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Natural Convection from Horizontal Non-Circular Annulus Partially Filled with Porous Sleeve

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In this paper natural convection heat transfer within a two-dimensional, horizontal, concentric cam shape cylinders that is partially filled with a fluid-saturated porous medium has been investigated. both cylinders are kept at constant and uniform temperatures with the outer cylinder being subjected relatively lower than the inner one. In addition, the forchheimer and brinkman effect are taken into consideration inside the porous sleeve. furthermore, the local thermal equilibrium condition is taken into account. the porosity factor is considered to be uniform and constant with ε = 0.9. the main objective of this study is to examine the effects of streamline shape, thermal conductivity ratio (ks/kf) and the porous layer thickness on the buoyancy induced flow motion under steady state condition. these effects are studied using the following dimensionless parameters: Ra = 10^4-10^6, Da = 10^{-3}-10^{-5}. The results show's that Nusselt number is affected mostly by porous sleeve thickness and Rayleigh number respectively.

Key words: Natural convection; Numerical investigation;Cam shape; Porous medium

1 Introduction

Buoyancy-driven flow and heat transfer between horizontal concentric and eccentric annulus filled with a porous medium has been the subject of many investigations in recent years. The motivation for these studies was taken from their technological applications such as thermal insulation, thermal storage systems [1]. Many significant aspects of transport phenomena in porous media were discussed in recent investigations by Nield and Bejan [2], Ingham and Pop [3], Vafai [4,5], Vafai and Hadim [6], Ingham et al. [7], and Bejan et al. [8]. Caltagirone [9,10] implemented an extensive numerical solution of steady state free convection in an annulus filled with a porous medium using both a perturbation method and a finite difference technique.
It was reported that a fluctuating three-dimensional regime in the upper part of the porous layer was observed although the lower part remained strictly two-dimensional.

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In a similar study, Burns and Tien [11] investigated natural convection in concentric and horizontal cylinders filled with a porous medium.
Stewart and Burns [12] used numerical method to analyzed the effect of a permeable inner boundary on the maximum temperature and the convective flows for a two-dimensional horizontal annulus with a uniformly heat generating porous media. They reported that multicellular flows happen at the highest Rayleigh numbers investigated. In addition, inverted symmetry in flow patterns and temperature distributions were observed when the heated isothermal wall condition changed from one cylinder to another. Vasseur et al. [13] presented a numerical study of two-dimensional laminar natural convection in annulus filled with a porous layer with internal heat generation using Darcy–Oberbeck–Boussinesq equations.
Bau [14,15], by using Darcy–Oberbeck–Boussinesq's equations in the considered range of employed parameters, obtained analytical solution for thermal convection in a horizontal, eccentric annulus containing a saturated porous medium using a regular perturbation expansion in terms of the Darcy–Rayleigh number.
More over, the Nusselt number was obtained as a power series of Darcy–Rayleigh number for range of eccentricity values. . Khanafer et al. [16] studied natural convection heat transfer within horizontal annulus that is partially filled with porous medium. The main objective of their sudy was the effect of porous sleeve on natural convection heat transfer. Leong and Lai [17] obtained analytical solutions for natural convection in annulus with a porous sleeve using the perturbation method and Fourier transform technique. The sleeve was press-fitted to the inner surface of the outer cylinder. Small temperature difference between the cylinders as well as small Rayleigh number were assumed in their investigation, which produces the little convection activities attained within the cylinders. They employed the extended Brinkman–Darcy's law to describe the flow motion inside the porous layer. Their results show that for a sufficiently thin porous sleeve, the sleeve behaves as if it were imperm- eable. The study, however, did not recognize that the rigid matrix resistance cannot be determined by Darcy's law at high velocities owing to the appreciated increase in inertial effects.

Sankar et al. [18] investigated natural convection heat transfer and flows in a vertical annulus filled with a fluid-saturated porous medium. Their results show that placing of the heater in lower half of the inner cylinder wall causes better heat transfer rather than placing the heater near the top and bottom portions of the inner wall. Baragh et al. [19] investigated the effect of different arrangment of porous media in heat transfer of air circle pipe. The results show that the fully filled pipe of porous media had better heat transfer. Wang et al. [20] studied the laminar natural convection in a vertical annulus with inner wall coverd by a porous layer numerically. The effect of Rayleigh number, Darcy number, thermal diffusity ratio and thickness of porous layer on overall heat transfer have been investigated. Their ruslts show that darcy number and thermal diffusity ratio have gearter impact on heat transfer. Siavashi et al. [21] studied the effect of nanofluid on natural convection heat transfer in horizontal circular annulus. They presented the influence of conductivity ratio, Rayleigh number and Darcy number on heat transfer.

The current study is focused on the analysis of the fluid flow and heat transfer within the concetric cam shap cylinders using generalized form of the momentum equation that accounts for the darcian and inertial effects. Further more, the undergoing investigation examines the effects of relevant dimensionless parameters on the flow motion and heat transfer characteristics in the cylinders. These parameters are Rayleigh number, Darcy number and conductivity ratio. The novelty of this work is to examine the combined effect of porous sleeve thickness and thermal conductivity ratio in various Rayleigh number. Also for expressing the correlation for average nusselt number the effect of Rayleigh number, Darcy number, thermal conductivity ratio and porous sleeve thickness are considered.

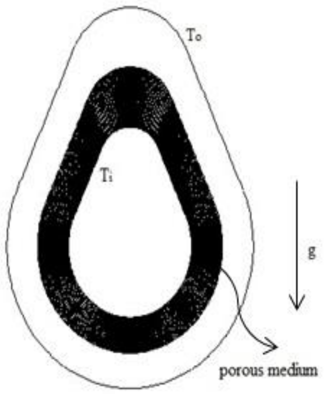

Figure 1 Schematic of the physical model

2 Governing Equation

A schematic diagram of a laminar two-dimensional natural convection heat transfer in a horizontal concentric cam shape cylinders filled partially with a porous medium is shown in Figure(1). In addition, the inner cylinder is of a equivalent radius R_i and the outer cylinder of a radius R_o are both maintained at a uniform and constant temperatures T_i and T_o, respectively, while maintaining $T_i > T_o$. also the porous medium is viewed as a continuum with the solid and fluid phases in thermal equilibrium, isotropic, homogene- ous, and saturated with an incompressible Newtonian fluid.

