [8] used HDTFD to solve nonlinear Burger's equation. Yen et al [9] used the HDTFD to analyze large deflections of orthotropic rectangular plate problem. Chen and Ju [10] applied HDTFD to 1D transient advective-dispersive transport equation. Chu and Chen [11] used HDTFD to solve 1D nonlinear heat conduction problem in Cartesian coordinate system. All mentioned papers have reported the results only for the case with constant time interval of differential transform and the effect of time interval of the method on stability of results has not been reported yet.

In a recent research by Chu and Chen [11], it has been shown that the following 3 parameters control the stability of HDTFD,

1. The number of spatial segments (*dx*),
2. The number of time segments (*dt*), and
3. The order of differential transfer (*k*).

In the above paper, the value of *H* has been assumed to be *H*=1 and the convergent results have been achieved by selecting appropriate values for *dx*, *dt* and *k*, so the effect of variation of *H* on the stability of the method has not been investigated. To achieve this effect the value of *H* has been assumed to vary in this study. This variation has brought us in the opportunity to show that in addition to the aforementioned parameters, *H* has also a considerable effect on the stability of HDTFD.

In the present study, we apply differential transform-finite difference method to solve 2D nonlinear heat transfer in an annular fin. We have improved the research by Chu and Chen [11] in the following directions. 1- Improve 1D Cartesian equation of [11] to a 2D cylindrical fin equation. 2- Investigate the effect of time interval of differential transform on the results. The rest of the paper is organized as follows: in section 2 the basic idea of DTM is presented. Section 3 involves mathematical formulation and discretization of the problem. The results and conclusion are presented in section 3 and 4, respectively.

2. Basic idea of DTM

The basic definition and operations of DTM are introduced in [12-19] as follows:

Definition 1:

Let $x(t)$ be an analytical function in domain D, then the Taylor series expansion of $x(t)$ is written in the following form:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!} \left(\frac{d^k x(t)}{dt^k}\right) \qquad for \quad \forall t \subset D \tag{1}$$

If we put $t_i = 0$, Eq. (1) is called the Maclaurin series of $x(t)$ with the form

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left(\frac{d^k x(t)}{dt^k} \right) \qquad for \quad \forall t \subset D \tag{2}$$

The differential transform of the function $x(t)$ is defined a s:

$$X(k) = \mathscr{T}(x(t)) = \frac{H^k}{k!} \left(\frac{d^k x(t)}{dt^k} \right) \qquad \forall k \subset K \tag{3}$$

Where $\mathscr{T}$ is differential transform operator, k is set of nonnegative integer denoted as K domain, $X(k)$ is the transformed function or the spectrum of the $x(t)$ in the K domain, k is the transformation parameter and H is the time interval of the differential transform method, which in most of the previous paper has been set to $H = 1$ and in this paper its effect on the stability of results will be examined.

Definition 2:

The inverse transformation of the differential transformation method is in the +form of:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H} \right)^k X(k) \tag{4}$$

In most previous papers [8, 9, 10 and 11], the value of H has been selected to H = Constant, but in this paper its effect on the results will be presented. In practical point of view, Eq. (4) can be rewritten by finite terms series plus a residual term in the following form

$$x(t) = \sum_{k=0}^{n} \left(\frac{t}{H} \right)^k X(k) + R_{n+1}(t) \tag{5}$$

Definition 3:

By applying Eq. (3) the following equation for differential transform of $\dot{x}(t)$ is deduced:

$$\mathscr{T}(\dot{x}(t)) = \frac{k+1}{H} X(k+1) \tag{6}$$

The fundamental operations performed by differential transform can be easily obtained and are listed in Table 1.

In solving the partial differential equation by differential transformation method, the equation is transformed into an algebraic equation in the K domain. The spectrum of the unknown function $x(t)$, $X(k)$, is obtained by solving the resulting algebraic equation and consequently $x(t)$ can be obtained by applying inverse differential transform according to Eq.(5). In order to have an accurate and fast method, the transformed domain D is split into sub-intervals, and the final values of the first sub-interval are used as initial values of next sub-interval, then the original differential equation is solved under these new initial values. The same process is repeated until the entire D domain is covered.

