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Time Delay Estimation and PID Controller Design using Smith Predictor for Lever Arm Platform

Zahra Ahangari Sisi*
Ph.D. Candidate

Mehdi Mirzaei†
Professor

Sadra Rafatnia‡
Professor

This paper addresses the design and experimental evaluation of a proportional-derivative-integral (PID) controller, employing a Smith predictor, for a lever arm platform with time-delay. The primary focus is on identifying the system transfer function with time-delay, which is then utilized to predict the actual delay-free output of the system using the Smith estimator. Consequently, the PID controller parameters can be established based on the delay-free portion of the model. The performance of different versions of the proposed controller is assessed through various experiments on the lever arm platform. The results obtained demonstrate good tracking performance for the arm position when operating under the designed controller, even in the presence of a delay caused by the DC motor acting as the system actuator.

Keywords: Time-delay system, PID controller, Smith predictor, Lever arm

1 Introduction

Robotic manipulators/arms are devices used to assist people in different areas such as repetitive and insecurity tasks [1]. Advantages such as reliability, accuracy, and speed make these devices widely used in aerospace, medicine, industrial automation, and satellite industries [2]. The main tasks of robotic manipulators/arms consist of trajectory tracking, reaching positions, and picking and dropping objects [3]. The control laws are used to satisfy the aims noted above in the presence of uncertainties. The control problem under uncertainties caused by actuators, vibration during operation, tip oscillation, and backlashes is a challenging problem for researchers worldwide. One important uncertainty is related to the time delay existing in the actuator.

*Ph.D. Candidate, Faculty of Mechanical Engineering, Sahand University of Technology, Tabriz, Iran, z_ahangari98@sut.ac.ir
†Corresponding Author, Professor, Faculty of Mechanical Engineering, Sahand University of Technology, Tabriz, Iran, mirzaei@sut.ac.ir
‡Assistant Professor, Faculty of Mechanical Engineering, Sahand University of Technology, Tabriz, Iran, sa_rafatnia@sut.ac.ir

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Various control methods have been suggested by the researchers to control the robotic manipulators/arms. Classical methods such as linear quadratic regulator (LQR) [4], state feedback control [5], sliding mode control (SMC) [6], optimal super-twisting sliding mode control [1], H∞ control [2], and various types of intelligent controllers such as fuzzy logic control [7] are the most widely used methods in recent years. Despite the success of modern control theory, the proportional-derivative-integral (PID) controller, is still used in industrial implementations because of its simple structure. Combining the PID control with Fuzzy logic, optimal control, and neural networks has improved the tracking performance in some applications. Also, the robust performance of this method is given for a broad class of systems and operational conditions [8]. However, the stability of the simple PID controller is highly related to the plant structure and the actuator used. The long-time delays may cause an additional phase lag and the instability of the closed loop system with a simple PID controller [9]. Therefore, the controller should be reliable in the presence of time delay during operation. Several structures have been proposed in the literature to compensate for the time delay problem.

In these studies, the Smith predictor-based methods have been proposed as an effective delay-time compensator [10-12].

Naoto et al. [13] proposed the internal model control (IMC) combined with the Smith predictor. Kaya [14] extended a modified Smith predictor-based PI-PD controller.

Truong et al. [15] presented a Smith predictor to improve the fractional-order PI controller performance.

Alrishan et al. [16] have graphically presented an approach to determine all PID controllers combined with the Smit estimator. In the other research, an IMC-PID with the Smith estimator using fractional and integer order filters is proposed [17].

Zhang et al. [18] used a Smith predictor to deal with the time delay in a robotic arm. In the proposed method, a neural network model has been developed to support the Smith predictor algorithm. Accordingly, a PID controller with neural network-based gains are adjusted to control the proposed structure. In the other study [19], a Smith predictor is designed based on neural network to adjust PID controller in the presence of time delay. While the neural network methods have been used to provide a black-box model for smith-predictor based methods, their performance is contingent on the quality and quantity of training data. As a result, they often rely on certain assumptions and may not always yield accurate and universally accepted results. On the other hand, the most important problem of these black-box models is their computational cost in real-time systems.

This study addresses the design and implementation of a Smith predictor-based PID controller to overcome the time delay of the lever arm platform actuated by a DC motor. After design and manufacture of the platform, which is representative of single-link robotic manipulators, the dynamic model of the system is identified by the classical identification method using the measured outputs of the platform.

Finally, various trajectory tests are conducted on the platform to show the performance of the proposed control method in compensating for the time delay caused by the actuator. In the results, different versions of PID controller based on Smith predictor are examined and their performances are compared.

The paper is organized as follows: After Introduction, in the second section, the control system design is presented.

In this section, the kinematic model of the lever arm is presented and the time-delayed transfer function of a second-order system is identified using the classic method. In the following, the proposed prediction method based the PID control method is presented.

The results of the practical implementation are discussed in section three. Finally, the conclusion is given in the last section.

2 Control system design

Figure (1) provides an outline of the envisioned control system. The physical setup encompasses an arm linked to a DC motor via pulleys and a conveyor tail. The encoder and gyroscope sensors are attached to the platform to provide the angular displacement and velocity of the arm, respectively. Facilitating communication, a serial connection interfaces the platform with a computer, which handles the execution of the control algorithm. In this study, the parameters of dynamic model of the system are estimated by the classical identification method and the obtained transfer function is used in the proposed method.