Hence, the porous medium has a unique porosity ε and permeability K values. Furthermore, in this study viscous heat dissipation in the fluid is assumed to be negligible. Also, it is assumed that the thermophysical properties of the fluid are temperature independent except for the density in the buoyancy term, which is changed according to the Boussinesq approximation. The governing equations in the porous region are obtained by using the volume-average method. By incorporating mentioned points, the system of the equations can be conducted as:

Fluid layer:
Continuity equation:

$$\nabla . V = 0 \tag{1}$$

Momentum equation :

$$\partial V/\partial \tau + V . \nabla V = -\nabla P + (1/\sqrt{Gr})\nabla^2 V + \theta(\cos\emptyset e_r - \sin\emptyset e_r) \tag{2}$$

Energy equation :

$$\partial\theta/\partial\tau + V . \nabla\theta = (1/Pr\sqrt{Gr})\nabla^2\theta \tag{3}$$

Porous layer:
Continuity equation:

$$\nabla . < V > = 0 \tag{4}$$

Momentum equation :

$$1/\varepsilon\ [\ \partial < V >/\partial t + \ < (V\ .\ \nabla)\ V >]\ = -\ \nabla< P^f >\ +\ (\ 1/\varepsilon\sqrt{\mathrm{Gr}})\ \nabla^2< V > - (< V >)/Da\sqrt{Gr} - \mathrm{F}\varepsilon/(\sqrt{\mathrm{Da}}\ )\ [< V >.< V >]\ \mathrm{J} + \theta\ (\cos\emptyset e_{\mathrm{r}} - \sin\emptyset e_{\varphi}) \qquad (5)$$

Energy equation :

$$\sigma\partial\theta/\partial\tau\ + V.\nabla\theta =\ (\ k_{eff}/k_f\ )\ 1/(\mathrm{Pr}\sqrt{Gr})\ \nabla^2\theta \qquad (6)$$

Where

$$k_{eff} = \varepsilon k_{\mathrm{f}} + (1 - \ \varepsilon)k_{s} \quad \text{and} \quad \sigma = [\varepsilon\big(\rho c_{\mathrm{p}}\big)_f + (1 - \ \varepsilon)\ (\rho c_{\mathrm{p}})_s\ ]\ /\ ((\rho c_{\mathrm{p}})_f) \qquad (7)$$

The above equation were normalized using the following dimensionless parameters:

$$V = \mathrm{v}\ /\ \sqrt{(\mathrm{g}\beta\Delta\mathrm{TRi}\ )}\quad ,\quad P =\ \mathrm{P}\ /\ (\rho(\mathrm{g}\beta\Delta\mathrm{TR_i}))\quad ,$$

$$\tau = (\mathrm{t}\ \sqrt{(\mathrm{g}\beta\Delta\mathrm{TRi}\ )})\ /\ \mathrm{R_i}\quad ,$$

$$\theta =\ (\mathrm{T}-\mathrm{T_o})\ /\ (\mathrm{T_i}-\mathrm{T_o}\ )\ ,\ R_i = \mathrm{R_i}\ /\ \mathrm{R_i}\ = 1 \qquad (8)$$

$$R_o = \mathrm{R_o}\ /\ \mathrm{R_i}\quad ,\quad b =\ \mathrm{R_{porous}}\ /\ \mathrm{R_i}$$

Where β is the thermal expansion coefficient, ρ is the fluid density, g is the gravitational acceleration, J = $V_P/|V_P|$ is a unit vector oriented along the pore velocity vector, P is the dimensionless pressure, V is the dimensionless velocity vector, e_{r} and e_{φ} are the unit vectors in the radial and angular dimensions in polar coordinate system, respectively, Da = K/R_i^2 is the Darcy number and R_{porous} is the radius of the porous layer.

The boundary conditions for the problem under consideration are represented as:

$$R_i = 1\ :\ V = 0,\ \theta = 1 \qquad , R_o = \mathrm{R_o}\ /\ \mathrm{R_i}\ :\ V = \theta = 0 \qquad (9)$$

At the interface ($b =\ \mathrm{R_{porous}}/\mathrm{R_i}$)

$$(\partial\theta\ /\ \partial\mathrm{R})_{\mathrm{f}} = [\ \mathrm{k_{eff}}\ /\ \mathrm{k_f}\ \ (\partial\theta\ /\ \partial\mathrm{R})]_{\mathrm{p}}\ ,\ \ (\partial\mathrm{V}\ /\ \partial\mathrm{R})_{\mathrm{f}} = [\ \mu_{\mathrm{eff}}\ /\ \mu_{\mathrm{f}}\ \ (\partial\mathrm{V}\ /\ \partial\mathrm{R})]_{\mathrm{p}},\ \theta_{\mathrm{f}} =\ \theta_{\mathrm{p}}\ \text{ and }\ V_{\mathrm{f}} =\ V_{\mathrm{p}} \qquad (10)$$

Where $\mu_{eff} = \mu_f / \varepsilon$.

The local Nusselt number distributions are calculated as the actual heat transfer divided by the heat transfer for pure conduction in the absence of fluid motion as follows:

$$Nu(\emptyset) = \mathrm{Q}\ /\ \mathrm{Q_{cond}} \qquad (11)$$

Meanwhile, the average Nusselt numbers calculated at the inner and outer cylinders by means of below equation:

$$\overline{Nu} = (1/2\pi\ )\int_0^{2\pi} Nu(\emptyset)\ d\emptyset \qquad (12)$$

Under steady-state conditions, both expressions in Eq.(12), should converge to the same result.

3 Method and Grid Generation

A two-dimension numerical simulation of the natural convection in annulus filled with porous medium is performed by using commercial CFD software FLUENT TM. The non-dimensionalized partial equations (Eq.(2)-(6)) together with the boundary conditions are discretized by using Finite Volume Method (FVM). The convection term and diffusion terms in equations are discretized by using second order upwind Central diffrence scheme respectively. For pressure and velocity coupling in (Eq.(1)-(2),(4)-(5)) the SIMPLE algorithm has been used. The resulting algebric equations are solved by iterative method (Gauss-Seidel) until the iterative converged solution is obtained. The convergence criteria for mass,momentum and energy equations are set as 10^{-4}, 10^{-8}, 10^{-8} respectively. The solution grid created in Gambit and is shown in Figure (2) . The gradient in the boundary layer around the cylinders surface and porous sleeve surface is more intensive and complicated than rest of the solution domain. Hence, computational cell were created with the fine mesh at these area. The grid independence test is performed for high Rayleigh number with three different sizes 16800, 60000 and 163000. Clearly, a grid size 60000 can be expected to yield acceptable accurate results. The present numerical solution was first validated against the numerical results of khanafer et al [17] for natural convection in concentric cylinders with a porous sleeve. Figures (3) and (4) illustrate a comparison of the streamlines and isotherms betw een the present solution and the results of khanafer et al [17] for various Rayleigh number, Darcy number, and thermal conductivity ratio. Both results were found in excellent agreement as depicted in Figures (3) and (4).

