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Coupler-curve Optimization of Planar Four Bar Mechanism Based on an Analytical Method

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In this paper the optimal dimensional synthesis of a planar four bar mechanism is demonstrated based on an analytical method. The aim of the optimal dimensional synthesis of the mechanism is to design a mechanism so that a certain point of the coupler can cross the fitted curve over a certain number of points. For this reason, the kinematics relations of the mechanism are obtained first and continuous coupler-curve equation coefficients is considered as a function of the design parameters. In this study, the optimization method of multi-variable functions is considered and design parameters are easily calculated. The results of this research show that the performance of the proposed method is very favorable and there is a significant difference between the primitive coupler-curve and the optimized one, and this comparison is presented both graphically and tabular.

Keywords: Four bar mechanisms, Synthesis, Coupler curve, Optimization

1 Introduction

Four bar mechanisms are widely used in industrial applications. One of the most important problems in designing these types of mechanisms is the synthesis of the coupler-curve or path-curve. Numerous researches concentrated on three types of problems in the field of coupler-curve generation:

I) Exact synthesis based on some precision points

In the problems of type (I), the goal is the synthesis of the four-bar mechanism with a limited number of precision points. The problems with four or five precision points have been studied by researchers. Morgan and Wampler [1], considered the problem of synthesizing a planar four-bar linkage with given pivots such that the coupler-curve passes through five precision points.

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They showed that the design parameters must satisfy a system of 4 fourth-degree polynomial equations in 4 unknowns which has at most 36 nonzero real solutions. Subbian, and Flugrad [2], presented a different approach for the synthesis of four-bar planar path generating mechanisms. They used a continuation method to solve the system of nonlinear equations derived for the path generating problem. Cabrera *et al.* [3], presented a solution method of optimal synthesis of planar mechanisms. They defined a searching procedure which applies genetic algorithms based on evolutionary techniques and the type of goal function. Wampler *et al.* [4], presented the problem of finding all four-bar linkages whose coupler-curve passes through nine prescribed points. They solved the problem using a combination of classical elimination, multi homogeneous variables, and numerical polynomial continuation.

II) Continuous coupler-curve synthesis

In the problems of type (II), there is a continuous coupler-curve, and the main issue is to find the mechanism parameters in such a way that the continuous coupler-curve is produced. This kind of synthesis is a special problem of path synthesis. Since in this type of problem, the coupler-curve is assumed to be known, the design parameters can be obtained by comparing this curve and parametric equation of the coupler-curve. Wu *et al.* [5], developed a fully analytical method to find the exact solutions of coupler-curve equations for the planar four-bar linkages. Their developed method is based on an algebraic formulation of the system of coefficient equations, which yields a small system of four equations and four variables about the positions of pivoting joints. With any given algebraic coupler-curve of a planar four-bar linkage, the position parameters of pivoting joints can be easily solved, while the other parameters can be calculated by the remaining equations. Soong and wu [6] presented a method for designing a variable continuous coupler curve four-bar mechanism with one link replaced by an adjustable screw-nut link and driven by a servomotor.

III) Synthesis according to a number of desired prescribed points

In this case, there is no necessity that coupler-curve passes through all of the prescribed points and the goal is to find the optimized dimensions for the linkages in such a way that the coupler-curve is the best fitted curve through these points. Li *et al.* [7], presented a novel analytical approach for synthesizing a path-generation mechanism without any limitation on the number of precision points.

Huang *et al.* [8], proposed a repellency evolutionary algorithm (REA) to solve complex dimensional synthesis problems. Hernandez *et. al.* [9], proposed the optimal dimensional synthesis methodology of a four-bar linkage based on path assessment and reformulating the error function. Cabrera *et al.* [10], described an evolutionary algorithm for path synthesis of mechanisms. They named the algorithm MUMSA (Malaga University Mechanism Synthesis Algorithm). To that end, they have obtained the error between the desired and the target coupler-curve in a four-bar mechanism and in a six-bar mechanism, showing that the found solutions by the MUMSA algorithm were accurate and valid for all cases. Ortiz *et al.* [11], presented how an algorithm based on Differential Evolution (DE) with no constant control parameters solves the dimensional synthesis of four and six-bar mechanisms for path generation. Bulatovi´c *et al.* [12], considered the application of the modified Krill Herd (KH) algorithm for obtaining optimal solutions in dimensional synthesis of a four-bar linkage as a path generator. Lin *et. al.* [13], presented a differential evolution algorithm with a combined mutation strategy for optimum synthesis of path-generating four-bar mechanisms. Li and Chen [14], developed a parametrization-invariant fourier approach to planar linkage synthesis for path generation. Sharma *et al.* [15], considered an optimal non-uniform parametrization for fourier descriptor based-path synthesis of four bar mechanisms. Sharma *et al.* [16], proposed a fourier descriptor (FD)-based path synthesis algorithm for generation of planar four-bar mechanisms. They presented a non-uniform parametrization scheme in conjunction with an objective function. Kang and Kim [17], proposed a gradient-based topology optimization method to synthesize a planar linkage mechanism consisting of links and revolute/prismatic joints. Yim [18], developed topology optimization for mechanism synthesis for the simultaneous determination of the number and dimension of mechanisms. They considered a gradient-based topology optimization method which can be used to synthesize mechanisms consisting of both linkages and gears. Hadizadeh Kafash and Nahvi [19], presented a circular proximity function for the optimization of path-generator four-bar linkages. Sleesongsom, and Bureerat [20], proposed a new variant of teaching-learning based optimization (TLBO) for optimal path synthesis of a four-bar linkage. Eqra *et al.* [21], applied adaptive inertia weight PSO (AIW-PSO) to the path synthesis problem in order to compare the performance of an adaptive solution technique to the so far implemented ones and to investigate the effect of adaptability on the efficiency of the solution method. Jong-won kim *et al.* [22] presented a new design methodology for crank-rocker four-bar linkages based on differential objective functions. Ruby Mishra *et al.* [23] presented a path synthesis of plannar four-bar mechanisms with joint clearances and using differential evolution algorithm. Dillip Datta *et al.* [24] presented a simple numerical method for approximating non-intersecting closed curves through four-bar linkages.

