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The Effect of Support Parameters on the Force Transmissibility of a Flexible Rotor

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Rotating machinery support design with the aim of reducing the force transmitted to the foundation has significant importance regarding the various applications of these machineries. In this paper presents a rapid approximate method for calculating the optimum support flexibility and damping of flexible rotors to minimize force transmissibility in the vicinity of the rotor first critical speed. First, the governing equations for the Jeffcott rotor model mounted on flexible supports are derived and the optimal parameters for the supports have been analytically achieved. Next with consideration of the complexity and tedious of the analytic equations, a numerical algorithm for determination of the optimal support design parameters is introduced which may be applied to any rotor model regardless of the model complexity and number of degrees of freedom. The simulation results show the effect of optimal parameters on the considerable reduction of the force transmitted to the foundation. The method has the advantage of being quickly and easily applied and can reduce analysis time by eliminating a time consuming search for the approximate optimum damping using more exact methods.

Keywords: Force Transmissibility, Support, Jeffcott Rotor, Rotor designs, Optimal Damping.

1 Introduction

In rotating machinery such as turbojet engines, compressors, and turbines, the rotors often encounter large forces transmitted to the support structure. The problem becomes more severe as machinery is designed to be lighter and, hence, more flexible. Large force transmissibility may be due to several causes and may be roughly grouped under the headings of synchronous and non-synchronous response. Synchronous response is usually associated with unbalance in the rotor. This unbalance may result from either the manufacturing process or the assembly of the components. Even if a rotor is well balanced initially, the balance degrades with rotor use. Thermal gradients can cause warping of the shaft. Erosion of compressor or turbine blades can alter the balance of the rotor. Therefore, in the design of machinery provisions should be made so that the increase of unbalance with operation will not overload the bearings or cause excessive rotor amplitudes.

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A large body of knowledge and sophisticated analytical methods exists for analyzing the unbalance response and stability [1-3]. Barrett et al. presented the effects of bearing support flexibility on rotor stability and unbalance response [4]. Gunter examined the influence of flexible damped supports on rotor amplitudes and forces transmitted over a speed range encompassing several critical speeds. An oil squeeze-film damper support was then designed to provide the required damping at the assumed unbalance conditions [5]. Ishida et al. studied the passive vibration control of nonlinear rotor systems using a dynamic damper. The Newton-Raphson method is used to determine the parameters of the dynamic damper for the nonlinear rotor, and passive vibration control utilizing the dynamic damper is achieved in the nonlinear rotor system [6].

The successful design of rotor-bearing system needs to be analyzed with respect to the optimal parameters of the dynamic support [7-9]. Natraj and Ashrafiuon studied the effect of rotor spin speed and the unbalance value on the optimal value of the bearing parameter. They derived analytical formulas based on the analysis of two-degree-of-freedom systems [10]. A. E1-Shafei et al. presented a study of the optimal design of SFDs for multi-mode rotors. The optimal design program obtained the best possible damper parameters for a given rotor to satisfy the minimization requirements for maximum vibration amplitude function [11]. Lin and Cheng studied the optimal design of complex flexible rotor support systems. Optimization using system strain energy is shown to be a convenient way to handle such systems. Multiple constraints such as the damped critical speeds, limitations on transmitted forces and the amplitudes of the deflection of shafts and disks, and stability considerations, are used to meet the engineering requirements [12].

A variety of new techniques and applications of optimization have been developed over the past years. Because of the complexity of rotor bearing system analysis and time consuming nature of process, an optimization procedure need to be employed that the design would be time-efficient and find the satisfactory design parameters to meet particular performance requirements. On the other hand, there is a range of support damping and stiffness values which will improve stability and minimizing the rotor amplitude. At present, there is a lack of information available in the literature on the support design in order to reduce force transmitted. It is, therefore, highly desirable to have an easily applied method to obtain an estimate of the optimum bearing damping for forces transmitted when operating above the first critical speed. The purposes of the current research are to evaluate the influence of flexible damped supports on rotor amplitudes and forces transmitted, and demonstrate design procedure for rotor model regardless of the model complexity and number of degrees of freedom. An analytic study was undertaken to determine the influence of flexible supports on the synchronous unbalance response of the single-mass Jeffcott rotor, and to optimize the support system characteristics so as forces transmitted. The approach method of the tuned damper support system is similar to that employed by Gunter [5] for minimizing the rotor amplitude. Although, the optimization procedure may be readily extended to more complex rotor-bearing systems by using the proposed outlined flow chart. Simulation shows that the system optimization design can effectively improve the transmitted force.

2 Flexible Rotor Dynamics Equation

A typical rotating system is composed of various components, such as rotors, disks, bearings and supports. The dynamic response of a rotor-bearing system can be obtained by the set of linear differential equations from the Lagrangian method. The system parameters including the inertia properties of rigid disk, stiffness of rotating shaft, coefficients of bearing and supports all of which have significant influence on the dynamic characteristics of the rotor-bearing system.