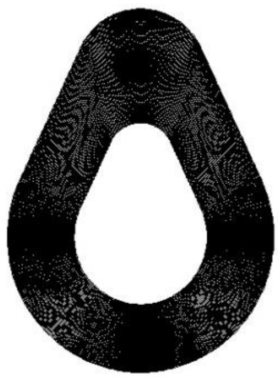

Figure 2 Solution grid in GAMBIT

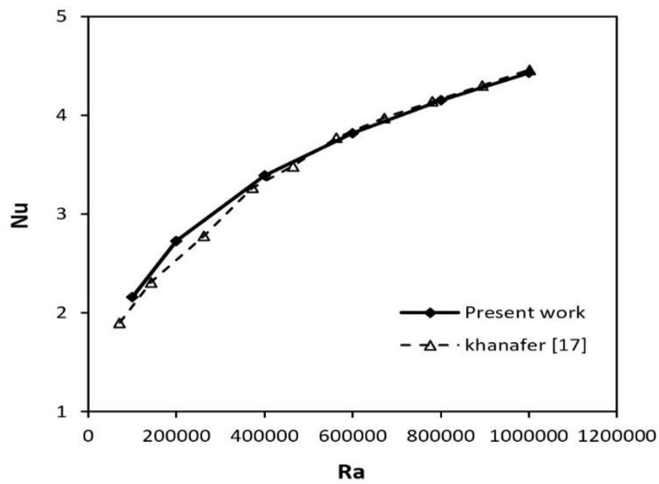

Figure 3 Compare present work and Khanafer work [17] using b= 1.5, ks/kf = 1 and Da=10^{-3}

4 Results and Discussion

The default case study in this work carried the following values: ε = 0.9, b = 1.5, Da = 10^{-3}, Ks/Kf = 1, Ra = 10^5. The results are reported in terms of the contour lines for the temperature in annulus. In addition, average Nusselt number are also documented for some cases.

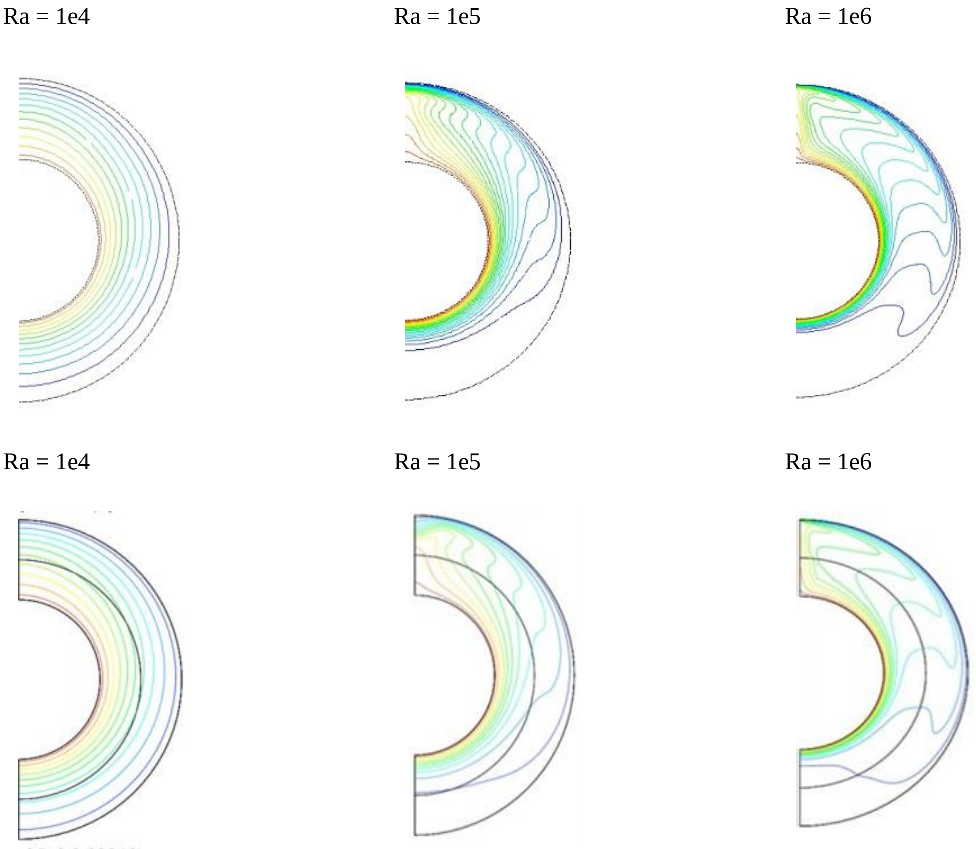

Figure 4 Compare present work and Khanafer work [17]Using b= 1.5, ks/kf = 1 and Da=10^{-3}

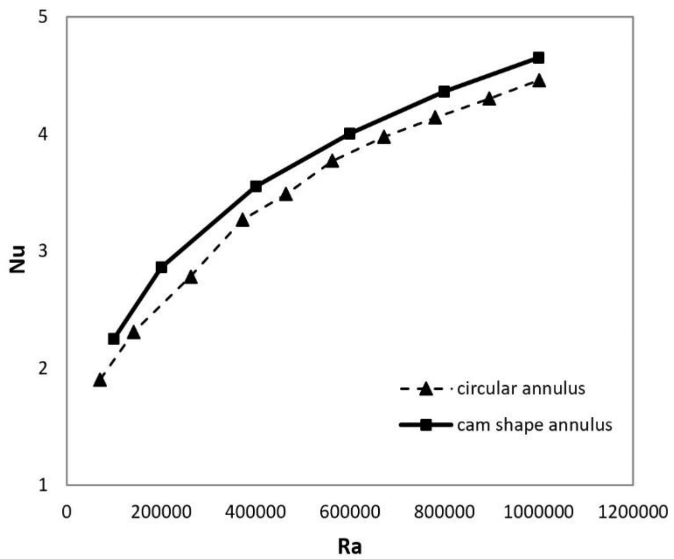

Figure 5 Effect of stream line shape on the average Nusselt number using b= 1.5, ks/kf = 1 and Da=10^{-3}

4.1 Effect of stream line shape

At the first step Figure (5) compare heat transfer in cam shape annulus and circular annulus partially filled with porous medium in the same equivalent diameter at different Rayleigh number. Figure(5) demonstrate that heat transfer in cam shape annulus is better than circular annulus. The reason of this improved is cam shape annulus is stream line shape than circular annulus so that smooth seperation and smaller wake region produce, so the pressur drop and heat transfer are improved.