In fact, the real art of differential transform method is to present the solution in the form of finite series of polynomials in each subinterval which leads to a reduction in computation costs, compared with pure numerical approaches.

3. Mathematical formulation

The geometry of the annular fin studied in the present paper is shown in Fig. 1. For this fin, variable thermal conductivity and constant convective heat transfer coefficient to the environment are assumed.

The governing heat balance of the fin in 2D cylindrical coordinate system with boundary/initial conditions are:

$$\rho c\frac{\partial T}{\partial t}=\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right)+\frac{k}{r}\frac{\partial T}{\partial r}+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

$$I.C.\ \{T=T_\infty \quad at \quad t=0$$

$$B.C.\begin{cases} T=T_b \quad at \quad r=r_i \\ \dfrac{\partial T}{\partial r}=0 \quad at \quad r=r_o \\ \dfrac{\partial T}{\partial z}=0 \quad at \quad z=0 \\ k\dfrac{\partial T}{\partial z}+h(T-T)_\infty=0 \quad at \quad z=a \end{cases} \qquad for \quad r_i\le r\le r_o \ and \ t>0 \tag{7}$$

Where ρ and c are density and specific heat of the fin, respectively. h and T_∞ are convective heat transfer coefficient and ambient temperature, respectively. k is variable thermal conductivity of the fin which varies linearly with temperature according to the following equation:

$$k=k_a(1+\lambda(T-T_\infty)) \tag{8}$$

Where k_a is thermal conductivity at ambient temperature and λ is a parameter describing the variation of thermal conductivity with temperature.

The following dimensionless parameters are considered:

$$R=\frac{r}{r_o},\ G=\frac{r_i}{r_o},\ Z=\frac{z}{r_o},\ E=\frac{a}{r_o},$$
$$\theta=\frac{T-T_\infty}{T_b-T_\infty},\ \mathrm{Bi}=\frac{hr_o}{k_a},\ \tau=\frac{k_a t}{\rho c r_o^{\,2}},\ \varepsilon=\lambda(T_b-T_\infty) \tag{9}$$

Substituting the dimensionless parameters (9) into Eqs. (7)-(8) and after some mathematical manipulations, the following dimensionless equations are obtained:

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial}{\partial \boldsymbol{R}}\left((1+\varepsilon\theta)\frac{\partial\theta}{\partial \boldsymbol{R}}\right)+\frac{(1+\varepsilon\theta)}{\boldsymbol{R}}\frac{\partial\theta}{\partial \boldsymbol{R}}+\frac{\partial}{\partial \boldsymbol{Z}}\left((1+\varepsilon\theta)\frac{\partial\theta}{\partial \boldsymbol{Z}}\right)$$

$$\boldsymbol{I.C.}\ \{\theta=0\ \ \boldsymbol{at}\ \ \tau=0$$

$$\boldsymbol{B.C.}\begin{cases}\theta=1\ \ \boldsymbol{at}\ \ \boldsymbol{R}=\boldsymbol{G}\\ \dfrac{\partial\theta}{\partial \boldsymbol{R}}=0\ \ \boldsymbol{at}\ \ \boldsymbol{R}=1\\ \dfrac{\partial\theta}{\partial \boldsymbol{Z}}=0\ \ \boldsymbol{at}\ \ \boldsymbol{Z}=0\\ (1+\varepsilon\theta)\dfrac{\partial\theta}{\partial \boldsymbol{Z}}+\mathrm{Bi}\theta=0\ \ \boldsymbol{at}\ \ \boldsymbol{Z}=\boldsymbol{E}\end{cases}\qquad \boldsymbol{for}\quad \boldsymbol{G}\le \boldsymbol{R}\le 1\ \boldsymbol{and}\ \ \tau>0 \tag{10}$$

The above dimensionless governing equation can be rewritten in the following form:

$$\frac{\partial\theta}{\partial\tau}=\left(\frac{\partial^2\theta}{\partial \boldsymbol{R}^2}+\varepsilon\left(\frac{\partial\theta}{\partial \boldsymbol{R}}\right)^2+\varepsilon\theta\frac{\partial^2\theta}{\partial \boldsymbol{R}^2}\right)+\frac{(1+\varepsilon\theta)}{\boldsymbol{R}}\frac{\partial\theta}{\partial \boldsymbol{R}}+\left(\frac{\partial^2\theta}{\partial \boldsymbol{Z}^2}+\varepsilon\left(\frac{\partial\theta}{\partial \boldsymbol{Z}}\right)^2+\varepsilon\theta\frac{\partial^2\theta}{\partial \boldsymbol{Z}^2}\right) \tag{11}$$

Equation (11) shows the effect of variable $\boldsymbol{k}\,(\lambda\neq 0)$ in transforming the linear fin equation to nonlinear one.

