Fracture Analysis of a FGM Strip Containing Multiple Interface Cracks Sandwiched between Two Homogeneous Layers

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A FGM layer sandwiched between two isotropic layers weakened by several interface cracks under antiplane loading is studied. This paper examines the modelling of cracks by distribution of strain nuclei along crack lines. In this investigation, the Volterra-type screw dislocation employed between FGM and an elastic layer. To solve the dislocation problem, the complex Fourier transform is applied. One merit of this technique is the possibility to determination of the stress intensity factors for multiple cracks. The system of equations is derived by considering the distribution of line dislocation on the crack. These equations are of Cauchy singular type at the location of dislocation, which can be solved numerically to obtain the dislocation density on the faces of the cracks. Several examples are solved and the stress intensity factors are obtained. The effect of the properties and cracks geometries on the mode III stress intensity factor are studied and the validity of analysis is checked.

Keywords: Multiple interface cracks, Functionallay graded layer, Singular integral equations, Stress intensity factors, Distributed dislocation tecchnique

1 Introduction

The failure of composites and coated materials, are associated with the cracks occurring near the interface between bonded dissimilar materials. The Functionally graded materials are composite materials with nonhomogeneous micromechanical structure. These materials have been widely offered in the environments with extremely high temperature. The distinguishing feature of these materials is that the material constants are continuous and have differentiable functions. The stress analysis of FGM layers sandwiched between two isotropic layers with interface cracks deals with the design of safe structures due to the fact that the application of nonhomogeneous materials has increased in industries recently.

Sills and Benveniste [1] considered the steady state propagation of a semi-infinite crack between two dissimilar viscoelastic solids. The stress intensity factor was found to be a function of the crack tip velocity and the material parameters by means of the Weiner-Hopf technique. The basic crack problem which is essential for the study of subcritical crack propagation and fracture layered of structural materials was considered by Erdogan [2].

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The problem was formulated in terms of integral equations and the singular behavior of the solution near and at the ends of intersection of the cracks was investigated. Furthermore, a numerical method for solving the problem was described and stress intensity factors obtained from the solution for various crack geometry were presented by Erdogan [3]. The problem was intended to approximate the cracks perpendicular to and on the interface of the two layers in a composite beam or a plate. Erdogan [4] investigated the singular nature of the crack-tip stress field in a nonhomogeneous medium having a shear modulus with a discontinuous derivative. In this work, the problem was considered in the antiplane shear loading of two bonded half spaces in which the crack is perpendicular to the interface.

Delale and Erdogan [5] reconsidered the linear elasticity problem for an interface crack between two bonded homogeneous and nonhomogeneous half planes. The problem was solved for various values of the nonhomogeneous parameter. Chiang [6] determined the asymptotic stress and displacement fields of a propagation interface crack under mode III conditions by employing the eigen-function expansion technique. The dynamic energy release rate was found to be related to the dynamic stress intensity factor. Suo et al. [7] considered a semi-infinite interface crack between two infinite isotropic elastic layers under general edge loading condition. Champion and Atkinson [8] considered the stress singularity at the tip of a crack at the interface between two different power-law materials under mode III loading. Erdogan et al. [9] considered the mode III crack problem for two bonded homogeneous half planes. The problem was solved for various crack locations in and around the nominal interfacial region. Chung and Robinson [10] considered a transient solution of the problem of a mode III crack propagating along the interface between two different media. In this paper, the method of self-similar potentials was introduced to solve the self-similar problem. Erdogan and Wu [11] investigated the influence of the structure and thickness of the interfacial regions on the strain energy release rate in bonded isotropic or orthotropic materials containing collinear interface cracks.

They formulated the problem in terms of a system of singular integral equations of second kind which was solved by using a relatively simple and efficient technique. Ozturk and Erdogan [12] investigated the influence of the thickness and the properties of the interfacial region on such fracture related parameters as the strain energy release rates, the stress intensity factors, and the crack opening displacements. Jin and Batra [13] studied the interface cracking between ceramic and/or functionally graded coatings and a substrate under antiplane shear. It was noted that large modulus gradients in thin coatings might seriously restrict the application of stress intensity factors as the stress intensity factors dominant zone fell into the crack tip nonlinear deformation and damage zone.

Li et al. [14] considered a moving mode III crack at the interface between two dissimilar piezoelectric materials. In this paper, the integral representation of a general solution was given in terms of Fourier cosine integrals. The elastic and electric fields were obtained for a moving impermeable crack and for a moving crack where the electric displacement was continuous across the crack surfaces. Chan et al. [15] presented a displacement based integral equation formulation for the mode III crack problem in a nonhomogeneous medium, with a continuously differentiable shear modulus assumed to be an exponential function. They solved the problem for a finite crack. Dag et al. [16] considered interface crack problems in graded orthotropic media. An interface crack between a graded orthotropic coating and a homogeneous orthotropic substrate was considered. It was noted that under normal loading of the crack faces energy release rate decreased the function of the normalized nonhomogeneity parameter and increased the function of the shear parameter under uniform normal loading. Chen and Chue [17] dealt with the anti-plane problem of two bonded functionally graded finite strips. A system of singular integral equations was derived and then solved numerically by utilizing Gauss-Chebyshev integral formula.