4.2 Effect of rayleigh number

Figures (6) and (7) shows the effect of the Rayleigh number on the isotherms for different conductivity ratios. The Rayleigh number reflects the strength of the imposed temperature gradient between cylinders in the annulus configuration. For a small Rayleigh number and a conductivity ratio of ks/kf = 1, Figure (6) demonstrates that the isotherms in both regions resemble eccentric cams which indicates pseudo-conductive regimes for fluid and porous sleeves since the totall heat transfer by conduction and the fluid motion driven by the buoyancy force is very slow. On the other hand, as Rayleigh number increases gradually at the same time, the center of the circulation is driven upward and the isotherm contours are distorted resulting in an enhancement in the overall heat transfer. For the intermediate Rayleigh number used in this investigation, i.e., $Ra=10^5$, flow activities raises which leads to the better heat transfer.
As the Rayleigh number is increased further to $Ra=10^6$, thinner boundary layers are shown along the inner and outer sides of the cylinders which is shows the domination of the buoyancy forces as compared to viscous forces. This leads to the formation of a turbulent boundary layer on the outer side of the cam shape cylinders.
Also Figure (7) shows the effect of Rayleigh number on isotherms lines for higher thermal conductivity ratio(ks/kf =100). As shown in the figure the porous sleeve because of the higher thermal conductivity become more conductive than fluid layer. For this reason the porous sleeve have uniform temperature at low Rayleigh number but in the high Rayleigh number there is a temperature gradient in the porous sleeve .
This means that the temperature distribution in porous sleeve has strong dependency on conductivity ratio at low Rayleigh numbers, when the conduction mechanism is dominant the temperature in the porous layer is uniform but at high Rayleigh numbers this dependency be weak. Figure (8) shows the effect of varying Rayleigh number on the average Nusselt number. It is obvious at fixed Rayleigh number the Nusslet number increase with increasing in conductivity ratios.

4.3 Effect of Darcy number

The effect of the Darcy number on the isotherms is illustrates in Figure (9). For small values of the Darcy numbers, the fluid experiences a pronounced large resistance as it flows through the porous matrix causing the flow to cease in the porous sleeve as depicted in Figure (9). It can be seen from this figure that as the Darcy number decreases, the porous sleeve is considered less permeable to fluid penetration and consequently, the convective activities are suppressed in the porous sleeve. This subsequently results in hindering flow activities in the fluid layer as well. Meanwhile, the isotherm patterns indicate a pure conduction regime for the considered range of Darcy numbers as noted by the presence of thermal stratification in the radial direction of the fluid layer. Figure (10) shows the effect of Darcy number on Nusselt. It is clear from the table and physic of the flow the Nusselt number increase by increasing in Darcy number.

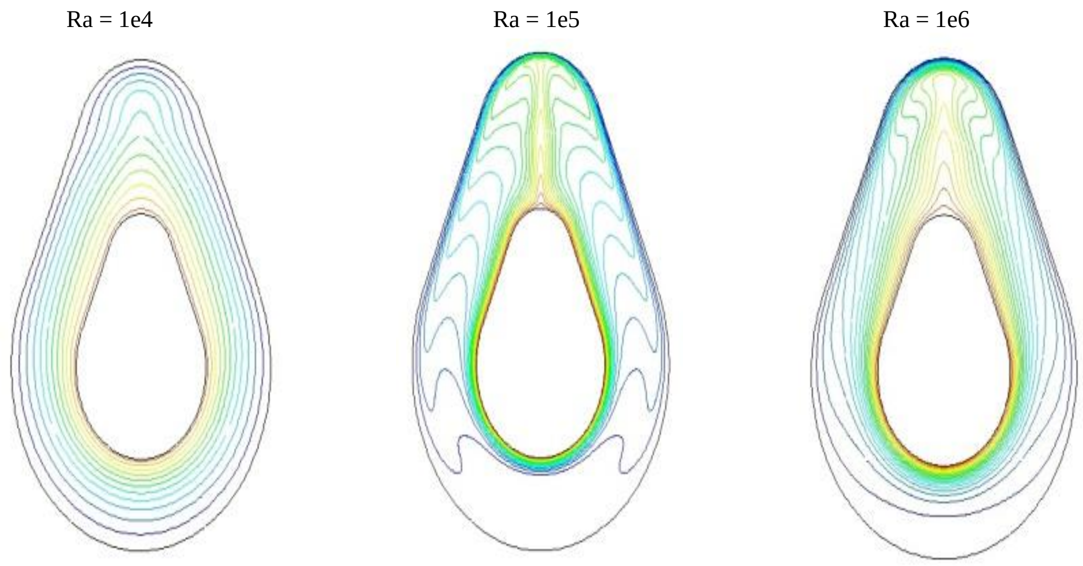

Figure 6 Effect of varying Rayleigh on the isotherms using b= 1.5, ks/kf = 1 and $Da=10^{-3}$

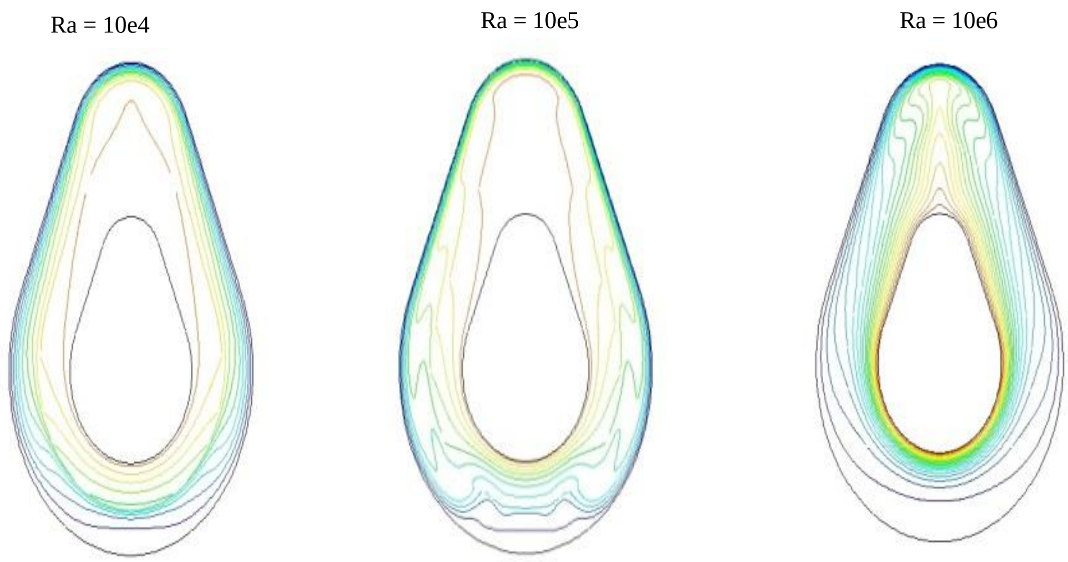

Figure 7 Effect of varying Rayleigh on the isotherms using b= 1.5, ks/kf = 100 and $Da=10^{-3}$

4.4 Effect of thermal conductivity ratio (ks/kf)

Figure (11) demonstrates the effect of diffrent conductivity ratio on the isotherms. It is clear that as the conductivity ratio increases, the porous sleeve becomes more conductive than the fluid layer. As a result, the level of the circulation activity in the fluid layer increases due to a large fluid temperature gradient.