By applying $\mathscr{T}$ operator and taking the differential transform of the variables: $\theta,\frac{\partial\theta}{\partial\tau},\frac{\partial\theta}{\partial R},\frac{\partial^2\theta}{\partial R^2},\frac{\partial\theta}{\partial Z}$ and $\frac{\partial^2\theta}{\partial Z^2}$ with respect to time, with the assumption of $H=1$, the governing equation in K domain becomes

$$\frac{(\boldsymbol{k}+1)}{\boldsymbol{H}}\vartheta(\boldsymbol{R},\boldsymbol{Z},\boldsymbol{k}+1)=\left(\frac{\partial^2\vartheta}{\partial \boldsymbol{R}^2}+\varepsilon\left(\frac{\partial\vartheta}{\partial \boldsymbol{R}}\right)^2+\varepsilon\theta\frac{\partial^2\vartheta}{\partial \boldsymbol{R}^2}\right)$$
$$+\frac{(1+\varepsilon\vartheta)}{\boldsymbol{R}}\frac{\partial\vartheta}{\partial \boldsymbol{R}}+\left(\frac{\partial^2\vartheta}{\partial \boldsymbol{Z}^2}+\varepsilon\left(\frac{\partial\vartheta}{\partial \boldsymbol{Z}}\right)^2+\varepsilon\theta\frac{\partial^2\vartheta}{\partial \boldsymbol{Z}^2}\right) \tag{12}$$

Where $\vartheta(R,Z,k)$ is differential transform of $\theta(R,Z,t)$, and in the same way the differential transform of the initial/boundary conditions are obtained:

$$\begin{aligned}&\vartheta(R,Z,0)=0\\ &\vartheta(0,Z,k)=1\quad for\ \ k=0\\ &\vartheta(0,Z,k)=0\quad for\ \ k=1,2,3\\ &\frac{\partial\vartheta}{\partial R}=0\ \ at\ \ R=1\\ &\frac{\partial\vartheta}{\partial Z}=0\ \ at\ \ Z=0\\ &(1+\varepsilon\vartheta)\frac{\partial\vartheta}{\partial Z}+\mathrm{Bi}\vartheta=0\ \ at\ \ Z=E\end{aligned} \tag{13}$$

Now, the spatial derivatives are approximated with second order central difference. The resulting differenced equation is as follows:

$$\frac{(k+1)}{H}\vartheta_{i,j}^{k+1}=\frac{\vartheta_{i+1,j}^{k}-\vartheta_{i,j}^{k}+\vartheta_{i-1,j}^{k}}{\delta R^{2}}+\varepsilon\left(\frac{\vartheta_{i+1,j}^{k}-\vartheta_{i-1,j}^{k}}{2\delta R}\right)^{2}+\varepsilon\vartheta_{i,j}^{k}\left(\frac{\vartheta_{i+1,j}^{k}-\vartheta_{i,j}^{k}+\vartheta_{i-1,j}^{k}}{\delta R^{2}}\right)$$
$$+\frac{(1+\varepsilon\vartheta_{i,j}^{k})}{R}\left(\frac{\vartheta_{i+1,j}^{k}-\vartheta_{i-1,j}^{k}}{2\delta R}\right)+\frac{\vartheta_{i,j+1}^{k}-\vartheta_{i,j}^{k}+\vartheta_{i,j-1}^{k}}{\delta Z^{2}}+\varepsilon\left(\frac{\vartheta_{i,j+1}^{k}-\vartheta_{i,j-1}^{k}}{2\delta Z}\right)^{2} \quad (14)$$
$$+\varepsilon\vartheta_{i,j}^{k}\left(\frac{\vartheta_{i,j+1}^{k}-\vartheta_{i,j}^{k}+\vartheta_{i,j-1}^{k}}{\delta Z^{2}}\right)$$

Where $\delta R,\delta Z$ are interval lengths in R and Z directions, respectively.