Bagheri et al. [18] investigated the linear steady state problem of several moving cracks in a functionally graded magneto-electro-elastic strip subjected to anti-plane mechanical and in-plane electric and magnetic loading. They obtained exact solution in closed form to this problem by combining the dislocation method and integral transform technique. Bagheri et al. [19] in another work investigated the behaviors of several moving cracks in a functionally graded piezoelectric strip subjected to anti-plane mechanical loading and in-plane electrical loading. It was observed that the stress fields in the functionally graded piezoelectric layer increased when the crack velocity increased. Bagheri et al. [20] studied the fracture problem for a medium composed of a cracked piezoelectric strip with functionally graded orthotropic coating. The layer was subjected to anti-plane mechanical and in-plane electrical loading. They addressed the problem of a screw dislocation located in a substrate imperfectly bonded to the coating and then constructed integral equations for the layer in order to model the cracked piezoelectric layer with the help of the dislocation solution.

In this work, we study the multiple cracks along the interface between FGM and elastic layers, using the distributed dislocation technique. also the complex Fourier transform is employed to evaluate the stress fields.

The solutions are then used to obtain singular integral equations for the dislocation density on the face of multiple cracks. These equations are solved numerically and the solutions are employed to designate SIF for cracks tip. Different examples are given to display the effects of the properties, cracks length, and thickness of coat on the stress intensity factor of cracks to demonstrate the advantage of this method.

2 Formulation of the problem

The stress analysis in a medium made up of a FGM layer with thickness h_2, bonded to isotropic layers under anti-plane loading, is plotted in Figure (1). A dislocation may be created by first making a slit in a medium, from the core of the dislocation in the origin of Cartesian coordinates (x,y,z) to any infinitely remote point. A cut has made along the positive x-axis, pull the material apart in the z-direction, and insert a thin strip of thichness b_z, before re-joining. For a medium under anti-plane deformation, the only nonzero displacement component is the out of plane component $w(x, y)$. Consequently, the constitutive equations for all three layers in a Cartesian coordinate system are:

$$\sigma_{zx1}(x,y)=\mu_1\frac{\partial w_1(x,y)}{\partial x},\qquad \sigma_{zy1}(x,y)=\mu_1\frac{\partial w_1(x,y)}{\partial y},\qquad -h_1\le y\le 0$$
$$\sigma_{zx2}(x,y)=\mu_2(y)\frac{\partial w_2(x,y)}{\partial x},\qquad \sigma_{zy2}(x,y)=\mu_2(y)\frac{\partial w_2(x,y)}{\partial y},\qquad 0\le y\le h_2$$
$$\sigma_{zx3}(x,y)=\mu_3\frac{\partial w_3(x,y)}{\partial x},\qquad \sigma_{zy3}(x,y)=\mu_3\frac{\partial w_3(x,y)}{\partial y},\qquad h_2\le y\le h_3 \tag{1}$$

where, $w_i(x, y)$, σ_{zki} and $\mu_i\ (i=1,2,3; k=x,y)$ are the z-component of the displacement vector, the stresses and the shear modulus of the material i, respectively. To simulate the problem of FGM layer bonded between two homogeneous layers, it should be supposed that μ_1 and μ_3 are known constants and $\mu_2=\mu_2(y)$ is a given function satisfying Figure (1).

$$\mu_2(0)=\mu_1,\qquad \mu_2(h_2)=\mu_3 \tag{2}$$

In order to simplify the complexity of mathematics, we will focus this study on a special class of FGMs in which the variations of the elastic shear modulus has the same material gradient parameter. Therefore, we assume:

$$\mu_2(y) = \mu_1 e^{2\beta y} \tag{3}$$

where β is the inhomogeneity parameter.
From (2) and (3) it may be seen that

$$\beta = \frac{1}{2h_2} Ln(\frac{\mu_3}{\mu_1}) \tag{4}$$

From Eqs.(1) , and using the equilibrium equations, the governing equations for layers can be expressed in the following form:

$$\begin{aligned}
&\nabla^2 w_1(x,y) = 0, && -h_1 \le y \le 0 \\
&\nabla^2 w_2(x,y) + 2\beta \frac{\partial w_2(x,y)}{\partial y} = 0, && 0 \le y \le h_2 \\
&\nabla^2 w_3(x,y) = 0, && h_2 \le y \le (h_2 + h_3)
\end{aligned} \tag{5}$$

where ∇^2 is the two dimentional Laplace operator. Note that body forces are not considered in the present work.
For this dislocation problem, the upper and lower surfaces of structure are stress free and can be described by:

$$\begin{aligned}
&\sigma_{zy1}(x,-h_1) = 0 \\
&\sigma_{zy3}(x,(h_2 + h_3)) = 0
\end{aligned} \tag{6}$$