In this investigation, a typical Jeffcott rotor is considered to be mounted in linearized, flexible, damped supports. Figure (1) represents the single-mass Jeffcott rotor mounted in damped elastic supports. In the Jeffcott model, the shaft is considered as a massless elastic member and the rotor mass is concentrated in a disk mounted at the center of the span. The shaft is supported in linear bearings which are mounted in damped flexible supports. The symmetric rotor system may be described for dynamic simulation by six coordinates or independent degrees of freedom. The support motion of either end of the rotor is given by the absolute coordinates (X_a, Y_a) and the bearing motion is denoted by (X_j, Y_j) in the fixed coordinate system. The rotor motion at the mid-span is described in the coordinate system by (X_d, Y_d). The support and bearing characteristics are assumed to be symmetric to simplify the analysis of this rotor-bearing system. Neglecting rotor acceleration and the disk gyroscopic, the governing equations of motion for the rotor, bearings, and support system in complex notation reduce to the following

$$M\ddot{Z} + C\dot{Z} + KZ = F \tag{1}$$

Where M, C and K are the system mass, damping coefficient and stiffness matrices respectively, can be represented by:

$$M = \begin{bmatrix} m_a & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} c_a + c_b & 0 & -c_b \\ 0 & c_s & 0 \\ -c_b & 0 & c_b \end{bmatrix}, K = \begin{bmatrix} k_a + k_b & 0 & -k_b \\ 0 & k_s & -k_s \\ -k_b & -k_s & k_s + k_b \end{bmatrix} \tag{2}$$

in which, $\mathrm{M_d}$ and $\mathrm{M_a}$ are the disk and support masses respectively. Damping and stiffness coefficients for the rotor shaft, bearings and supports are denoted by $\mathrm{c_s}, \mathrm{k_s}$ and $\mathrm{c_b}, \mathrm{k_b}$and $\mathrm{c_a}, \mathrm{k_a}$, respectively.

The rotor displacement vector, Z and Force vector, F according to the rotor model become

$$F = \begin{Bmatrix} 0 \\ me\omega^2 \\ 0 \end{Bmatrix} e^{i\omega t} \ , Z = \begin{Bmatrix} z_a \\ z_d \\ z_j \end{Bmatrix} \tag{3}$$

After the initial transient motion has damped out, the steady-state unbalance response may be assumed $Z_k = A_k e^{i\omega t}$ in which A_k is in general complex. The differential equations of motion may be reduced to a set of algebraic equations for the determination of the rotor steady-state motion. It is assumed that the damping coefficient $\mathrm{c_b}$ is equal to zero for simplicity. This assumption may be suitable for rolling bearing in common engineering practice rotordynamics, therefore one can write.

$$\begin{aligned} &(-m_a\omega^2 + i\, c_a\omega + k_a + \ k_b)A_a - k_bA_j = 0 \\ &(-m_d\omega^2 + i\, c_a\omega + k_s)A_d - k_sA_j = m_d e\, \omega^2 \\ &-k_bA_a - k_sA_d + (k_b + \ k_s)A_j = 0 \end{aligned} \tag{4}$$

In order to simplicity and better interpretation (more convenient explanation) of governing equations, the following dimensionless parameters are defined.

$$\begin{aligned} &\beta = k_b/k_a \quad , \quad \mu = k_s/k_b \\ &\omega_n^2 = k_s/m_d \quad , \quad \omega_a^2 = k_a/m_a \\ &\Omega_a = \omega/\omega_a \quad , \quad \Omega = \omega/\omega_n \\ &c_{ca} = 2\sqrt{k_a m_a} \quad , \quad \zeta = c_a/c_{ca} \ , \ \zeta_s = c_a/c_{ca} \end{aligned} \tag{5}$$