Applying the finite difference to boundary conditions results in:

$$\vartheta_{i,j}{}^{k}=1 \quad \textbf{for} \quad k=0$$
$$\vartheta_{i,j}{}^{k}=0 \quad \textbf{for} \quad k=1,2,3$$
$$\vartheta_{i+1,j}{}^{k}-\vartheta_{i-1,j}{}^{k}=0 \quad \textbf{at} \quad i=NR+1$$
$$\vartheta_{i,j+1}{}^{k}-\vartheta_{i,j-1}{}^{k}=0 \quad \textbf{at} \quad j=1$$
$$(1+\varepsilon\vartheta_{i,j}{}^{k})\frac{\vartheta_{i,j+1}{}^{k}-\vartheta_{i,j-1}{}^{k}}{2\delta Z}+Bi\vartheta_{i,j}{}^{k}=0 \quad \textbf{at} \quad j=NZ+1 \quad (15)$$

where $NR=\frac{1-G}{\delta R}$ and $NZ=\frac{E}{\delta Z}$.

4. Results and discussion

As the first case, linear heat conduction equation ($\varepsilon=0$) with $H=1$ is considered. The results for non-dimensional temperature distribution obtained by HDTFD are compared with exact analytical results reported in [3]. As shown in Fig. 2, good agreement is achieved.

Negligible difference between the results is the outcome of considering the Taylor series as series with finite terms in the t direction, and differencing the governing equation in the r direction.

For the second case, variation of non-dimensional temperature distribution for different values of ε and Bi is considered in figure 3. As the figure shows, an increase in ε which means higher thermal conductivity, leads to more uniform temperature distribution in the medium, as expected. The figure also compares non-dimensional temperature distribution for different values of Bi. As prospected, increase in Bi number leads to lower temperature distribution.

For the next case the effect of dimensionless time on dimensionless temperature distribution is considered. As time goes on, fin temperature approaches a steady state situation. As Fig.4 indicates, very small difference in temperature distribution is observed between $\tau=0.1$ and $\tau=10$.

In order to illustrate the steady state fin temperature distribution, 3D plot is presented in Fig. 5.

In this paper the results of the problem will be presented for different values of H in order to investigate the effect of H on the stability of results. The effect of variation of H on the stability of the results has been presented in Fig. 6. The results have been obtained by Matlab 7.6 (R.2008.a) software and Intel (R), Xeon(TM), CPU 3.2GHZ PC. The figure shows minimum time segments which do not cause divergence for different values of H for two sets of spatial segments. Set 1 $NZ = 6, NR = 12$ and set 2 $NZ = 5, NR = 10$. As the figure shows with decrease in the value of H , minimum time segments required to get converged results decreases, and it reaches a constant value for $H < 0.01$ in the cases shown. By combination of equations (3) and (4) it can be seen that for converged numerical solution, H has no effect on the results but for special number of time and spatial segments H can convert a diverged solution into a converged acceptable one. This has been shown in Fig. 7. The figure shows how the decrease in the value of H can lead to converged numerical solution. In the figure temperature distribution with $\mathrm{Bi} = 1$ $\varepsilon = 0.01, \mathrm{NR} = 10, \mathrm{NZ} = 5, \mathrm{N}\tau = 382, \tau = 0.1$ have been plotted for different values of H . As the figure shows by decrease in the value of H from $H = 0.1$ to $H = 0.01$, converged acceptable numerical result have been approached. In the figure it has been shown that for the aforementioned values of NR , NZ and $\mathrm{N}\tau$ the value of H should be $H = 0.01$. This means that by appropriate selection of H , for definite number of spatial segments, number of time segments can be reduced without appearance of divergence problem and reduction in number of time segments means reduction in computational cost of the method.