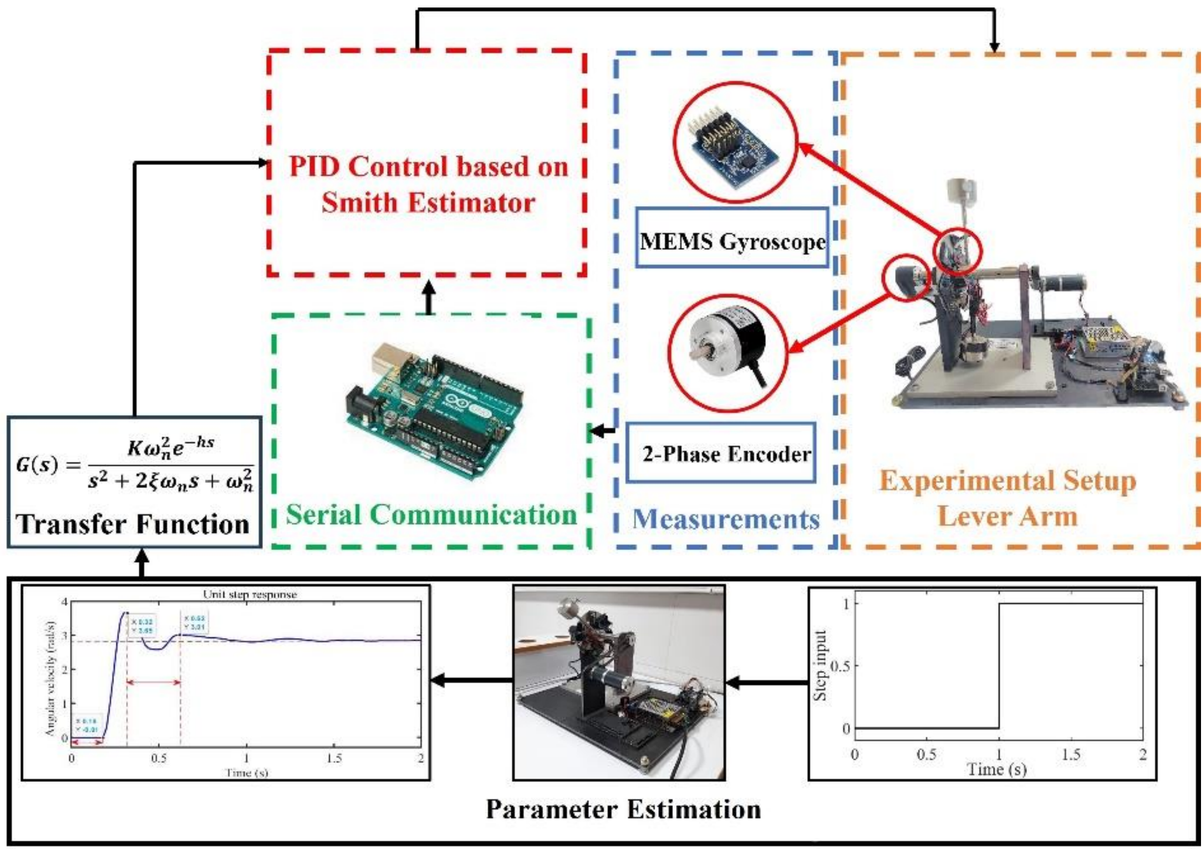

Figure 1 The overview structure of the control system

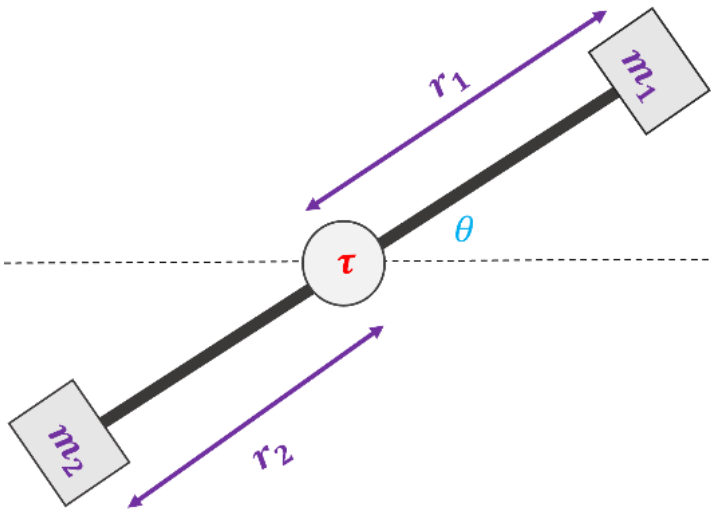

Figure 2 Schematic of the lever arm

2.1 Dynamic modelling and parameter estimation

Figure (2) shows a schematic of a lever arm it is linked to a DC motor. The proposed structure consists of a massless arm with two lumped masses positioned at the both endpoints.
In order to derive the governing equations of the system, we adopt the Lagrange approach. The kinetic and potential energies due to gravity of the system are formulated as:

$$T_k = \frac{1}{2} I_o \dot{\theta}^2, \tag{1}$$

$$V_g = (m_1 r_1 - m_2 r_2) g \, sin\theta \,, \tag{2}$$

where I_o is the moment of inertia. m_1 and m_2 denote the end-point masses and the angular displacement of the arm is presented with θ. The Lagrange equation for the lever arm is presented as:

$$\frac{d}{dt}\left(\frac{\partial T_k}{\partial \dot{\theta}}\right) - \frac{\partial T_k}{\partial \theta} + \frac{\partial V_g}{\partial \theta} = \tau \tag{3}$$

where, T_k and V_g are kinetic and potential energies of the system, respectively. Consequently, the dynamics of the lever arm can be expressed as:

$$I_o \ddot{\theta} + H(\theta) = \tau, \tag{4}$$

where $I_O = m_1 r_1^2 + m_2 r_2^2$ denotes the inertia of the lever arm, $H = (m_1 r_1 - m_2 r_2) g \, cos\theta$ is the gravity term and τ is the motor torque. By considering $\theta = x_1$, $\dot{\theta} = x_2$, the state-space model of the lever arm is represented as:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -I_0^{-1} H(x_1) + I_0^{-1} u, \end{cases} \tag{5}$$

where $u = \tau$ is the control input and $y \in R$ is the output of the lever arm. The time delay and friction are ignored in derivation of Eq. (5). By considering $m_1 r_1$ close to $m_2 r_2$, the lever arm shows the behavior of a linear second order system. In order to model the time delay and friction of the system, the general form of delayed transfer function for a second-order system is defined as follows:

$$\tilde{G}(s) = \tilde{G}_0 e^{-\tilde{h}s}, \tag{6}$$

in which

$$\tilde{G}_0 = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}. \tag{7}$$

In Eqs. (6) and (7), K, ξ, ω_n and $\tilde{h}$ indicate the gain of the system, the damping ratio, the natural frequency, and the time delay, respectively. These parameters will be identified from the measured data. Various methods in the literature are available for calculating the parameters of transfer function model using input-output data. In this study, the step response analysis is employed as a classical identification method. The output of a continuous-time system is called the step response when a unit step signal is considered as the input of the system.