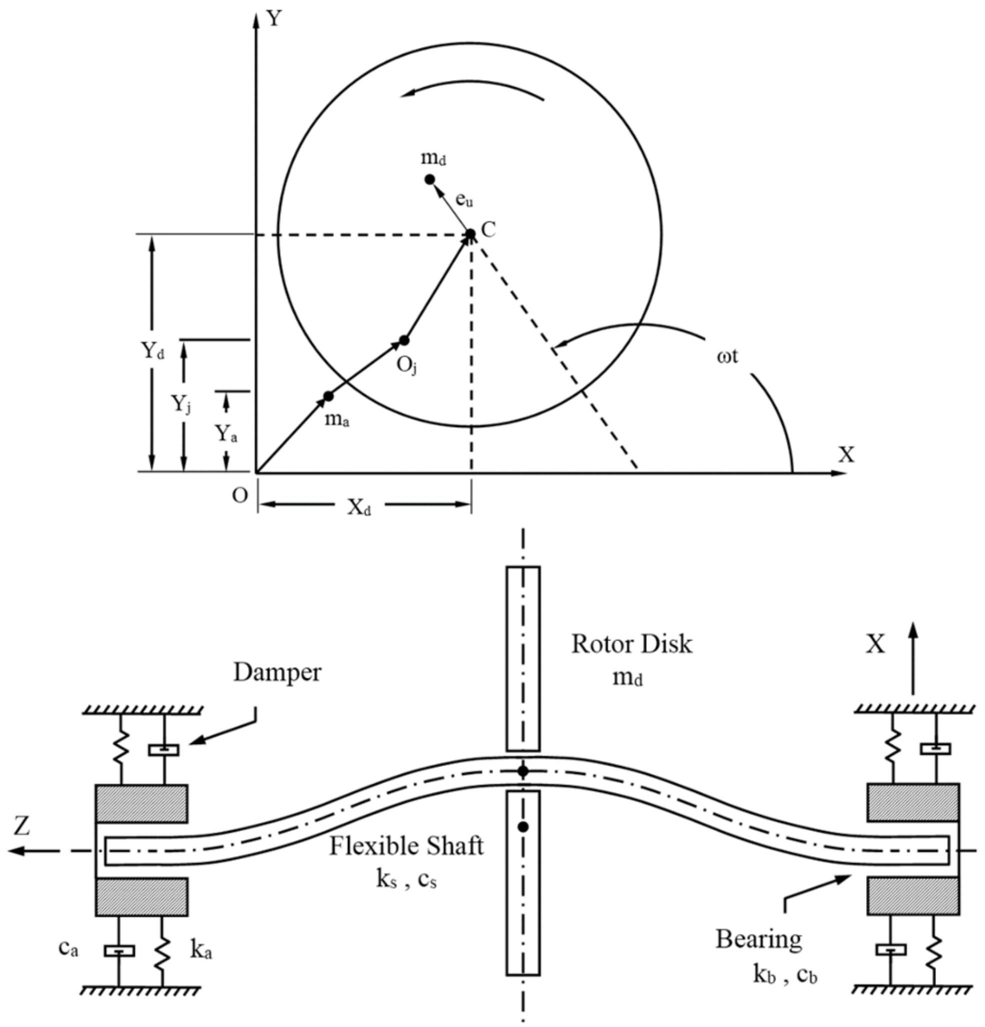

Figure 1 Schematic Diagram of Jeffcott Rotor on Damped Flexible Supports.

Where ω is the rotating speed. Equations (4) can be written in terms of non-dimensional quantities as follow

$$(-\Omega_a^2 + i\,2\,\zeta\,\Omega_a + 1 + \beta)A_a - \beta\,A_j = 0$$
$$(-\Omega^2 + i\,2\,\zeta_s\,\Omega + 1)A_d - A_j = e\,\Omega^2 \tag{6}$$
$$-A_a - \mu\,A_d + (1 + \mu)A_j = 0$$

Solving algebraic Equation (6) for the rotor amplitude yields

$$\frac{A_d}{e} = \left[\frac{D_1 + i\,D_2}{[H_1(1-\Omega_a^2) - \mu\beta\Omega^2 - 2\,\zeta_s\,\Omega\,D_2] + 2\,i[\zeta\,\Omega_a H_1 + \zeta_s\,\Omega\,D_1]}\right]\Omega^2 \tag{7}$$

Since the support amplitude in the frequency domain is given by

$$\frac{A_a}{e} = \frac{\mu\beta\Omega^2}{[H_1(1-\Omega_a^2) - \mu\beta\Omega^2 - 2\,\zeta_s\,\Omega\,D_2] + 2\,i[\zeta\,\Omega_a H_1 + \zeta_s\,\Omega\,D_1]} \tag{8}$$

Where $H_1 = [1 - \Omega^2(1+\mu)]$, $D_1 = (1-\Omega_a^2)(1+\mu) + \mu\beta$ and $D_2 = (2\,\zeta\,\Omega_a)(1+\mu)$.
By using the support amplitude, the magnitude of the force transmitted through the supporting structure and foundation are derived, in order to determine support parameters.

The force transmitted through the support system can be represented by

$$F_a = A_a\sqrt{k_a^2 + (\omega c_a)^2} \tag{9}$$

An indication of the effectiveness of the support system in attenuating the forces transmitted to the foundation is the support dynamic transmissibility factor TRD which will be defined as the ratio of the magnitude of the transmitted support force $F_a$to the rotating unbalance load $m_d e\omega^2$.
The dynamic transmissibility for the support is defined as

$$TRD = \frac{F_a}{m_d e\omega^2} = \frac{A_a\sqrt{k_a^2 + (\omega c_a)^2}}{m_d e\omega^2} \tag{10}$$

By substituting Eq. (8) into Eq. (10) and Simplifying, the transmissibility factor becomes

$$TRD = \left\{\frac{1 + [2\,\zeta\,\Omega_a]^2}{[H_1(1 - \Omega_a^2) - \mu\beta\Omega^2 - 2\,\zeta_s\,\Omega\,D_2]^2 + [2\,\zeta\,\Omega_a H_1 + 2\,\zeta_s\,\Omega\,D_1]^2}\right\}^{1/2} \tag{11}$$

The above expression leads to the well-known conclusion that to minimize the forces transmitted through the support for supercritical speed operation in the Jeffcott model, the support damping should be zero and the support stiffness should be as light as possible. This is a highly undesirable design practice for several reasons since large rotor amplitudes and forces transmitted may be encountered when passing through the rotor critical speeds, and also the rotor system would be extremely shock sensitive and particularly susceptible to self-excited whirl instability under such conditions. A compromise support damping coefficient should be selected to minimize the rotor amplitudes and the forces transmitted over the operating speed range and also be sufficient to insure adequate rotor stability.

3 Optimal Design Procedure

Modern high speed rotor bearing systems are complex. With increasing performance criteria, the design process of these systems usually requires the integration of the design and analysis. Typically, design objectives for rotor systems include placement of critical speeds, minimization of response amplitudes and bearing loads, optimal choice of balance planes, and maximization of the onset of instability speed. In this work, synchronous response, stability, and transmitted load to the supporting structure in the operational speed range is the objective function. Many numerical optimization methods have been developed and used for design optimization of rotor-bearing systems. Most of these optimization methods are cumbersome. Therefore, the approach method is highly desirable and easily applied method to obtain an estimate of the optimum bearing stiffness and damping for amplitude rotor and forces transmitted.