The stress and displacement continuity conditions are expressed as:

$$\begin{aligned}
&\sigma_{zy2}(x,h_2^-) = \sigma_{zy3}(x,h_2^+) \\
&w_2(x,h_2^-) = w_3(x,h_2^+) \qquad |x| < \infty
\end{aligned} \tag{7}$$

Let a Volterra-type screw dislocation with Burgers vector b_z be situated in the interface functionally graded layer and isotropic layer at a point with coordinates $(0,0)$. it should be Noted that, a sudden jump, or discontinuity, in the displacements in the z-direction has experienced by traversing a path from the negative to the positive side of the cut, around the dislocation core. Therefore, The conditions representing the screw dislocation are:

$$\begin{aligned}
&\sigma_{zy1}(x,0^-) = \sigma_{zy2}(x,0^+) \\
&w_2(x,0^+) - w_1(x,0^-) = b_z H(x) \qquad |x| < \infty
\end{aligned} \tag{8}$$

where $H(x)$ is the Heaviside-step function, $y = 0^+$ and $y = 0^-$ designate upper and lower edges of the cut, respectively. The former relation, denotes the traction continuity condition

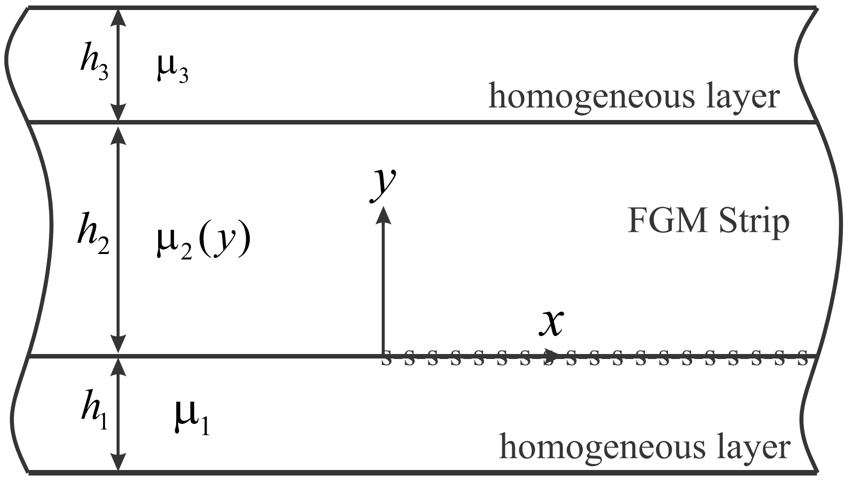

Figure 1 Schematic view of the medium with screw dislocation.

while the latter implies the multivaluedness of displacement, In above conditions. It is noteworthy that the above conditions for screw dislocation were employed by several investigators, e. g., Weertman [21]. By means of the complex Fourier transforms, the general solution of Eqs.(5) is obtained as follows:

$$
\begin{aligned}
w_1^*(s,y) &= A_1(s)\sinh(sy) + A_2(s)\cosh(sy) && -h_1 \le y \le 0 \\
w_2^*(s,y) &= A_3(s)e^{(-\beta+\lambda)y} + A_4(s)e^{-(\beta+\lambda)y} && 0 \le y \le h_2 \\
w_3^*(s,y) &= A_5(s)\sinh(sy) + A_6(s)\cosh(sy) && h_2 \le y \le (h_2+h_3)
\end{aligned}
\tag{9}
$$

where $\lambda = \sqrt{\beta^2 + s^2}$ and the functions $A_i(s), i = 1,2,3,4,5,6$ are unknown. The application of conditions (6-8) gives the unknown coefficients. After determining coefficients, referring to (9) and using the constitutive equations with the aid of inverse Fourier transform, the stress field associated with a single dislocation in the interface functionally graded layer and homogeneous layer can be written as:

$$\sigma_{zy1}(x,y) = \frac{\mu_1 b_z}{\pi}\int_0^\infty \frac{K_1(s)}{K_2(s)}\sin(sx)ds \tag{10}$$

where

$$K_1(s) = \sinh(s(y+h_1))[\sinh(\lambda h_2)\cosh(sh_3)s + \lambda\sinh(sh_3)\cosh(\lambda h_2) + \beta\sinh(sh_3)\sinh(\lambda h_2)$$

and

$$K_2(s) = \sinh(\lambda h_2)\cosh(s(h_1+h_3))s + \beta\sinh(\lambda h_2)\sinh(s(h_3-h_1)) + \lambda\cosh(\lambda h_2)\sinh(s(h_1+h_3)) \tag{11}$$

Considering the asymptotic behavior of the integral in Eq. (10) for $s \to \infty$, one may prove that $\sigma_{yz1\infty}$ has the following relation:

$$\sigma_{zy1\infty}(x,y) = \lim_{s\to\infty}\sigma_{zy1}(x,y) = -\frac{\mu_1 b_z}{\pi}\int_0^\infty \frac{e^{sy}}{2}\sin(sx)ds \tag{12}$$