5. Conclusion

In this paper, heat transfer in an annular fin with variable thermal conductivity has been studied. The problem was formulated and discretized based on HDTFD. For the case of constant thermal conductivity, results were verified with analytical solution and good agreement achieved. For other cases, the effect of different dimensionless parameters on fin temperature was investigated. Following results have been achieved

1. The ability of HDTFD in solving 2D heat transfer problems in cylindrical coordinate system has been shown for the first time.
2. For a definite number of spatial segments, $(\mathrm{NR}, \mathrm{NZ})$, reduction of H decreases the minimum number of time segments, (NT), required to get acceptable results. This reduction in the number of required time segments can lead to reduction in the CPU time of the method.
3. It has been also shown that reduction of the value of H can convert diverged results into acceptable converged one.

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Nomenclature

a :	Half of the fin thickness
c :	Specific heat capacity
D :	domain
G :	Ratio of inner radius to outer radius
h :	convective heat transfer coefficient
H :	Time interval of differential transform
k :	Transformation parameter
k :	Variable thermal conductivity
k_a :	thermal conductivity at ambient temperature
K :	domain of the transform function
NR :	Number of spatial segments in R direction
NZ :	Number of spatial segments in Z direction
$N\tau$:	Number of time segments in τ
R :	Dimensionless radius
r :	Radius in cylindrical coordinates
r_i :	Inner radius
r_o :	Outer radius
T :	Temperature
T_b :	Base temperature
T_∞ :	Ambient temperature
$x(t)$:	Analytical function
$X(k)$:	Transformed function

Greek symbols

δR : Grid size in R direction
δZ : Grid size in Z direction
ε : Dimensionless parameter describing the variation of thermal conductivity with temperature
λ : parameter describing the variation of thermal conductivity with temperature
θ : dimensionless temperature
z : Height in cylindrical coordinates
ϑ : Transformed dimensionless temperature
$\mathscr{T}$: Differential operator

Tables

Table 1. The fundamental operations of one-dimensional differential transform method

Original function	**Transformed function**
$y(x)=u(x)\pm v(x)$	$Y(k)=U(k)\pm V(k)$
$y(x)=cw(x)$	$Y(k)=cW(k)$
$y(x)=dw(x)/dx$	$Y(k)=(k+1)W(k+1)$
$y(x)=d^{j}w(x)/dx^{j}$	$Y(k)=(k+1)(k+2)...(k+j)W(k+j)$
$y(x)=u(x)v(x)$	$Y(k)=\sum_{r=0}^{k}U(r)V(k-r)$
$y(x)=x^{j}$	$Y(k)=\delta(k-j)=\begin{cases}1, & k=j\\ 0, & k\neq j\end{cases}$