3.1 Tuned Support Parameters for Synchronous Response

To determine how effective the flexible support is in attenuating the rotor amplitude, a new ratio $\alpha = \frac{\omega_a}{\omega_n} = \frac{\Omega}{\Omega_a}$ is defined. Also, the effect of internal damping is not relevant to the theory of optimization of support parameters using the Gunter method. It is assumed that the damping ratio ζ_s is equal to zero. Then Eq. (7) become

$$\frac{A_d}{e}=\left\{\frac{[(\alpha^2-\Omega^2)(1+\mu)+\mu\alpha^2\beta]^2+[2\,\zeta\,\alpha\,\Omega(1+\mu)]^2}{[H_1(\alpha^2-\Omega^2)-\mu\beta\alpha^2\Omega^2]^2+[2\,\zeta\alpha\,\Omega\,H_1]^2}\right\}^{1/2}\Omega \tag{13}$$

This equation is plotted in Figure (2). Note that all curves pass through two points **A**, **B** on the graph, independent of the damping parameter ζ. These points are known as fixed points. Their locations are independent of the value of ζ if the ratio of the coefficient of ζ^2 to the term independent of ζ is the same in both numerator and denominator of Eq. (13).

$$\frac{[2\,\alpha\,\Omega(1+\mu)]^2}{[(\alpha^2-\Omega^2)(1+\mu)+\mu\alpha^2\beta]^2}=\frac{[2\,\alpha\,\Omega\,H_1]^2}{[H_1(\alpha^2-\Omega^2)-\mu\beta\alpha^2\Omega^2]^2} \tag{14}$$

The equation (14) is satisfied if

$$(2\,\alpha\,\Omega)^2=0\,,$$
$$\frac{(1+\mu)}{(\alpha^2-\Omega^2)(1+\mu)+\mu\alpha^2\beta}+\frac{H_1}{H_1(\alpha^2-\Omega^2)-\mu\beta\alpha^2\Omega^2}=0\,, \tag{15}$$
$$\frac{(1+\mu)}{(\alpha^2-\Omega^2)(1+\mu)+\mu\alpha^2\beta}-\frac{H_1}{H_1(\alpha^2-\Omega^2)-\mu\beta\alpha^2\Omega^2}=0$$

The first two solutions are trivial. The third yields the equation

$$(1+\mu)^2\,\Omega^4-b\Omega^2+(1+\mu+\mu\,\beta/2)\alpha^2=0\,,$$
$$b=\ (1+\mu)[\alpha^2(1+\mu+\mu\,\beta)+1] \tag{16}$$

The solution of this equation gives two values of Ω, designated Ω_c, one corresponding to each fixed point. The amplitude of motion at each fixed point may be found by substituting each value of Ω_c given by Eq. (16) into Eq. (13). Since the amplitude is independent of ζ, the value that gives the simplest calculation (namely, ζ = ∞) can be used for the calculation as bellow.

$$\frac{A_d}{e}=\left\{\frac{(1+\mu)^2}{[1-\Omega_c^2(1+\mu)]^2}\right\}^{1/2}\Omega_c \tag{17}$$

In order to determine the optimal support parameters in limiting the value of A_d/e over a full range of excitation frequencies, it is necessary to select the spring and damping constants of the support system as given by the parameters α and ζ, respectively; hence the amplitude A_d of the rotor is a minimum.

First consider the influence of the ratio α. As α is varied, the values of Ω_c computed from Eq. (16) are substituted in Eq. (17) to obtain values of A_d/e for the fixed points **A** and **B**. The optimum value of α is that for which the amplitude A_d at **A** is equal to that at **B**. Assuming that the two roots of Eq. (16) be Ω_{c1}^2 and Ω_{c2}^2, where Ω_{c1}^2 is less than 1 and Ω_{c2}^2 isgreater than 1. When A_d/e has the same value for both Ω_{c1} and Ω_{c2} in Eq. (17), whereupon

$$\Omega_{c1}^2+\Omega_{c2}^2=\frac{2}{1+\mu} \tag{18}$$

In an equation having unity for the coefficient of its highest power, the sum of the roots is equal to the coefficient of the second term with its sign changed, thereby

$$\Omega_{c1}^2+\Omega_{c2}^2=[\alpha^2(1+\mu+\mu\,\beta)+1]/(1+\mu) \tag{19}$$

From the two preceding equations, the optimum tuning α that required giving the same amplitude of motion at both fixed points, is obtained as follow

$$\alpha_{opt}=\sqrt{\frac{1}{1+\mu\,(1+\beta)}} \tag{20}$$

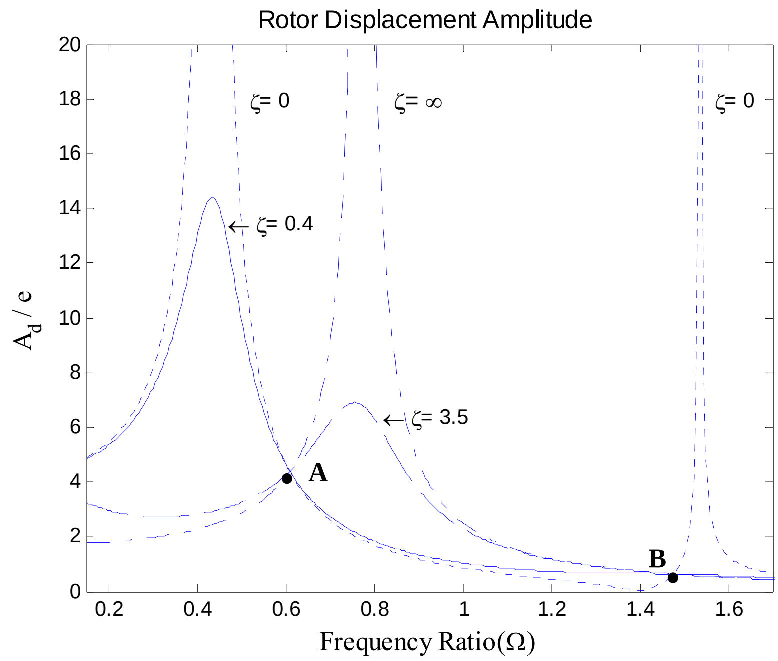

Figure 2 Absolute rotor motions with $\mu = 0.67$ and $\beta = 4$ for various values of support damping