The singular parts can be evaluated by the use of the following identities:

$$\int_0^\infty e^{sy}\sin(sx)ds = \frac{x}{x^2+y^2}, \qquad y<0$$
$$\int_0^\infty e^{sy}\cos(sx)ds = -\frac{y}{x^2+y^2}, \quad y<0 \tag{13}$$

After performing the appropriate asymptotic analysis using a symbolic manipulator and separating the singular parts of the kernels, we obtain:

$$\sigma_{zy1}(x,y) = \frac{\mu_1 b_z}{\pi}[\int_0^\infty (\frac{K_1(s)}{K_2(s)} - \frac{e^{sy}}{2})\sin(sx)ds + \frac{x}{2(x^2+y^2)}] \tag{14}$$

There may be observed that stress componenty exhibit the familiar Cauchy type singularity at the dislocation location.

3 Cracks formulation

In this section, the basic concepts of the technique is introduced, such as the dislocation density and the fundamental function. In this part, the method will be expanded to deal with multiple interface cracks by considering a FGM layer bonded between two isotropic layers weakened by N interface cracks. The cracks configuration are presented in parametric form as:

$$x_i = x_i(s)$$
$$y_i = y_i(s) \qquad i \in \{1,2,...,N\} \qquad -1 \le s \le 1 \tag{15}$$

Two orthogonal coordinate systems *s*-*n* are chosen on the i-th crack such that their origin is located on the crack while the *s*-axis remains tangent to the crack surface. The anti-plane traction on the surface of the i-th crack in terms of stress become:

$$\sigma_{zn}(x_i, y_i) = \sigma_{zy} \tag{16}$$

Suppose dislocations with unknown dislocation density $B_{zj}(t)$ are distributed on the infinitesimal segment $\sqrt{[x'_j(t)]^2 + [y'_j(t)]^2}dt$ on the surface of the *j*-th crack. The traction components on the surface of the i-th crack in the presence of dislocations distribution on the surfaces of all *N* cracks mentioned above, yield:

$$\sigma_{zn}(x_i(s), y_i(s)) = \sum_{j=1}^{N}\int_{-1}^{1} K_{ij}(s,t)B_{zj}(t)\sqrt{[x'_j(t)]^2 + [y'_j(t)]^2}dt \tag{17}$$

The kernel of integral equations (17) takes the fllowing form

$$K_{ij} = \frac{\mu_1 b_z}{\pi}\{\int_0^\infty ([\sinh(\lambda h_2)\cosh(sh_3)s + \lambda\sinh(sh_3)\cosh(\lambda h_2) + \beta\sinh(sh_3)\sinh(\lambda h_2)]$$
$$\times \frac{\sinh(s((y_i - y_j) + h_1))}{K_2(s)} - \frac{e^{s(y_i - y_j)}}{2})\sin(s(x_i - x_j))ds + \frac{(x_i - x_j)}{2[(x_i - x_j)^2 + (y_i - y_j)^2]}\} \tag{18}$$

The left hand side of the Eqs. (17) represent stress component at the presumed location of the cracks with negative sign. Employing the definition of dislocation density functions, the equations for the crack opening displacement across *j*th crack become:

$$w_j^+(s) - w_j^-(s) = \int_{-1}^{s} B_{zj}(t)\sqrt{[x_j'(t)]^2 + [y_j'(t)]^2}\,dt \qquad j = 1,2,...,N \tag{19}$$

The singled-value conditions of displacement field can be determined by closure requirements:

$$\int_{-1}^{1} B_{zj}(t)\sqrt{[x_j'(t)]^2 + [y_j'(t)]^2}\,dt = 0, \qquad j = 1,2,...,N \tag{20}$$

The singular integral equations (17) are solved numerically using (19) and an appropriate collocation technique to determine the dislocation density functions [22]. The stress fields near a crack tips having square-root singularity, can be expressed as:

$$B_{zj}(t) = \frac{g_{zj}(t)}{\sqrt{1-t^2}}, \quad -1 \le t \le 1,\ j = 1,2,...,N \tag{21}$$

The function $g_{zj}(t)$ are obtained via solution of the system of equations. The stress intensity factors for the i-th interface crack can be calculated as:

$$K_{Rj}^{M} = -\frac{\mu_1}{2}\left[[x_j'(1)]^2 + [y_j'(1)]^2\right]^{\frac{1}{4}} g_{zj}(1)$$

$$K_{Lj}^{M} = \frac{\mu_1}{2}\left[[x_j'(-1)]^2 + [y_j'(-1)]^2\right]^{\frac{1}{4}} g_{zj}(-1) \qquad j = 1,2,...,N \tag{22}$$

The details of the derivation of fields intensity factors to reach (22) are not given here.