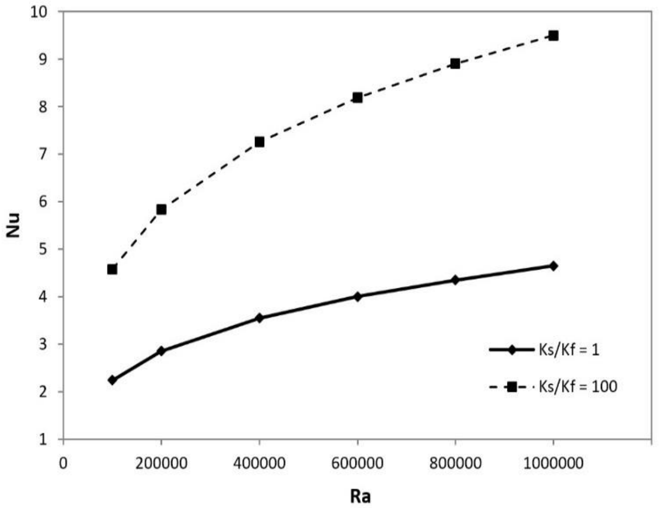

Figure 8 Effect of varying Rayleigh number on the average Nusselt number for different conductivity ratio using b= 1.5 and Da=10^{-3}

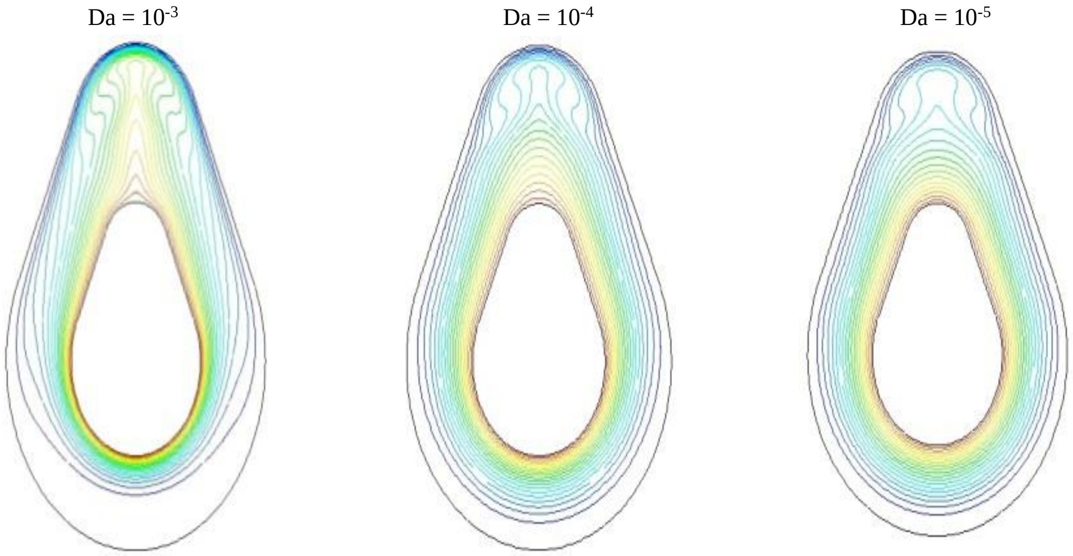

Figure 9 Effect of varying Darcy number on the isotherms using b= 1.5, ks/kf = 100 and Ra=10^5

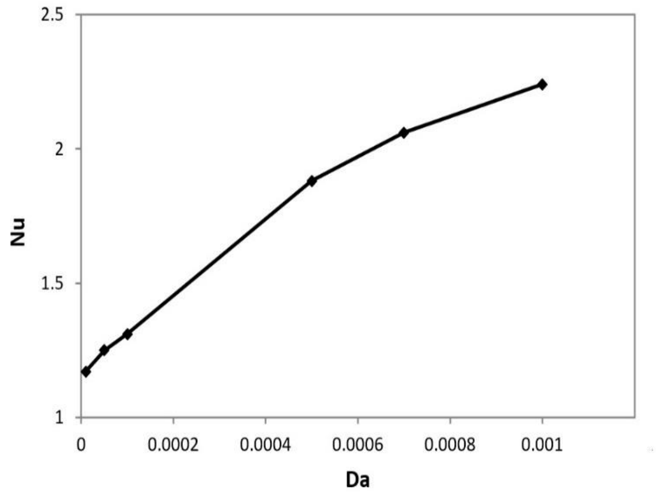

Figure 10 Effect of varying Darcy number on the average Nusselt number using b= 1.5, ks/kf = 1 and Ra=10^5

On the other hand, the strength of the convective flow in the porous sleeve decreases with an increase in the conductivity ratio leaving the porous sleeve almost isothermal as demonstrated in Figure (11). It is obvious that as the conductivity ratio increases, the spacing between the isotherms in the fluid layer decreases. Moreover, Figure (12) illustrates the effect of the increase in the conductivity ratio on the average Nusselt number. The overall heat transfer rate is raised with an increase in the thermal conductiv ity ratio due to large temperature gradients in the fluid layer. As such, the average Nusselt number increases dramatically for ks/kf < 25 as depicted from large slope of the average Nusselt number increases at a lower rate as indicate by the slope of the average Nusselt number. After that, for ks/kf > 25, the average Nusselt number increases slightly as indicate by the slope of the average Nusselt number.

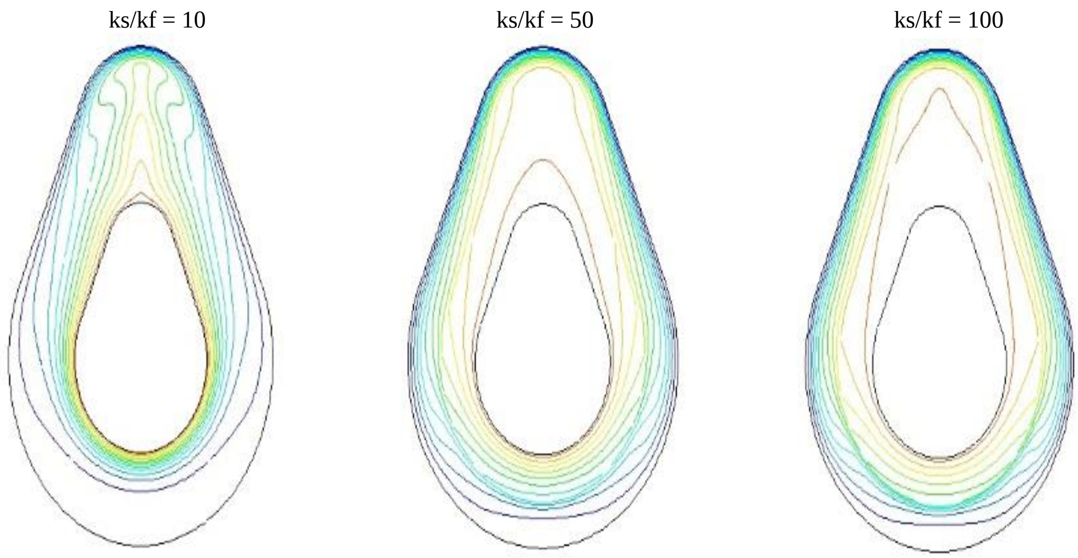

Figure 11 Effect of varying the conductivity ratio on the isotherms using b = 1.5, Da= 10-3 and Ra= 10^5