Three possible responses of second-order system to unit step input, depending on ξ, are underdamped response $\xi < 1$, critically damped response $\xi = 1$, or overdamped response $\xi > 1$ [20]. The time response of the lever arm platform to unit step input, shown in Figure (3), demonstrates an underdamped behavior. Note that, a MEMS-grade gyroscope is attached to the platform to provide the time response of the platform. The system gain K is obtained from the steady-state response as:

$$y_{ss} = \lim_{x\to\infty} y(t) = K. \tag{8}$$

From Figure (3), it is found that $M_p = 3.65$, $K = 2.96$, $T = 0.23$ and $h = 0.18$. Accordingly, the values of damping ratio and the natural frequencies are calculated as $\xi = 0.48$ and $\omega_n = 27.54$ from the following formulas [21]:

$$M = \frac{M_p}{K} - 1, \tag{9}$$

$$\omega_n = \frac{2}{T}[\pi^2 + (lnM)^2]^{1/2}, \tag{10}$$

$$\xi = \frac{-lnM}{[\pi^2 + (lnM)^2]^{1/2}}. \tag{11}$$

By using the calculated values of the parameters from the step response diagram, depicted in figure, the second-order transfer function with time delay, defined in (6), for the lever arm platform is obtained as follows:

$$\tilde{G}(s) = \frac{2242.8e^{-0.18s}}{s^2 + 26.43s + 757.9}. \tag{12}$$

The obtained transfer function (12) is used to design a Smith predictor-based PID controller to overcome the time delay in tracking the desired reference.

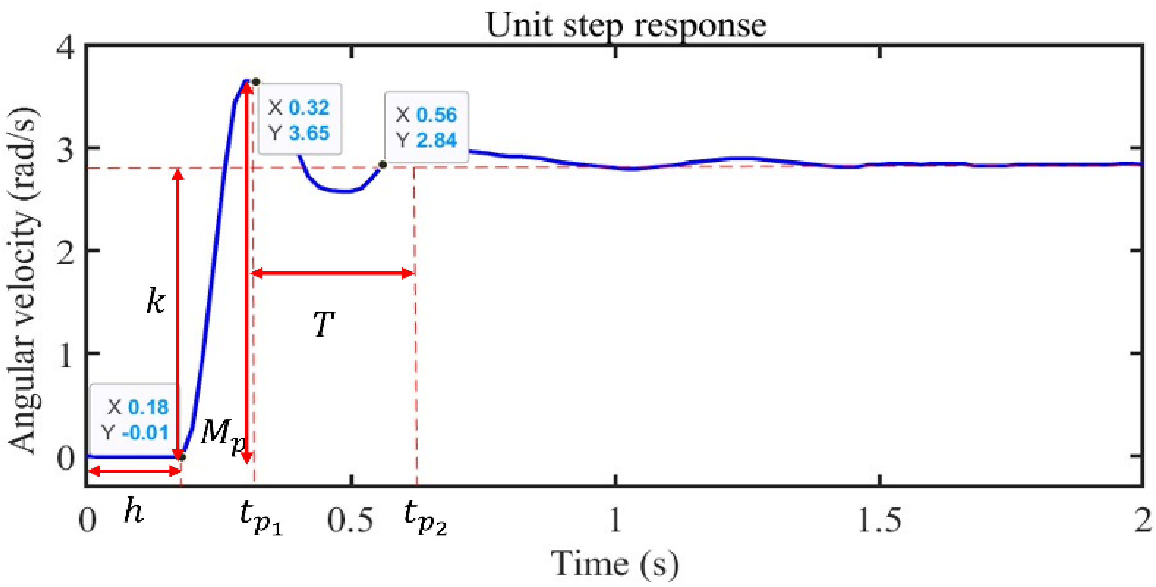

Figure 3 Unit step response of second order lever arm system

2.2 Design of Smith predictor-based PID control

In this study, to deal with the time-delay of the system, a Smith predictor-based PID controller is designed based on the system model including the time delay. The Smith predictor, often referred to as Smith estimator, is a strategy utilized to enhance the performance of control systems in the presence of time delay. To describe the idea of the Smith estimator, a plant with a time delay is considered as:

$$G = G_0 e^{-hs}, \tag{13}$$

where G_0 does not have any time delay and h is a positive constant signifying the time delay. In practice, G and h may not be available due to uncertainties and therefore the nominal $\tilde{G}$ and $\tilde{h}$ should be identified from the measured data. The Smith predictor and the PID controller will be designed based on the nominal $\tilde{G}$ and $\tilde{h}$.
The structure of Smith predictor-based control system including the controller and the predictive part is illustrated in Figure (4). This structure involves a comparison between the actual output and the output generated by the prediction. Any difference between these two outputs is mitigated through a feedback loop [22]. The identified $\tilde{G}$ and $\tilde{h}$ play a crucial role in the primary controller design.