4 Results and discussions

The analysis developed in latter section, allows the FGM strip bonded between two isotropic layers with multiple interface cracks subjected to constant anti-plane load, to be analysed. In the following examples, the length of straight crack is $2L$. The quantities of interest are the absolute values of non-dimensional stress intensity factors, K_{III}/K_0, where we take $K_0 = \tau_0/\sqrt{L}$.

The first example deals with a strip weakened by an interface crack. The problem is symmetric with respect to the y-axis. In this case h_1 and h_3 are infinite. Figure (2) shows the effect of modulus ratio μ_3/μ_1 on K_{III}/K_0 for various values of h_2/L. It is clearly seen that K_{III}/K_0 monotonically decrease as that the stiffness ratio μ_3/μ_1 increases. In this example, by asumming that $h_1 \to \infty$ and $h_3 \to \infty$, excellent agreement is observed with the results presented by Ozturk and Erdogan [12].

In the second example, we consider a strip weakened by an interface crack with different modulus ratio $\mu_3/\mu_1 = 3,1/3,22,1/22$. The absolute value of dimensionless stress intensity factor K_{III}/K_0, versus L/h_2 is depicted in Figure (3). In this case, by asumming that $h_1 \to \infty$ and $h_3 \to \infty$, excellent agreement is observed with the results presented by Ozturk and Erdogan [12].

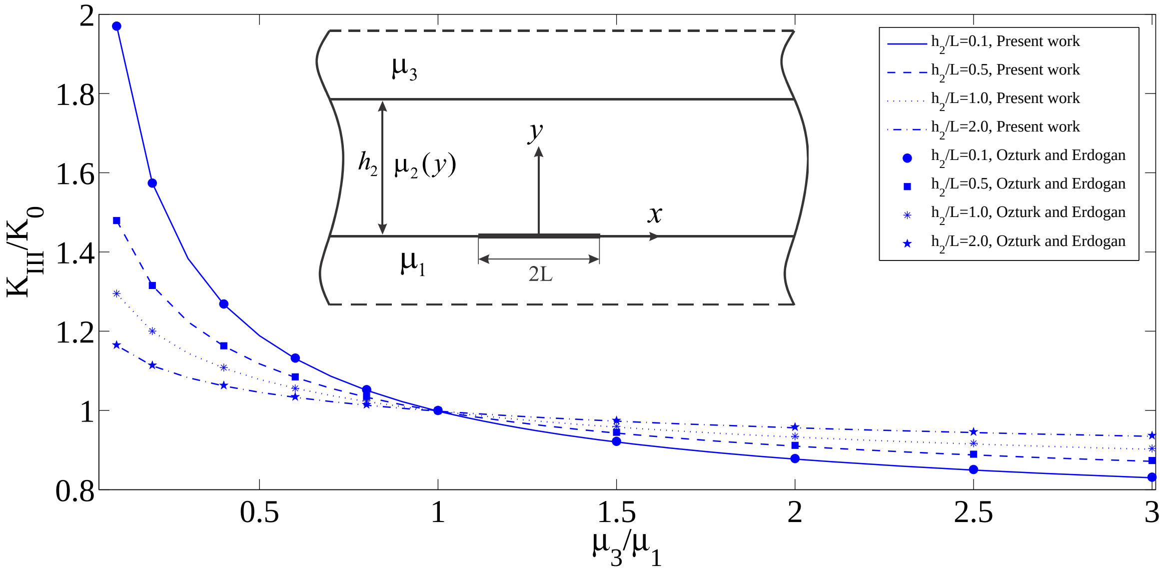

Figure 2 Variations of normalized stress intensity factors of crack tips versus the modulus ratio.