If the effect of the damping is considered, it is possible to choose a value of the damping parameter ζ that will make the fixed points nearly the points of greatest amplitude of the motion. The condition of points **A** and **B** being the maximum means that the rotor amplitude curve should pass through the two fixed points with a horizontal tangent, that is

$$\frac{\partial}{\partial \Omega^2}\left[\frac{A_d}{e}\right]^2 = 0 \tag{21}$$

Solving this equation for ζ^2 one obtains

$$\zeta^2 = \frac{H_1^2 H_2 - (1+\mu)[H_1(\alpha^2 - \Omega^2) - \mu\beta\alpha^2\Omega^2][H_1 + H_2]}{H_1(1+\mu)^2[2\,\alpha\,\Omega\,]^2} \tag{22}$$

That is

$$H_2 = (1+\mu)(\alpha^2 - \Omega^2) + \mu\beta\alpha^2 \tag{23}$$

A proper value for the maximum damping is obtained by solving for the value of ζ in Eq. (22) when $\Omega_{A,B}^2$ is given by Eq. (18) and α has the optimum value given by Eq. (20). This gives the following approximate value for the optimum damping parameter

$$\zeta_{A,B}^2 = \frac{3}{4}\frac{\mu\,\beta(1+\mu+\mu\,\beta)}{(1+\mu)\left[2(1+\mu+\mu\,\beta) \pm \sqrt{2\,\mu\,\beta(1+\mu+\mu\,\beta)}\right]} \tag{24}$$

Taking an average of ζ_A^2 and ζ_B^2 produces

$$\zeta_{opt}^2 = \frac{\zeta_A^2 + \zeta_B^2}{2} = \frac{3}{4}\frac{\mu\,\beta(1+\mu+\mu\,\beta)}{(1+\mu)[2(1+\mu+\mu\,\beta) - \mu\,\beta]} \tag{25}$$

In the more general, the single mass rotor theory can be applied to investigate optimum support flexibility α_{opt} and damping ζ_{opt} for multi-mass rotors operating below their second bending critical speed.

3.2 Design Optimization for Forces Transmitted

The properly designed support system cannot only greatly improve the rotor amplitude, but it can result in a substantial attenuation of the forces transmitted through the support structure due to imbalance. Substituting the ratio $\alpha = \frac{\omega_a}{\omega_n} = \frac{\Omega}{\Omega_a}$ in Eq. (11) leads to

$$TRD = \left\{ \frac{[\alpha^2]^2 + [2\,\zeta\,\alpha\,\Omega]^2}{[H_1(\alpha^2 - \Omega^2) - \mu\beta\alpha^2\Omega^2]^2 + [2\,\zeta\alpha\,\Omega\,H_1]^2} \right\}^{1/2} \tag{26}$$

This equation passes through two points **P**, **Q**, independent of the damping parameter ζ. Like the amplitude vibration, the characteristic equation (26) gives two values of Ω, designated Ω_c, one corresponding to each fixed point.

$$\begin{aligned}&(1+\mu)\Omega^4 - b\,\Omega^2 + 2\,\alpha^2 = 0\,,\\ &b = 1 + 2\,\alpha^2(1+\mu) + \alpha^2\mu\,\beta\end{aligned} \tag{27}$$

In order to determine the optimal support parameters, it is necessary to select the stiffness and damping constants of the support system as given by the parameters α and ζ, respectively; hence the force transmissibilityTRD of the rotor is a minimum.

First consider the influence of the ratio α. The optimum value of α is that for which the forces transmitted TRD at **P** is equal to that at **Q**. The optimum tuning α that required giving the same magnitude at both fixed points, is obtained as follow

$$\alpha_{opt} = \sqrt{\frac{1}{1+\mu\,(1+\beta)}} \tag{28}$$

The parameter of stiffness is equal to attenuate the amplitude of vibration and to decrease the dynamic forces transmitted, makes it an attractive occurrence to the design of supporting turbomachinery.

If the effect of the damping is taken into account, it is possible to select a value of the damping parameter ζ that will make points **P** and **Q** the maximum points on the TRD. The condition of points **P** and **Q** being the maximum means that the force transmitted curve should pass through the two fixed points with a horizontal tangent, that is

$$\frac{\partial}{\partial\Omega^2}[TRD(\Omega)]^2 = 0 \tag{29}$$

Solving this equation for ζ^2 one obtains

$$\zeta^2 = \frac{-[H_1(\alpha^2 - \Omega^2) - \mu\beta\alpha^2\Omega^2][(1+\mu)(\alpha^2 - \Omega^2) + H_1 + \mu\beta\alpha^2]}{H_1(1+\mu)[2\,\alpha\,\Omega\,]^2} \tag{30}$$

A proper value for the optimum damping is achieved by solving for the value of ζ in Eq. (30) when $\Omega^2_{P,Q}$ is given by Eq. (18) and α has the optimum value given by Eq. (28). It can be shown that there are two separate damping values that causes zero slopes at fixed-points **P** and **Q** separately, it can be written as

$$\zeta_P^2 = \frac{\mu\,\beta}{2(1+\mu+2\,\mu\,\beta)}\ ,\zeta_Q^2 = \frac{\mu\,\beta}{2(1+\mu)} \tag{31}$$

Taking an average of ζ_P^2 and ζ_Q^2 produces

$$\zeta_{opt}^2 = \frac{\zeta_P^2 + \zeta_Q^2}{2} = \frac{\mu\,\beta\,(1+\mu+\mu\,\beta)}{2(1+\mu)(1+\mu+2\,\mu\,\beta)} \tag{32}$$

The multi-mass flexible rotor is represented as an equivalent single mass rotor for analysis in the vicinity of the first flexible rotor critical speed. The optimum support flexibility α_{opt} and damping ζ_{opt}can be applied as an explicit expression for the optimum bearing damping of multi-mass rotors.
The approach technique needs to be developed mathematically for each particular rotor, and the mathematics can be cumbersome for complicated rotors. Because of the complexity of rotor bearing system analysis, an outlined flow chart need to be employed that the design would be time-efficient and find the satisfactory design parameters to meet particular performance requirements. The flow chart for determination of the optimal support design parameters is presented in Figure (3).