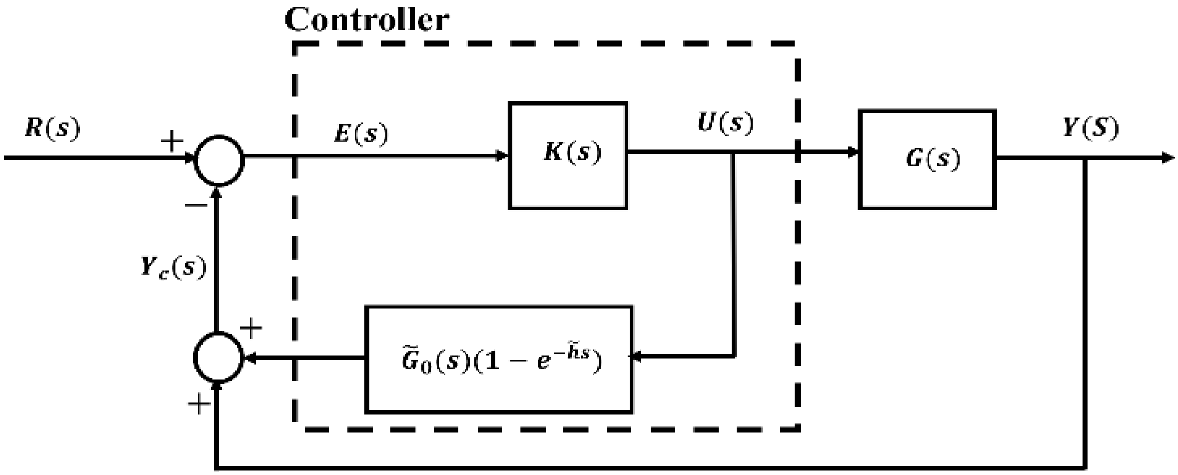

Figure 4 Smith predictor structure

According to Figure (4), the transfer function of the controller C, consisting of the controller $K(s)$ and the delayed transfer function $\tilde{G}_0(1 - e^{-\tilde{h}s})$, is driven as:

$$C = \frac{K(s)}{1 + K(s)\tilde{G}_0(1 - e^{-\tilde{h}s})}. \tag{14}$$

Following the standard design method for Smith predictor-based PID controller [23], the controller $K(s)$ is typically designed based on the nominal model $\tilde{G}_0$ that does not include the time delay (dead time) of the process. The Smith predictor is then used to compensate for the dead time.
The structure of PID controller for the nominal model $\tilde{G}_0$ is shown in Figure (5). The standard PID control algorithm is suggested to compute the control signal based on the following form:

$$U(s) = K_P E(s) + \frac{K_I}{s} E(s) + K_D s E(s), \tag{15}$$

in which $E(s)$ is the Laplace transform of the tracking error, $e(t) = x_{1d} - x_1$, where $e(t)$ is the angular displacement tracking error. Note that, the encoder and gyroscope sensors are attached to the platform to provide the angular displacement (x_1) and the velocity of the arm (x_2), respectively. In addition, K_P, K_I, and K_D are the gains of proportional, integral $\mathrm{K_d}$ and derivative terms, respectively.

For tuning the PID controller gains, there are various methods such as Ziegler Nichol tuning (Z-N) [24], internal model control (IMC) [17], and PI tuning [25]. In this paper, the Z-N method gives the initial PID controller gains that can be improved by trial and error for implementation. In the Z-N method, the values of the proportional gain K_P, the integral time T_I, and the derivative time T_D are determined using the closed-loop response of the system, where $K_I = k_P/T_I$ and $K_D = k_P/T_D$. First, by setting $T_I = \infty$ and $T_D = 0$, the proportional control gain K_P as shown in figure 6, increases from 0 to a critical value K_c at which the output first exhibits sustained oscillations. In this case, the system response shows a consistent and sustained waveform with a constant amplitude. Thus, the critical gain K_c and the period of the sustained oscillations P_c are experimentally determined. Accordingly, the parameters of the PID controller can be calculated according to Table (1).