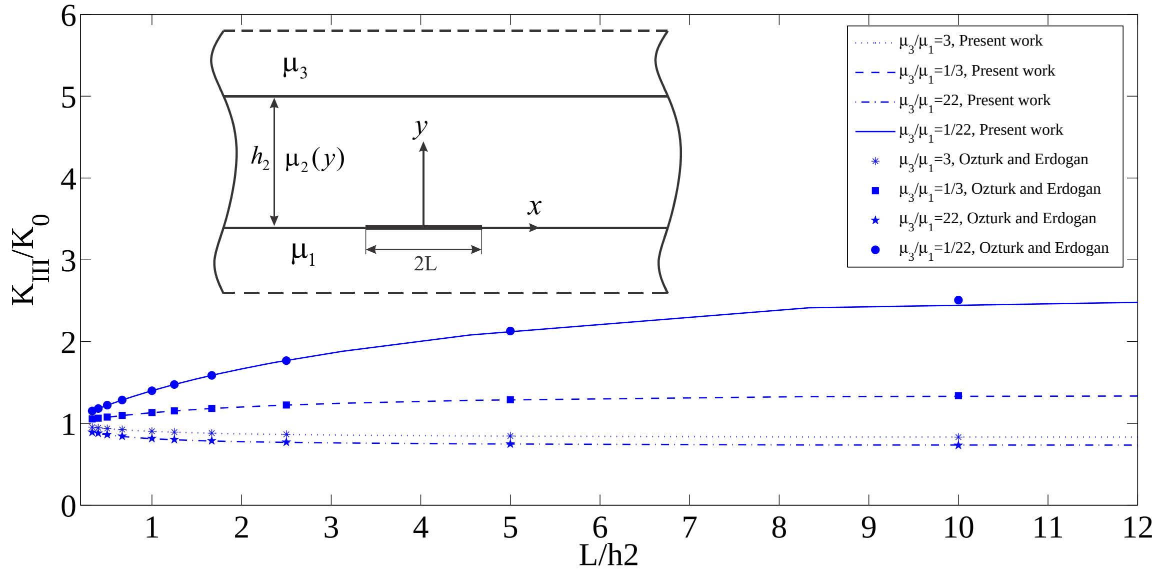

Figure 3 Variations of normalized stress intensity factors with L/h_2 for different modulus ratio μ_3/μ_1 .

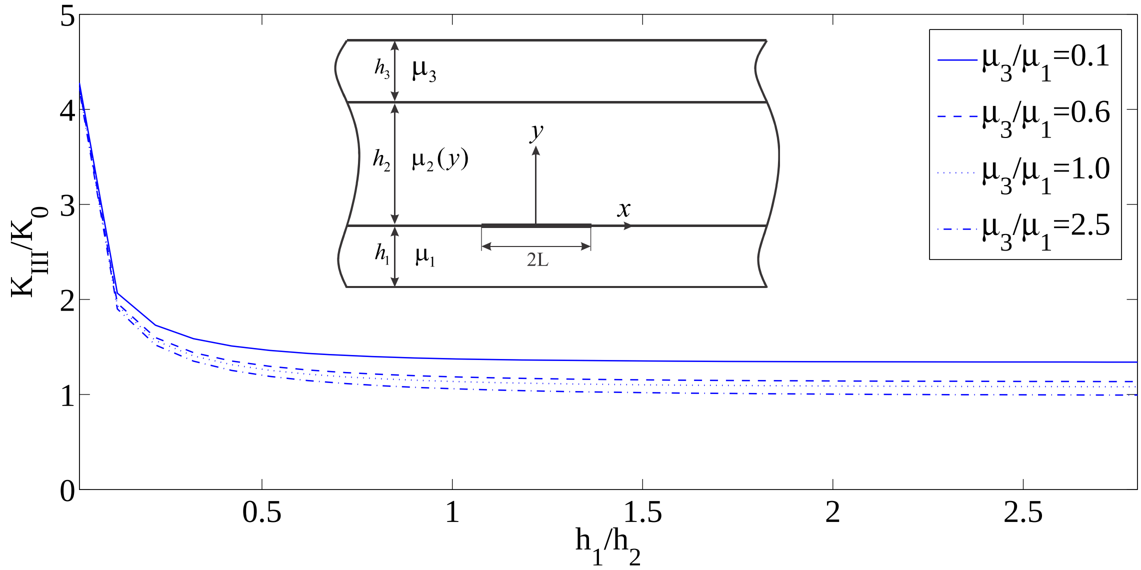

Figure 4 Variations of dimensionless stress intensity factor with h_1/h_2 for different modulus ratio μ_3/μ_1 .

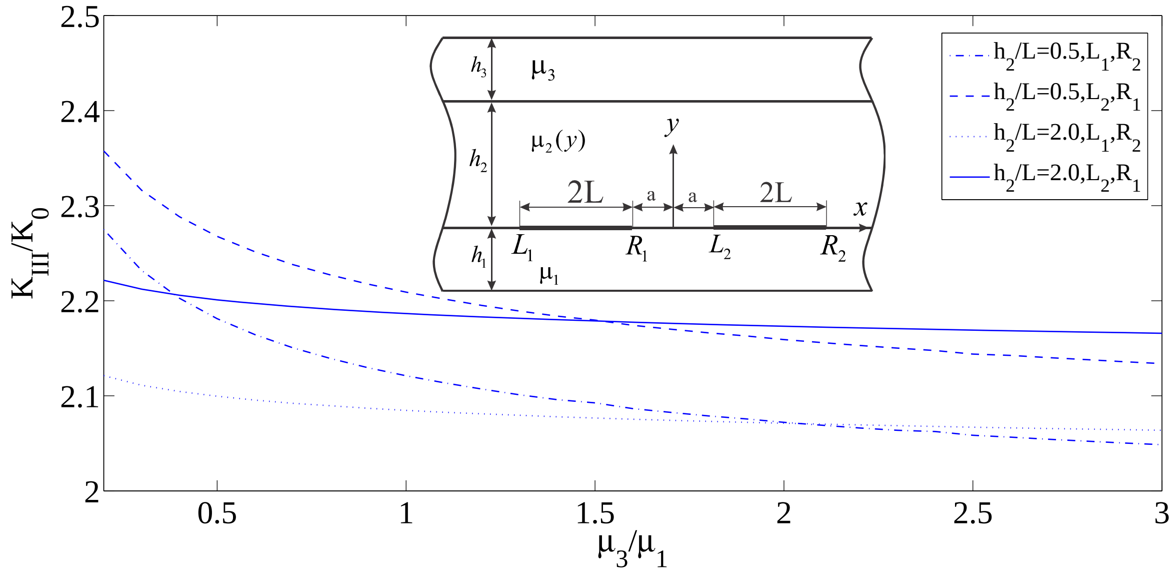

Figure 5 Dimensionless stress intensity factors of two interface cracks with μ_3/μ_1 for different values of the h_2/L .