3.3 Stability analysis of rotor

Stability is related to the solution of the damped eigen-value problem for the rotor system. The real part of eigen-value is called the growth factor, and must be negative for the system to be stable. The imaginary part of eigen-value is the damped critical speed. Stability analysis is necessary because of the effect of fluid forces in the rotor system. Incorrect bearing selection or presence of aerodynamic effects can produce the equivalent of negative damping, and give an unstable system. If a rotor is unstable at a given speed, any perturbation will cause the vibration amplitude to grow rather than decay.
A typical stability analysis that includes fluid-film bearings and the destabilizing interaction with process fluid-flow forces is customarily summarized graphically by plotting the stability parameter (namely the growth factor or logarithmic decrement) versus an increasing value of the destabilizing parameter. Logarithmic decrement is defined as the natural logarithm of the ratio of any two successive amplitudes as follow

$$Logarithmic\ Decriment = \delta = \ln\frac{A_1}{A_2} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad (33)$$

4 Simulation Studies

As an illustration of the suggested design procedure, consider the problem of providing a single Jeffcott model rotor on flexible damped supports, shown in Figure (1). The parameters used in simulation are given in Table (1).

Table 1 Parameters used for simulation

Parameter	*Rotor*	*Roller Bearing*	*Support (Absorber)*	*Unit*
Mass	*44*	-	*22*	(kg)
Damping Coefficient	*0*	*0*	*30*	$(N.s/m) \times 10^3$
Stiffness Coefficient	*58*	*87. 5*	*22*	$(N/m) \times 10^6$
unbalance load	*0. 045*	-	-	(N)
Disk Radius	*0. 254*	-	-	(m)

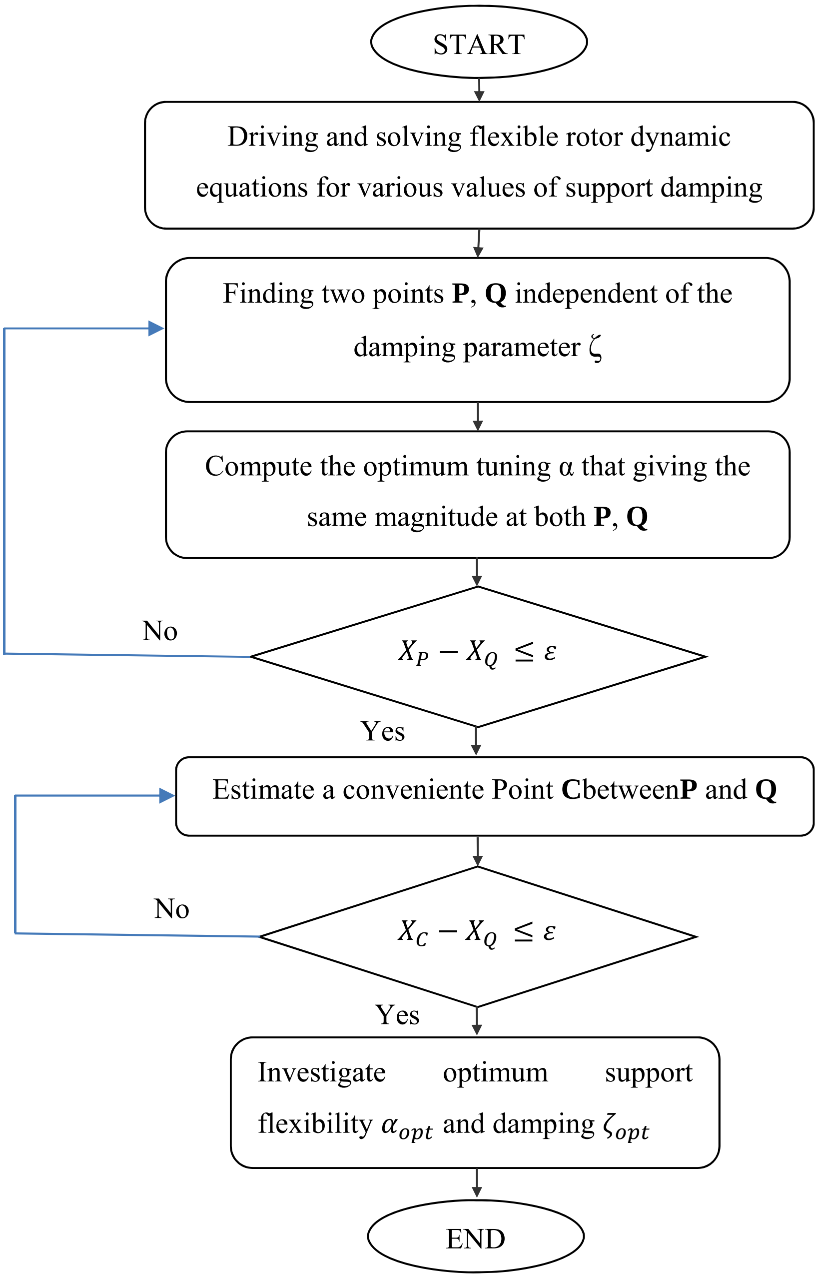

Figure 3 Flow chart for design optimization of rotor-bearing systems

Figure (4) represents the rotor amplitude versus the rotor speed for the optimum tuning α in which the tuned support will cause the same amplitude of motion at both fixed points ***A*** and ***B*** for several damping ratios. The support stiffness ratio is obtained $\alpha_{opt} = 0.4437$from equation (20). Also, the first loop of proposed algorithm in Figure (3) can be used to repeat calculations that the optimum support stiffness ratio will be equal to the value of equation (20).