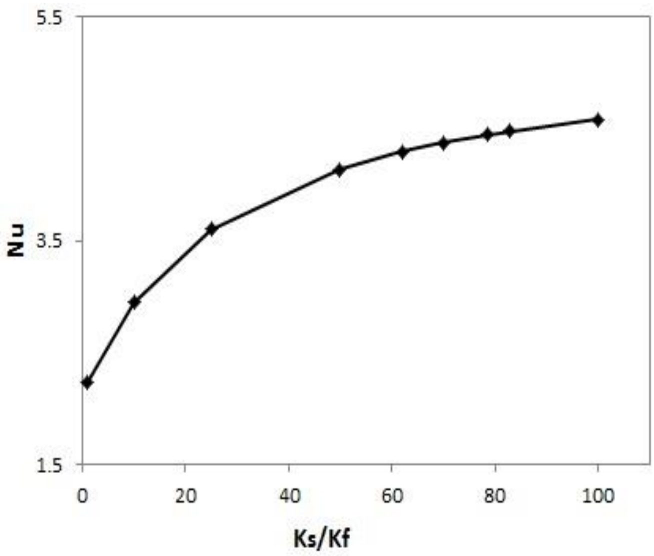

Figure 12 Effect of varying the conductivity ratio on the average Nusselt number using b = 1.5, Da= 10^{-3} and Ra= 10^5

4.5 Effect of porous layer thickness

The combined effects of porous layer thickness and thermal conductivity ratio on the isotherms for various Rayleigh numbers can be examined from Figures (13), (14), (15) and (16), respectively. For ks/kf=1, Figure (13) demonstrates depreciation in the convection currents as the porous thickness increases. This is attributed to the increase in the offered flow resistance in the entire annulus which accordingly results in higher energy being lost through the flow resistance that subsequently leads to a weak convective flow in the annulus. In a nutshell, for a thermal conductivity ratio of unity, a thinner porous sleeve improves convective activities within the fluid layer and also, permits better heat transfer. for a porous sleeve thickness of b= 1.9 indicating that the dominant heat transfer mechanism is conduction. This is comes from the fact that the presence of a porous medium within the annulus results in a force opposite to the flow direction which tends to hinder the flow motion. This consequently causes suppression in the thermal currents of the flow. This statement is obvious from the isotherm patterns as displayed in Figure (13).
When considering a large conductivity ratio, i.e.,ks/kf = 100, Figures (14) and (15) the fluid layer is less conductive than the porous sleeve and consequently the fluid layer will be subjected to a large temperature gradient as shown in Figure (15) while the temperature distribution is nearly uniform in the porous sleeve as depicted from the formation of a family of concentric isotherm lines. This is likely attributed to the fact that for a small Rayleigh number of $Ra=10^4$ and a Darcy number of $Da=10^{-3}$, the flow resistance becomes more important and, also, it becomes more difficult for the convective flow to penetrate the porous layes, which leads to weaker convective cells.

Figure (16) shows the combined effects of porous sleeve thickness and thermal conductivity ratio on the average Nusselt number for different Rayleigh numbers. Figure (16) displays that the average Nusselt number decreases with an increase in the porous layer thickness for a thermal conductivity ratio of unity, ks/kf = 1. In fact, the average Nusselt number approaches unity when a porous layer thickness greater than 1.75, which indicates that the overall heat transfer mechanism is conduction heat transfer. This observation means that thicker porous sleeve lends better insulation effect for ks/kf =1.
Also because the flow penetrate slightly through porous layer at porous layer thickness of $1.3<b<1.7$ the heat transfer changes slightly . On the other hand, it is observed from Figure (16) that For large conductivity ratio by increasing the porous sleeve thickness the Nusselt number raised so for these range of ratio for improving heat transfer it is good idea to extend porous sleeve. Moreover, At low Rayleigh number for high thermal conductivity ratio and for porous sleeve with thickness $b<1.7$ the average Nusselt number is increase slightly with increasing porous layer. This means that the flow resistance becomes more important and flow couldn't penetrate through the porous layer and as mentioned earlier the convective heat transfer become weaker than conduction.
In addition, for high Rayleigh number at high thermal conductivity ratio the behaviour of heat transfer for porous sleeve thickness $b<1.7$ is little different. As it shown in Figure (16), the Nusselt number be constant for $1.3<b<1.7$, at this level the strength of conductive and convection are the same but after $b>1.7$ the conductive heat transfer become dominant and heat transfer increase dramtically because of high thermal conductivy.
Finally, the average Nusselt number along the inner cam shape cylinder is correlated over a wide range of various pertinent dimensionless parameters such as Rayleigh number $Ra=10^3$–10^6, porous sleeve thickness (b= 1.1–1.9), Darcy number ($Da=10^{-4}$–10^{-6}), and thermal conductivity ratio (ks/kf =1–100). This correlation can be expressed as follows:

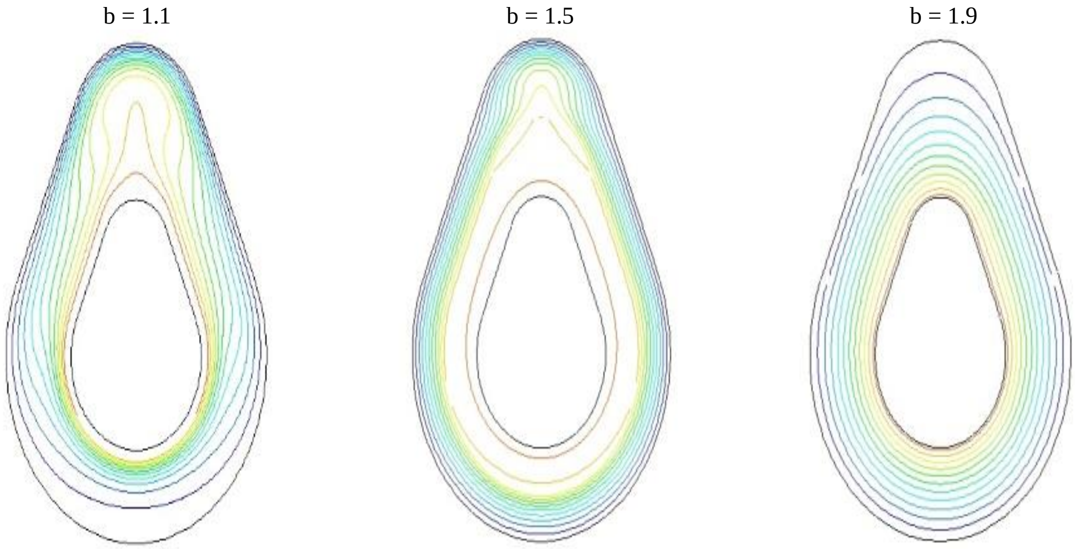

Figure 13 Effect of varying the porous thickness on the isotherms using ks/kf=1, Da= 10^{-3} and Ra= 10^5