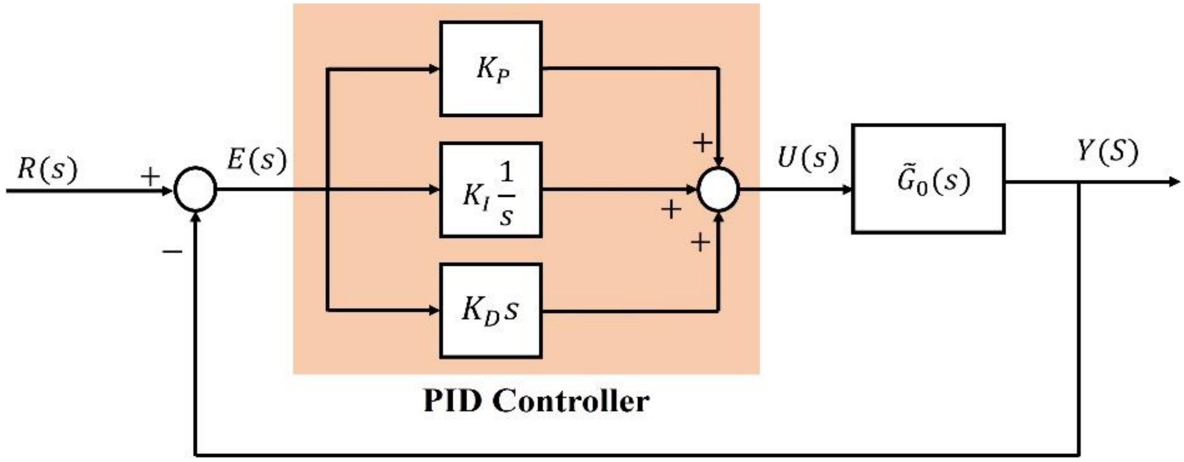

Figure 5 The PID control structure

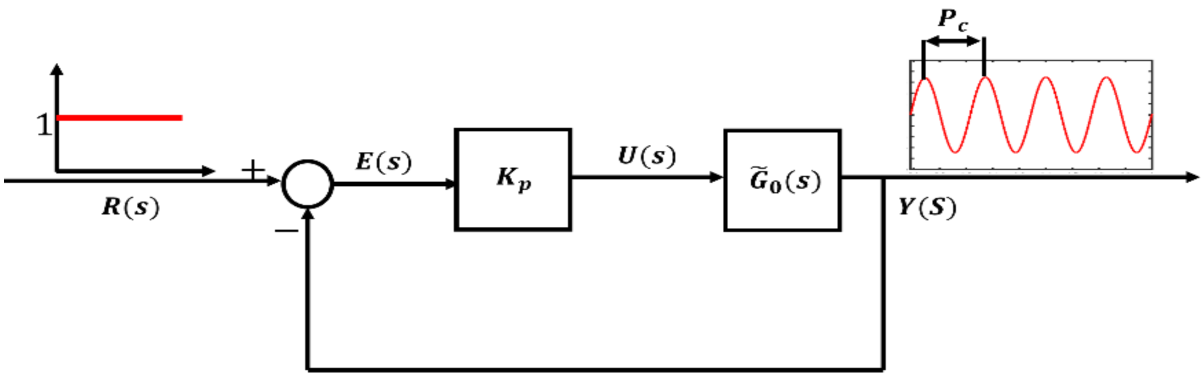

Figure 6 Closed loop system with proportional controller for unit step input

Table 1 Z-N tuning rules based on critical value K_c and critical period P_c

Control method	K_P	T_I	T_D
P	$0.5K_c$	∞	0
PI	$0.45K_c$	$1/1.2P_c$	0
PID	$0.6K_c$	$0.5P_c$	$0.125P_c$

For the PID controller, the proportional term, generating a control action proportional to the error, influences on the stability, steady-state error, and responsiveness of the system. The integral term accumulates the past error over time and produces a control action to address steady-state errors and eliminate the system bias. The derivative term anticipates future error trends by evaluating the rate of change of the error. It contributes to damping oscillations and improves the transient response. By adjusting the parameters defined in Eq. (15), the system will track the desired value, and the control error will approach zero.

3 Results and discussion

In this section, the Z-N tuning rules, explained in the previous section, are applied to calculate the parameters K_P, T_I and T_D , addressed in Table (1), for the lever arm platform. Considering only the proportional control action, the stable oscillation is started by increasing K_P up to 8.93 as demonstrated in Figure (7). Therefore, by taking the critical gain as $K_c = 8.93$ and $P_c = 0.36$, the parameters K_P, T_I and T_D are determined as noted in Table (2).

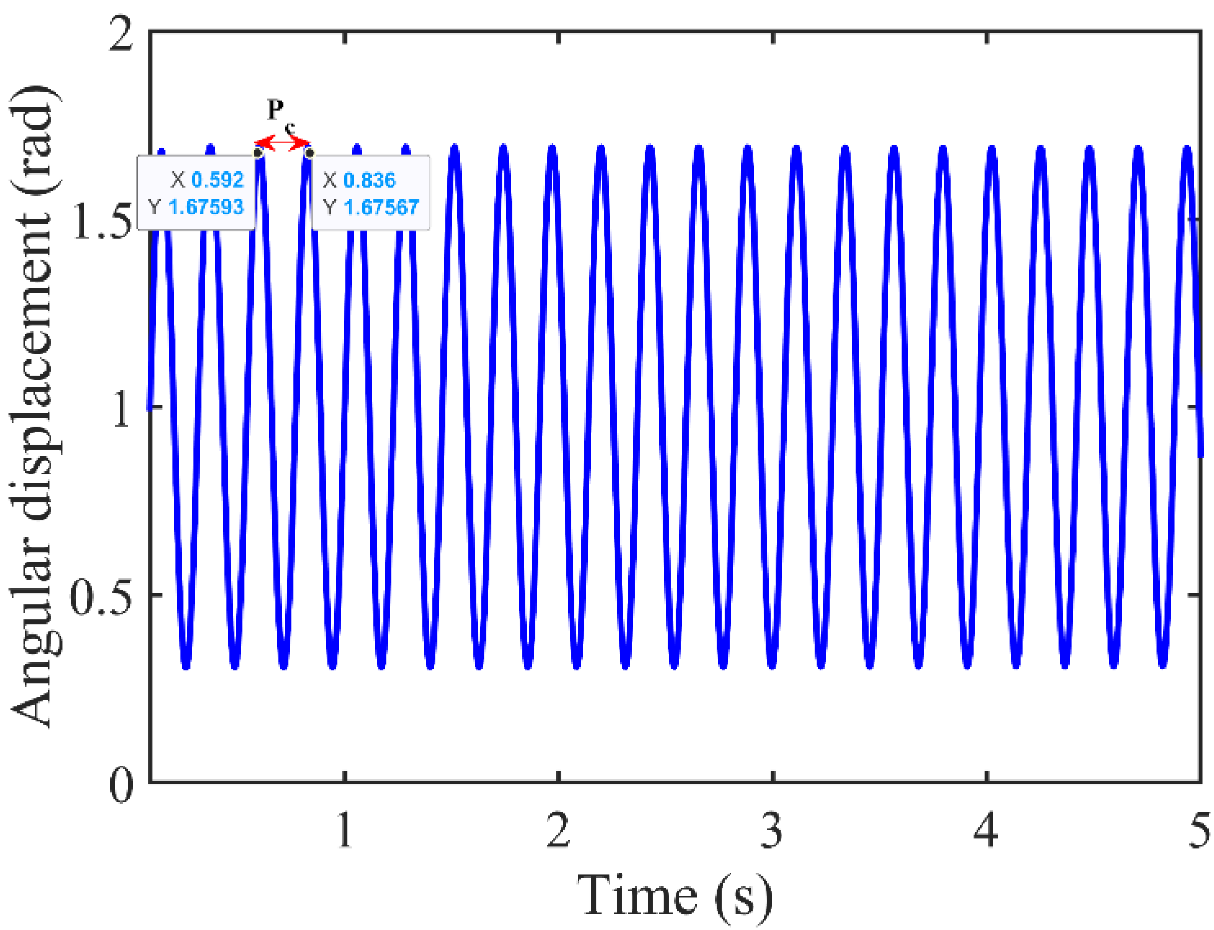

Figure 7 Unit step response of the lever arm in a closed loop with critical gain K_c

Table 2 Z-N tuning rules based on critical value K_c and critical period P_c

Control method	K_P	K_I	K_D
P	4.465	∞	0
PI	4.0185	0.3	0
PID	5.358	0.18	0.045

In the experimental results, the lever arm platform is controlled by the PID controller with smith predictor and the performances of P, PI and PID controllers are compared. For implementation of the proposed strategies, some dynamical tests have been carried out. The test equipment is shown in Figure (8).