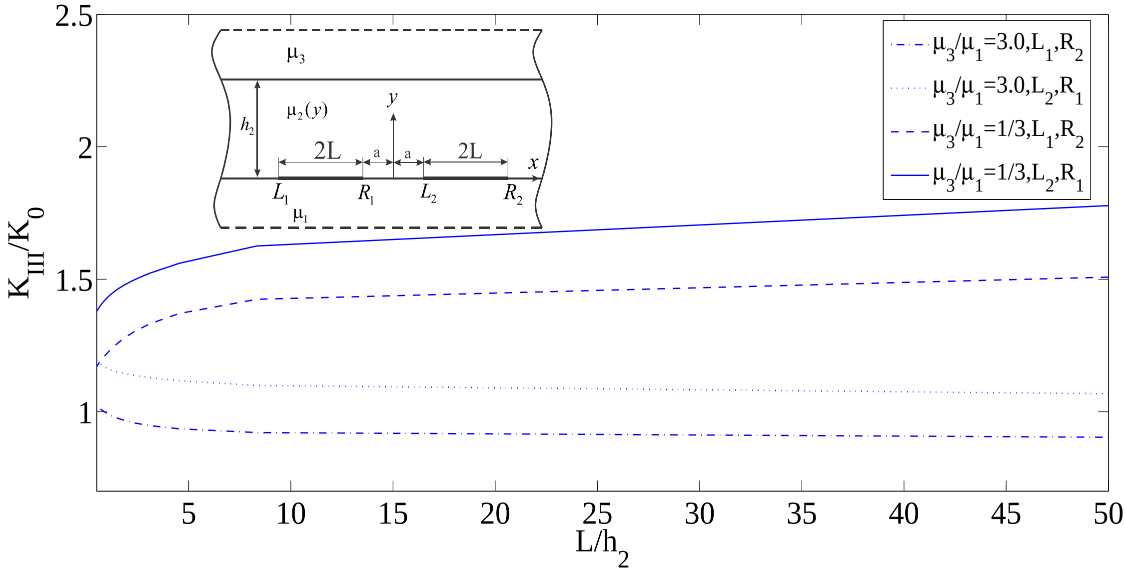

Figure 6 Dimensionless stress intensity factors of two interface cracks with L/h_2 .

Figure (4) illustrate the effects of the h_1/h_2 on the dimensionless stress intensity factors. We can see that the stress intensity factors decrease as the h_1/h_2 increases. And the values of stress intensity factors decrease with the increasing of the modulus ratio.

The next example deals with the interaction of a stationary two interface crack with a fixed center. Figure (5) shows the influence of the μ_3/μ_1 on the K_{III}/K_0 for different values of the h_2/L . It is apparent that the stress field around crack tip can be intensified by the interaction of cracks. Similar phenomena can be observed for variation of the stress intensity factors.

In Figure (6), we present the results of the identical growing interface cracks with fixed centers placed on the interface. The modulus ratio μ_3/μ_1 is chosen as $\mu_3/\mu_1 = 3,1/3$. The dimensionless stress intensity factors for $h_1 = h_3 = \infty$ with L/h_2 , are shown in Figure (6). As expected, the variations of stress intensity factors for the two approaching crack tips change rapidly.

As the last example, the variation of the normalized stress intensity factors of crack tips against h_1/h_2 is depicted in Figure (7), where $2a$ is the distance between two approaching crack tips. As it may be observed, with increasing thickness of underneath coat the stress intensity factors decrease.