Figures

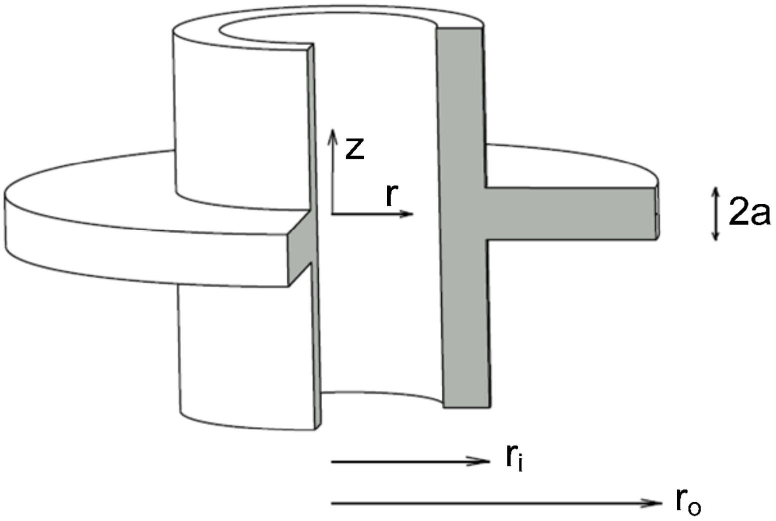

Figure 1 Schematic of straight annular fin

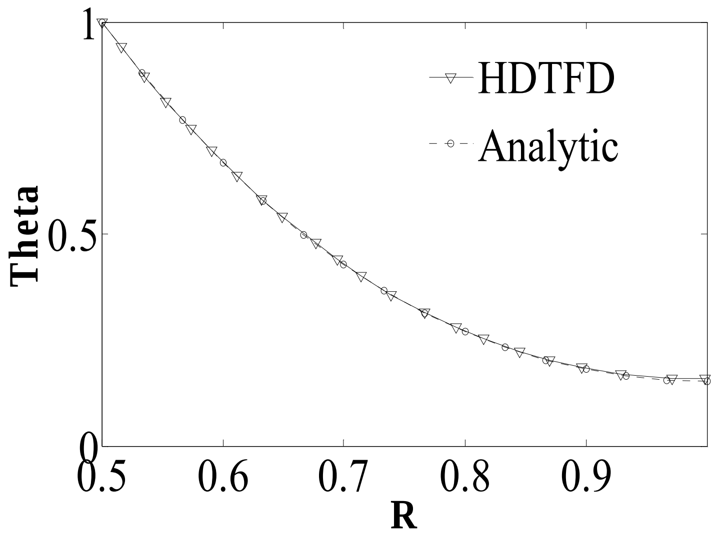

Figure 2 Comparison of the present study with analytic results

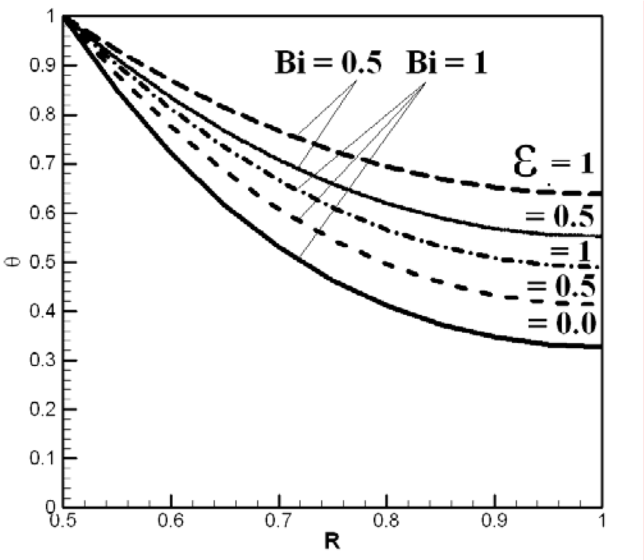

Figure 3 Effect of Bi and ε on fin temperature distribution

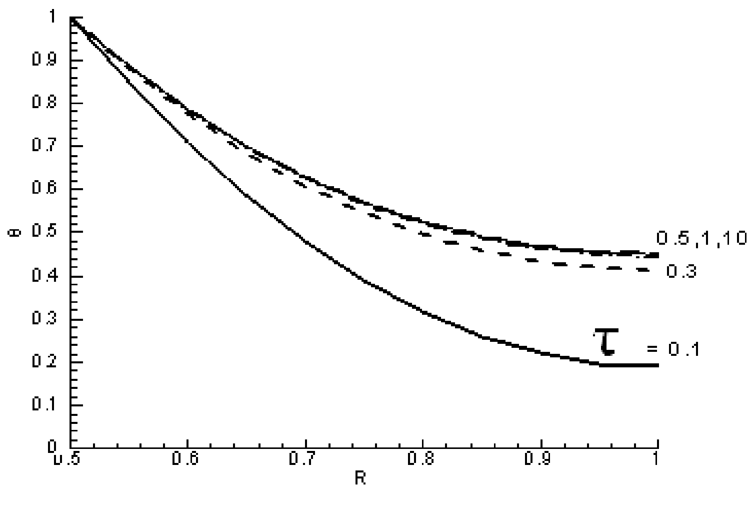

Figure 4 Variations of fin temperature distribution with time

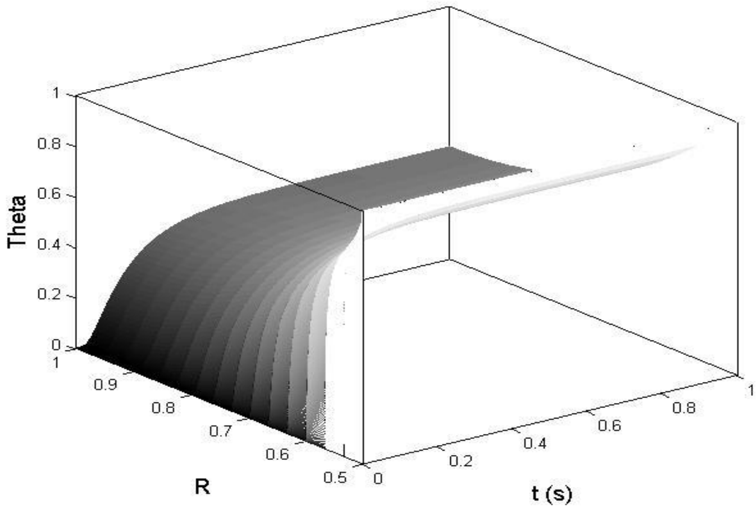

Figure 5 3D graph of temperature distribution

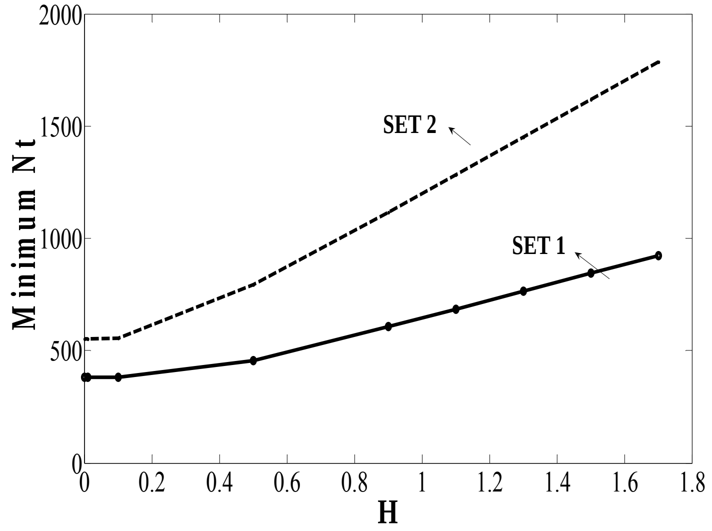

Figure 6 Effect of H on minimum required number of time segments to get converged solution

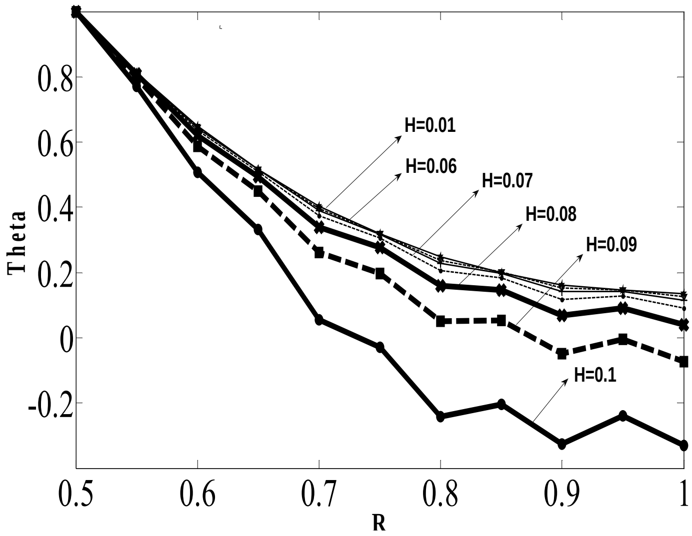

Figure 7 Effect of H on approaching a diverged solution into acceptable converged one

چکیده

در تحقیق حاضر، روش ترکیبی تبدیل دیفرانسیل- تفاضل محدود برای حل معادله فین گذرای ۲ بعدی غیر خطی مورد استفاده قرار گرفته است. برای حالت انتقال حرارت خطی نتایج در مقایسه با حل تحلیلی مورد اعتبار سنجی قرار گرفته اند. تاثیر پارامتر های مخلف بر توزیع دما مورد بررسی قرار گرفته است. تاثیر بازه زمانی تبدیل دیفرانسیل بر پایداری نتایج آزموده شده است. نتایج نشان دهنده قابلیت ممتاز روش ترکیبی تبدیل دیفرانسیل- تفاضل محدود در حل مسائل مهندسی می باشد. همچنین انتخاب مناسب بازه زمانی تبدیل دیفرانسل می تواند مشکل واگرایی روش را حل نموده و هزینه محاسبات آنرا کاهش دهد.