According to this figure, the MPU6050 inertial measurement unit (IMU) is used to provide the angular velocity of the arm with a frequency of 50 Hz. In addition, the HN3806 two-phase encoder measures the angular displacement of the arm with an accuracy of 0.05 (Deg) at the same frequency. After measuring the sensor output, the control signal is calculated, and the pulse width modulation (PWM) of the motor is sent to the motor driver. Finally, a 12-volt DC motor produces the proposed control input. To validate the proposed controller, the MATLAB-Simulink environment is used to read the sensors outputs, to calculate the control input and to send the control signals to the actuator.

The first desired trajectory is defined as follows:

$$x_{1d} = \begin{cases} \pi(1-e^{-0.8t}), t \leq 10 \\ \pi + \frac{2}{3}\pi\left(1-e^{-0.8(t-10)}\right), 10 \leq t \leq 20 \\ \frac{5}{3}\pi + \frac{1}{3}\pi\left(1-e^{-0.8(t-20)}\right), 20 \leq t \leq 30 \\ 2\pi - \pi\left(1-e^{-0.8(t-30)}\right), 30 \leq t \leq 40 \\ \pi - \frac{1}{2}\pi\left(1-e^{-0.8(t-40)}\right), 40 \leq t \leq 50 \\ \frac{1}{2}\pi - \frac{1}{2}\pi\left(1-e^{-0.8(t-50)}\right), 50 \leq t \leq 60 \end{cases} \tag{16}$$

The results of the proposed P, PI, and PID control methods for the first reference trajectory are shown in Figures (9) to (11). According to Figure (9), the controllers are focused on reducing the angular displacement tracking errors as the main aim. The results show that all three versions of the proposed controller can track the given desired path in the presence of time delay caused by the DC motor. Table (3) compares the performance of the proposed controllers quantitively. The results indicate that the root mean square (RMS) of the PID controller in tracking the desired trajectory is the lowest.