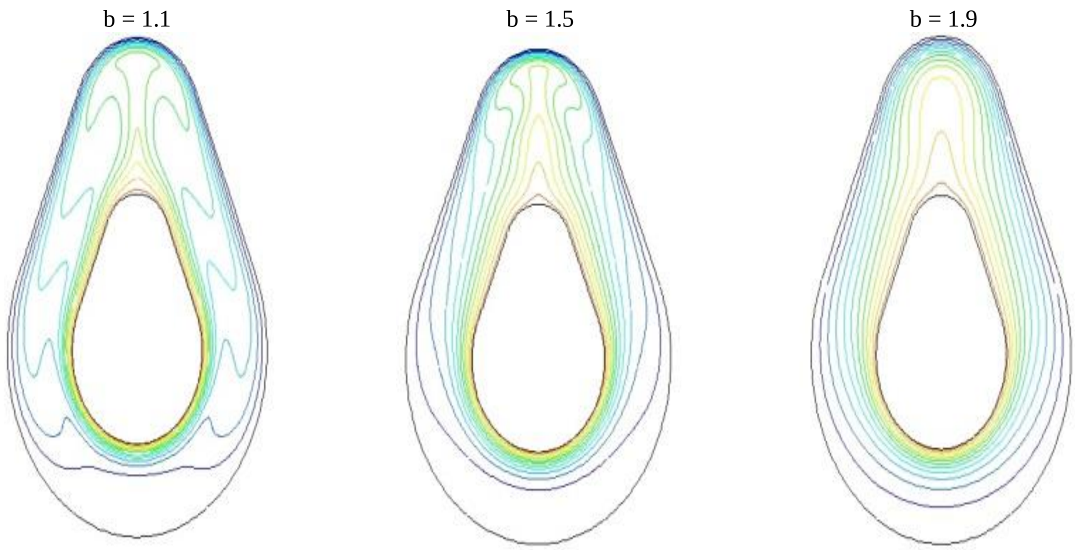

Figure 14 Effect of varying the porous thickness on the isotherms using ks/kf=100, Da= 10^{-3} and Ra= 10^5

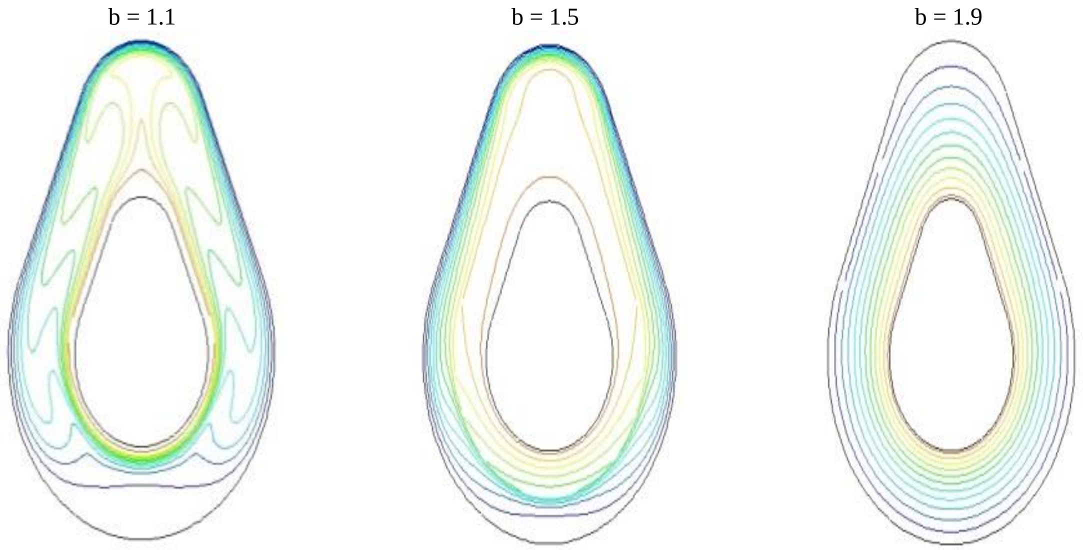

Figure 15 Effect of varying the porous thickness on the isotherms using ks/kf=100, Da= 10^{-3} and Ra= 10^4

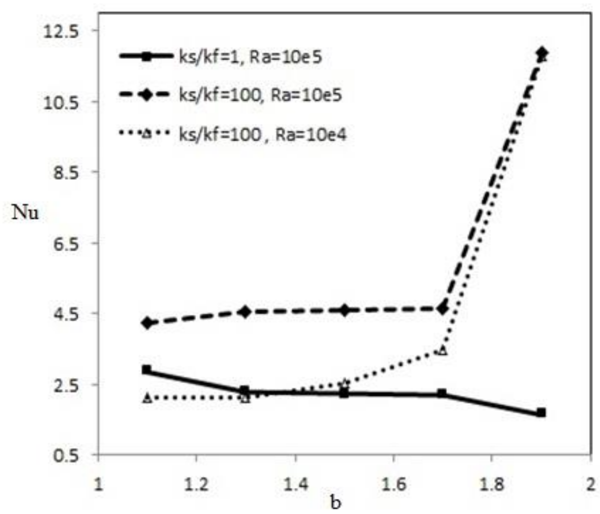

Figure 16 Effect of varying the porous thickness and conductivity ratio for different Rayleigh numbers on the average Nusselt number using Da= 10^{-3}

$$\overline{Nu} = 0.03427(23.95 + Ra^{0.38})Da^{0.1732}b^{1.332}({k_s}/{k_f})^{0.192} \quad , \text{for } {k_s}/{k_f} > 1 \qquad (13)$$

$$\overline{Nu} = 0.196Ra^{0.336}Da^{0.162}b^{-0.845} \quad , \text{for } {k_s}/{k_f} = 1 \qquad (14)$$

The correlations exposes the significance of Rayleigh number and the dimensionless porous sleeve thickness from the value of their respective exponents. Figure (17) demonstrates a good agreement between above correlations and numerical results.