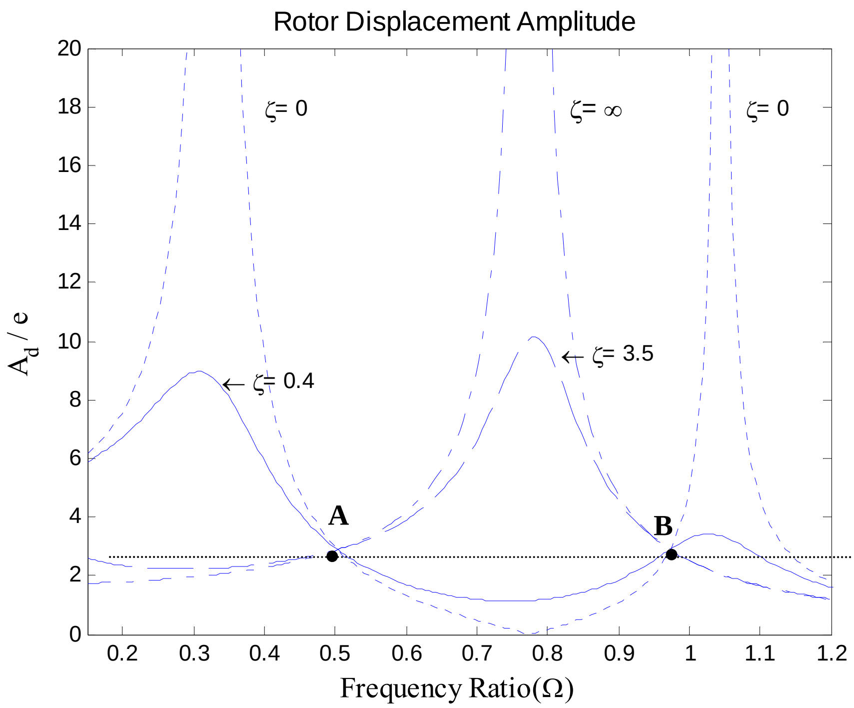

Figure 4 The rotor amplitude vs. speed ratio for various values of support damping for a tuned support system

The effect of optimum support damping that required to make the same value of A_d/e at a convenient point between A and B as at these fixed points, can be illustrated on the rotor amplitude in Figure (4). By selecting the optimum support stiffness and using equation (24), the magnitude of support damping coefficient will be $\zeta_{opt} = 0.962$ that also can be achieved from the second loop of proposed algorithm in Figure (3). The magnitude of damping required has been determined, and it is now necessary to design the damper bearing to produce this amount of damping. Figure (5) shows the resultant minimum rotor amplitudes with an undamped support, infinite damping and optimum damping ratio. It is obvious from the comparison that an optimum damping gives the same value of A_d/e at a convenient point between A and B. From Figures (4) and (5) for optimum support stiffness and damping ratio, a reduction in the maximum rotor amplitude to one-half that of a rotor running on rigid supports is represented.

In the proper design of a flexibly mounted rotor with damping, the dynamic transmissibility force should be considerably less. The problem to be considered now will be the selection of an optimum value of stiffness and damping to use in the support to minimize forces transmitted to the foundation. Figure (6) shows that $\alpha_{opt} = 0.4437$ leads to the same magnitude of force at both fixed points **Q** and ***P*** for several damping ratios. The stiffness value chosen to minimize transmissibility is equal optimum for minimizing rotor amplitude ($\alpha_{opt} = 0.4437$). The magnitude of support damping coefficient will be $\zeta_{opt} = 0.866$ that also can be achieved from the second loop of proposed algorithm in Figure (3). Thereby optimum support damping will cause decreased loads to be transmitted to the foundation.

The proper design of the damper then must take into consideration the forces, amplitudes of motion and stability criteria to be permitted throughout the operating speed range. The logarithmic decrement stability criterion for numerous values of support damping is shown in Table (2) to illustrate the system is stable because all the δ coefficients are greater than zero.

Table 2 Logarithmic decrement stability criterion

Objective	*Jeffcott Rotor Model*		
	α_{opt}	ζ_{opt}	*Stability* $\delta > 0$
Amplitudes of Motion	*0. 7748*	*0. 962*	δ *= 22, Stable*
Dynamic Transmissibility Force	*0. 4437*	*0. 866*	δ *= 11, Stable*