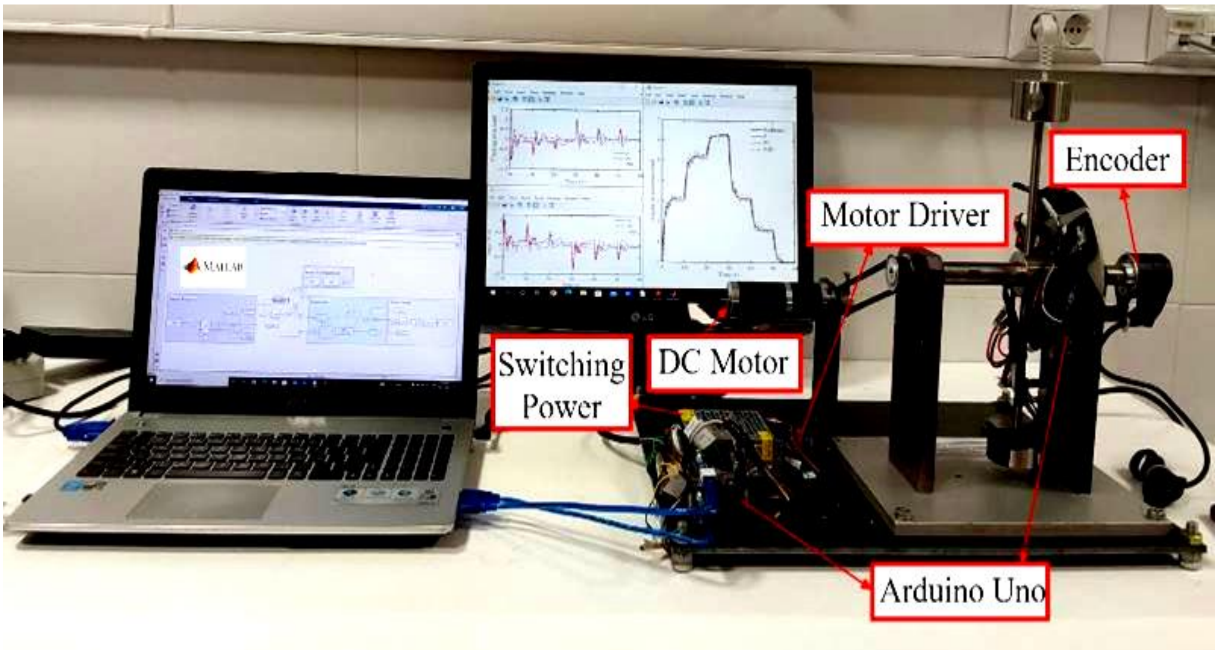

Figure 8 The experimental setup

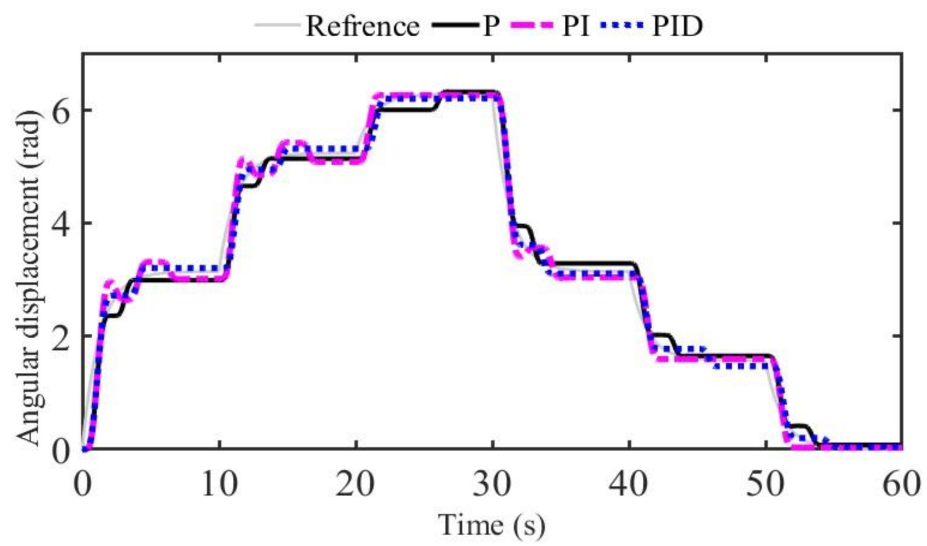

Figure 9 Angular displacement during test-1

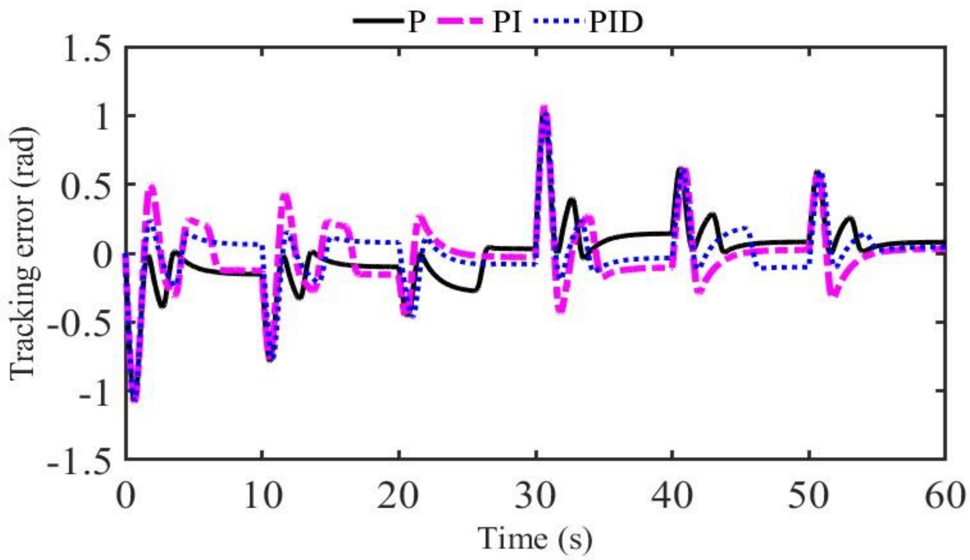

Figure 10 Tracking error during test-1

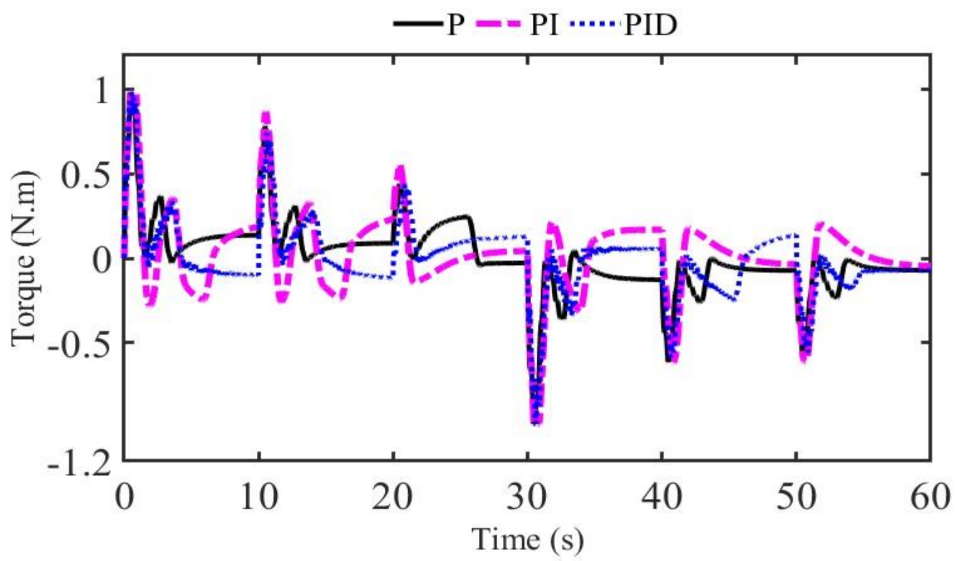

Figure 11 Control input during test-1

Table 3 Comparison of the RMS of the tracking errors and control inputs by different controllers during test-1

Control method	Tracking Error	Control input
P	0.2400	0.2167
PI	0.2455	0.2503
PID	0.2213	0.2174

To show the independency of the proposed method from the defined trajectory, the second desired trajectory is generated as follows:

$$x_{2d} = 2.2\sin(\pi t). \tag{17}$$

The results of the proposed control are shown in Figures (12) to (14). In addition, the RMS of the tracking error and control input are presented in Table (4). The results indicate superior performance of the proposed PID controller in tracking the desired path when the delay exist in the actuator.