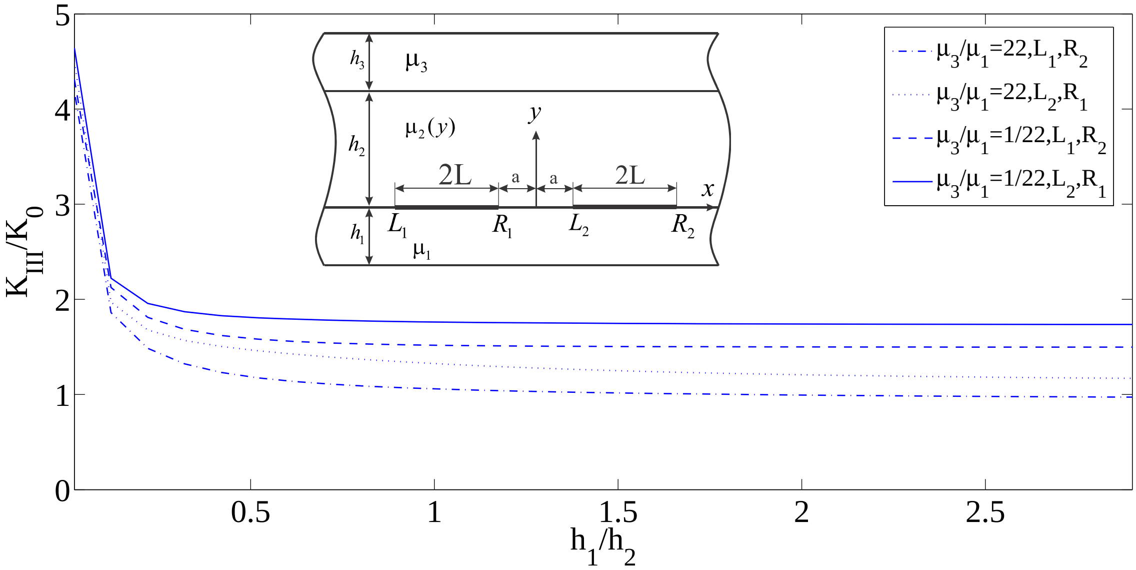

Figure 7 Variation of the normalized stress intensity factors of two interface cracks tips against h_1/h_2 .

5 Conclusion

The fracture problem of a functionally graded strip bonded between two isotropic layers weakened by multiple interface cracks under anti-plane mechanical loading is investigated. The solution of screw dislocation between FGM and an elastic layer is obtained by using the integral transform technique. The dislocation solution leads to Cauchy singular integral equations with unknown dislocation density function which can be solved by numerical method. The solutions are obtained in integral forms which may be considered as Green's functions for the medium with multiple interface cracks. The effects of crack geometry, material properties, interaction of cracks on the stress intensity factor are studied. Results from the present work can be reported as follows:

The stress intensity factors decrease with increasing the modulus ratio μ_3/μ_1 . It is observed that for $\mu_3/\mu_1 = 1.0$ or for $h_i \to \infty, (i = 1,2,3)$, the problem reduces to the dislocation solution of infinite isotropic plane and for $\mu_3/\mu_1 > 1.0$, the stress intensity factors increase with increasing the functionally graded thickness. It was also observed that for $\mu_3/\mu_1 < 1.0$, the stress intensity factors decrease with increasing the functionally graded thickness. In summary, the stress intensity factor decrease with increasing thickness of underneath coat and can be intensified by the interaction of cracks.

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Nomenclature

$A_i(s),\ i=1,2,...,6$	:	unknowns coefficients
b_z	:	Burgers vector
$B_{zj}(t)$	:	dislocation density
$g_{zj}(t)$	:	regular terms of dislocation density
h_1	:	thickness of homogeneous lower layer
h_2	:	thickness of FGM strip
h_3	:	thickness of homogeneous upper layer
$H(x)$	:	Heaviside step function
K_{Lj}^{M}	:	stress intensity factor of left side of crack
K_{Rj}^{M}	:	stress intensity factor of right side of crack
K_0	:	stress intensity factor of a crack in infinite plane
$K_{ij}(s,t)$	:	kernels of integral equations
L	:	half lengths of straight crack
N	:	total number of cracks
s	:	Fourier varible
$w_i(x,y), i=1,2,3$	:	out of plane displacement component
$x_i(s), y_i(s)$	:	functions describing the geometry of cracks
β	:	FGM exponent
μ_1, μ_3	:	elastic shear modulus
σ_{nz}	:	traction vector
σ_{zx}, σ_{zy}	:	out of plane stress components

چکیده

در این مقاله از روش توزیع نابجایی برای حل مساله مکانیک شکست در باریکه ساخته شده از مواد تابعی محدود شده توسط دو لایه الاستیک تحت بار خارج صفحه ایی استفاده شده است. رفتار محیط الاستیک خطی است و سطوح ترکها هموار در نظر گرفته شده است. یکی از مزیت های این روش، تعیین ضرایب شدت تنش برای مجموعه ایی از ترکها می‌باشد. با استفاده از تبدیل فوریه مختلط و حل معادله حاکم با توجه به شرایط مرزی، میدان تنش ناشی از نابجایی در فصل مشترک باریکه با رفتار تابعی و لایه الاستیک محاسبه می‌گردد. سپس با داشتن این حل، معادلات انتگرالی برای تحلیل مساله چندین ترک هموار بدست می‌آید. این معادلات دارای تکینگی از نوع کوشی هستند که با استفاده از روش عددی حل می‌گردند تا تابع توزیع نابجایی‌ها بدست آید. بعد از بدست آوردن توزیع نابجایی می‌توان ضرایب شدت تنش در نوک ترکها را بدست آورد. برای نشان دادن صحت روابط بدست آمده، نتایج با منبع مناسبی مقایسه شده و تطابق خوبی مشاهده شده است.