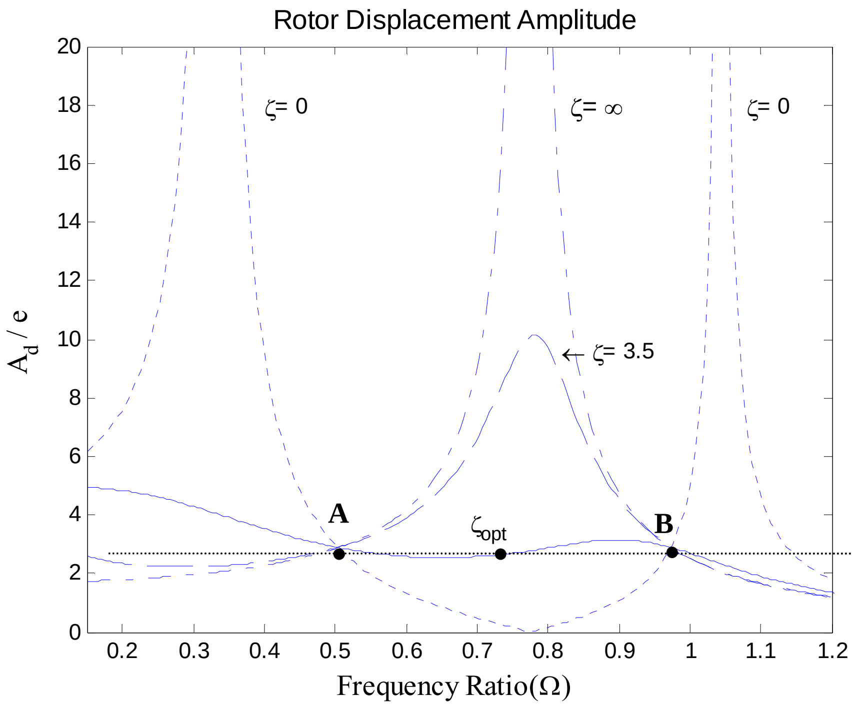

Figure 5 The rotor amplitude vs. speed ratio with optimum tuning

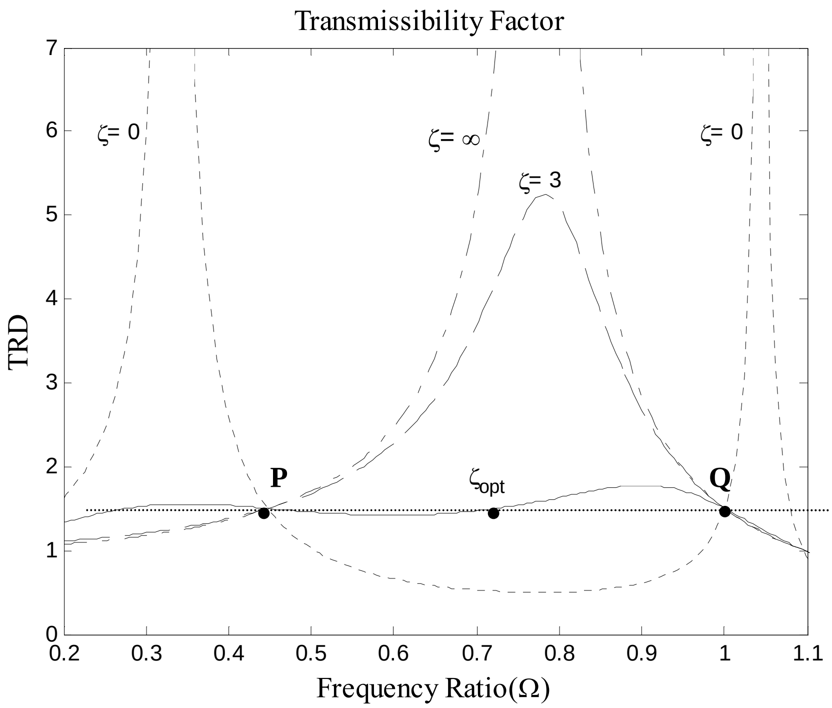

Figure 6 Force transmitted to foundation with optimum support stiffness and damping values

The effect of these parameters represents a tuned condition in which the designer enables to select the support appropriately. In general, theoretical data for a single-mass rotor can be used to determine flexible support properties to attenuate rotor amplitudes and dynamic transmissibility for a multi-mass rotor operating through the first bending critical speed.

5 Conclusion

The main objective of this investigation was to motivate and give an idea to designers who are willing to deal with optimization of rotor-bearing systems. Because of the complexity of rotor-bearing system analysis and time consuming nature of process, an optimization procedure need to be employed that the design would be time-efficient and find the satisfactory design parameters to meet particular performance requirements. The approach method was to determine the optimum support flexibility and damping of a flexible rotor to minimize vibration amplitude and the force transmissibility. The governing equations for the Jeffcott rotor model mounted on flexible supports are derived and the optimal parameters for the supports have been analytically achieved. Next with consideration of the complexity and tedious of the analytic equations, a numerical algorithm for determination of the optimal support design parameters is presented which may be applied to any rotor model regardless of the model complexity and number of degrees of freedom. The results obtained and presented in this study are to show the effect of optimal parameters on the considerable reduction of the maximum amplitude and the force transmitted to the foundation.

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Nomenclature

X_a, Y_a	Support Coordinates
X_j, Y_j	Bearing Coordinates
X_d, Y_d	Rotor Disk Coordinates
$\mathrm{M_d}$, $\mathrm{M_a}$	Disk and Support Masses
$\mathrm{c_s}, \mathrm{k_s}$	Damping and Stiffness Coefficients for the Rotor Shaft
$\mathrm{c_b}, \mathrm{k_b}$	Damping and Stiffness Coefficients for Bearings
$\mathrm{c_a}, \mathrm{k_a}$	Damping and Stiffness coefficients for Supports
A_k	Amplitude
ω	Rotating Speed
ζ	Damping Ratio
Ω	Dimensional Frequency

Subscript

a	Supports
d	Disk
s	Rotor Shaft
b	Bearings