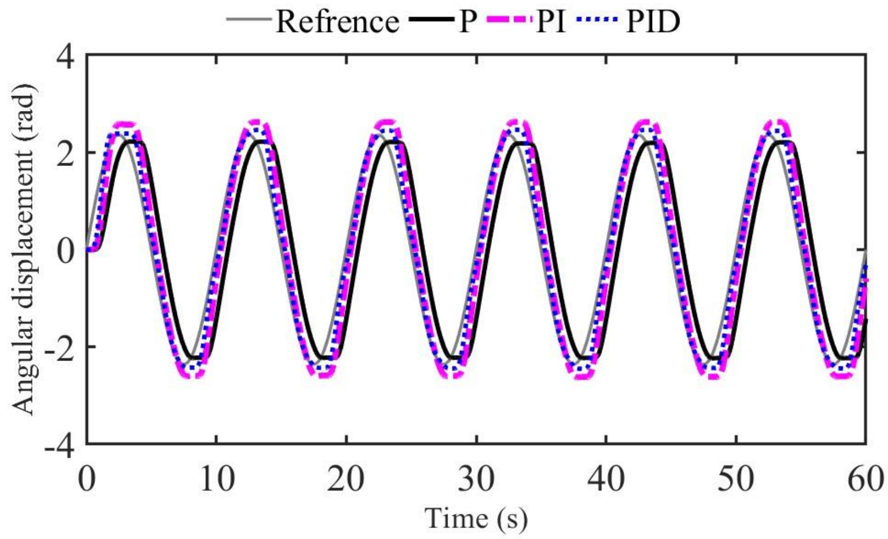

Figure 12 Angular displacement during test-2

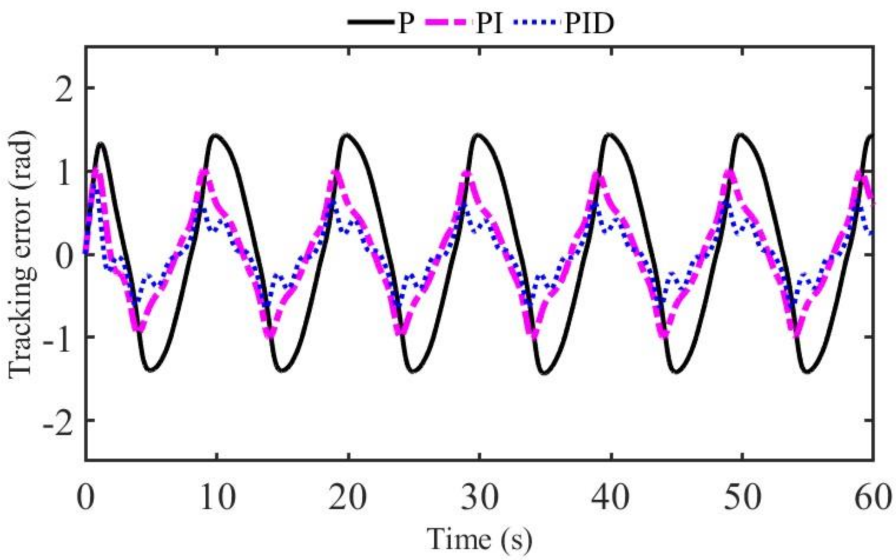

Figure 13 Tracking error during test-2

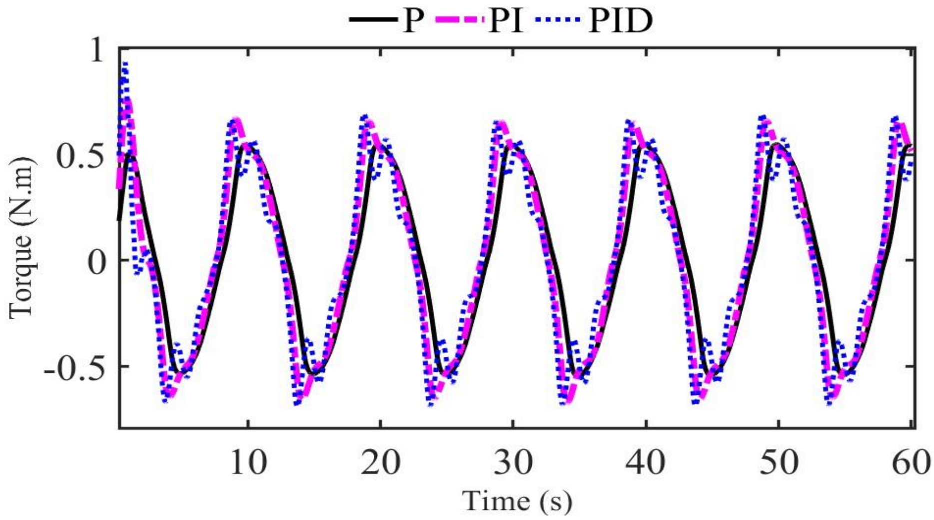

Figure 14 Control input during test-2

Table 4 Comparison of the RMS of the tracking errors and control inputs by different controllers during test-2

Control method	Tracking Error	Control input
P	0.9729	0.4718
PI	0.5658	0.4290
PID	0.3301	0.4095

4 Conclusion

In this paper, the parameters of system transfer function are estimated using the step response analysis. Accordingly, a Smith estimator-based PID controller is designed to overcome the time delay when tracking the desired trajectories for the lever arm. In this way, the controller $K(s)$ is designed for the lever arm without considering time delay, and the Smith estimator is connected to the $K(s)$. Through experimental results, it was demonstrated that the proposed algorithm provides a precise tracking performance and the RMS of tracking error is reduced by approximately 20% and 40% when PID is chosen as $K(s)$ compared to PI and P controllers.

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Nomenclature

L	Lagrange equation
T_k	Kinetic energy
V_g	Potential energy
m_1	Mass
m_2	Mass
r_1	Length of manipulator
r_2	Length of manipulator
g	Gravitational acceleration
K	Gain of the system
h	Time delay
G	Transfer function
$\tilde{h}$	Nominal value of time delay
$\tilde{G}$	Nominal value of transfer function
$e(t)$	Angular displacement tracking error
K_p	Proportional gain
K_I	Integral gain
K_D	Derivative gain
K_c	critical gain
P_c	oscillation period
T_I	Integral time
T_D	Derivative time
θ	Angular displacement
ξ	Damping ratio
ω_n	Natural frequency
τ	Motor torque