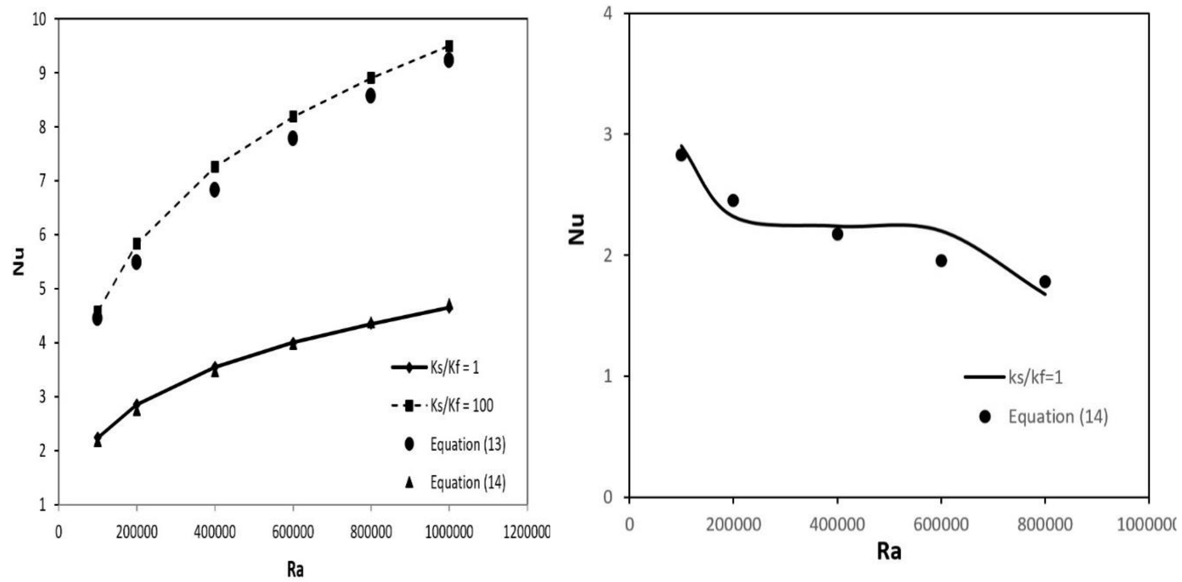

Figure 17 comparison of the average Nusselt number between the numerical results and Correlations

5 Conclusions

Natural convection heat transfer in an cam shape cylinders partially filled with a fluid-saturated porous medium is studied numerically under steady-state condition. The momentum and energy transport phenomena were explored for various pertinent dimensionless parameters such as the Rayleigh number, Darcy number and thermal conductivity ratio , and porous sleeve thickness. The inner and outer surfaces of the annulus were maintained at constant temperatures T_i andT_o, respectively, with $T_i > T_o$. In addition, the governing equations were solved using the finite volume method. The results of this investigation illustrate that the effect of stream line shape annulus were improved the heat transfer compared to circular annulus and Rayleigh number has a significant effect on the overall heat transfer. also the average Nusselt number are found to be dependent mostly on the thermal sleeve as they exhibit better conductivity ratio for thick porous layer. for shape factor and Darcy number, results show's that cam shape annulus increased the Nusselt number about 7 percent and by increasing the Darcy number the Nusselt number also increases. finally for porous sleeve thickness results shows that at large thermal conductivity ratio and at different Rayleigh number by increasing the porous sleeve thickness, Heat transfer improved Dramatically. How ever, for thermal conductivity ratio of unity, the overall heat transfer rate decrease by increasing the porous layer thickness.

Finally, the average Nusselt number calculated along the inner cam shape cylinder is correlated over a wide range of various pertinent dimensionless parameters. A results shows a good agreement between correlation and numerical results.

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Nomenclature

b	dimensionless porous sleeve thickness [-]
c_p	specific heat at constant pressure, [j m^{-1} k^{-1}]
Da	Darcy number, $(= K/R_i^2)$, [-]
e_r , $e_\emptyset$	unit vector in radial and angular direction, respectively [-]
F	Forchheimer costant [-]
g	gravitational acceleration, [m s^{-2}]
Gr	Grashof number, (= $g\beta\Delta T R_i^3/\nu^2$), [-]
J	unit vector oriented along the pore velocity vector [-]
k	thermal conductivity, [w m^{-1} k^{-1}]
K	permeability, [m^2]
L	length of the annulus, [m]
Nu	Nusselt number [-]
$\overline{Nu}$	average Nusselt number[-]
p	pressure, [N m^{-2}]
P	dimensionless pressure [-]
Pr	prandtl number (= ν/α)
Q	total heat transfer, [w]
r	radial coordinate
R	dimensionless radial coordinate
R_i	radius of the inner cylinder, [m]
R_o	radius of the outer cylinder, [m]
R_{porous}	radius of the porous sleeve, [m]
Ra	Rayleigh number, [Gr.Pr] [-]
t	time, [s]
T	temperature, [K]
v	dimensional pore velocity, [m s^{-1}]
V	dimensionless pore velocity, [-]

Greek symbols

α thermal diffusivity, [$m^2 s^{-1}$]
β thermal expasion coefficient,[K^{-1}]
ε porousity, [-]
∅ Angular coordinate, [-]
μ dynamic fluid viscosity [kg $m^{-1}s^{-1}$]
θ dimensionless temperature,(= T- T_o)/ (T_i- T_o)) , [-]
ρ density, [kg/m^3]
σ heat capacity ratio, (= $[\varepsilon(\rho c_p)_f + (1-\varepsilon)\ (\rho c_p)_s\ ]\ /\ ((\rho c_p)_f), [-]$
τ dimensionless time, (= (t(gβΔTRi $)^{1/2}$) / R_i , [-]

Subscripts

cond conductio
eff effective
f fluid
i inner
o outer
s solid

چکیده

در این مقاله، انتقال حرارت جابجایی طبیعی در فضای بین دو لوله بادامکی شکل، افقی و هم‌مرکز، که بخشی از آن با یک لایه محیط متخلخل اشباع از سیال پر شده است، به صورت دو بعدی مورد بررسی قرار گرفته است. هر دو سیلندر در دماهای ثابت و یکنواخت و با فرض اینکه دمای سیلندر بیرونی نسبتا پایین تر از سیلندر داخلی در نظر گرفته می شوند. علاوه بر این، اثر فورچهایمر و برینکمن در داخل لایه متخلخل مورد توجه قرار می‌گیرد. علاوه بر این، در لایه متخلخل شرط تعادل حرارتی محلی در نظر گرفته شده است.
ضریب تخلخل به صورت یکنواخت و ثابت فرض شده و اندازه آن $\varepsilon = 0.9$ در نظر گرفته شده است. هدف اصلی این مطالعه بررسی اثرات خطی جریانی شدن لوله، نسبت هدایت حرارتی‌ها (ks / kf) و ضخامت لایه متخلخل بر حرکت جریان ناشی از نیروی شناوری در شرایط پایا است. این اثرات با استفاده از پارامترهای بی‌بعد زیر مورد بررسی قرار می‌گیرد: $Ra = 10^4$-10^6 ، $Da = 10^{-3}$-10^{-5} .
نتایج نشان می‌دهد که عدد ناسلت عمدتاً تحت تاثیر ضخامت لایه متخلخل و عدد رایلی است.