In this paper, the four-bar mechanism is designed in such a way that the coupler-curve is optimally fitted on N points of the desired path. For this reason, the square of the coupler-curve equation, which is implicitly defined, is considered as the target function for optimization and it is calculated at the points on the desired path-curve and added together, and finally, using the optimization method of multivariable functions, the partial derivative of the last term is taken with respect to the design parameters and they are considered equal to zero. After solving the resulting equations, the optimized design parameters are easily calculated. The good thing about this method is that there is no restriction on choosing the number of the prescribed points. In addition, a limited number of design parameters can be selected. The proposed method does not have the limitations of the precise synthesis method with a limited number of accuracy points and the synthesis based on the continuous coupler curve. The innovation of this research compared to other researchers' methods is the use of the parametric coupler curve and the optimization method of multivariable functions.

2 Kinematic relations

A typical four-bar linkage is shown in Figure (1), where the fixed coordinate system on O denotes by {O - XY}, and {D - xy} is the movable coordinate system attached to the mechanism as shown in Fig. (1). The origin of the {O - XY} can be arbitrarily located and the origin of the {D - xy} is located at the center of the revolute pair D and the x -axis in this coordinate system corresponds to link DC.

Considering Figure (1), the vector relation $\overrightarrow{\boldsymbol{DA}} + \overrightarrow{\boldsymbol{AB}} + \overrightarrow{\boldsymbol{BC}} = \overrightarrow{\boldsymbol{DC}}$ can be written as follows:

$$l_2 e^{i(\theta+\xi)} + l_3 e^{i\alpha} - l_4 e^{i\psi} = l_1 e^{i\xi} \tag{1}$$

The above vector relation can be expressed as the following two algebraic relations:

$$l_2 \sin(\theta+\xi) + l_3 \sin\alpha = l_4 \sin\psi + l_1 \sin\xi \tag{1a}$$

$$l_2 \cos(\theta+\xi) + l_3 \cos\alpha = l_4 \cos\psi + l_1 \cos\xi \tag{1b}$$

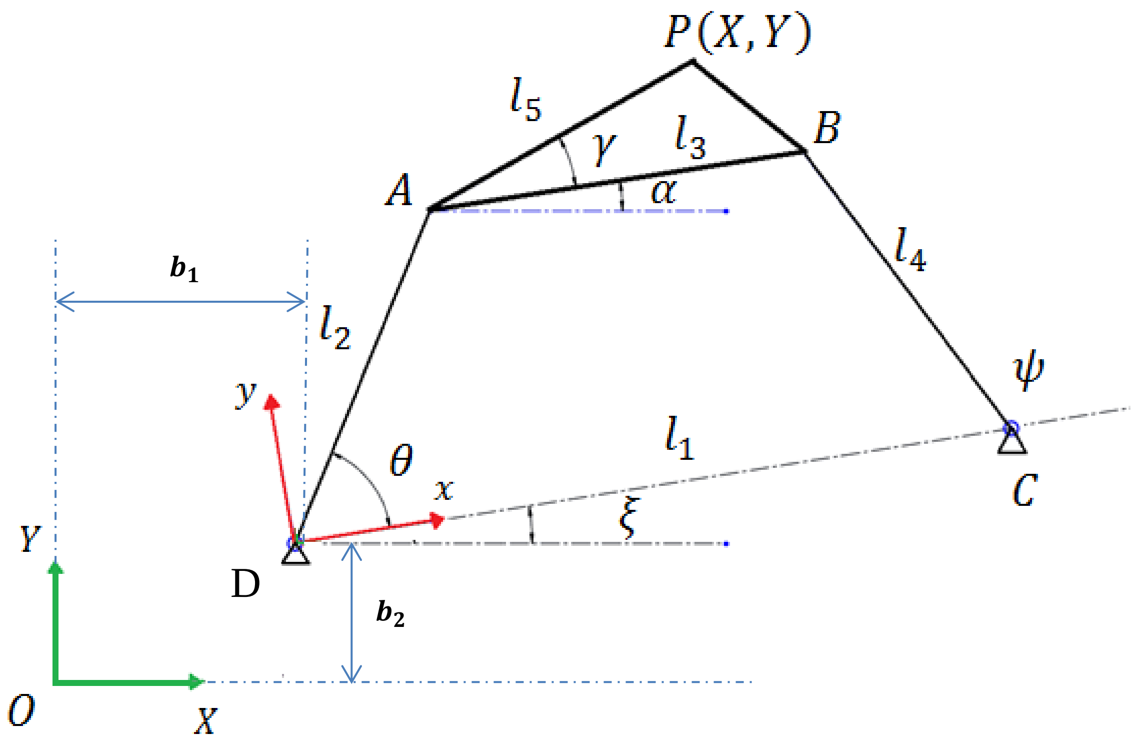


Figure 1 Configuration of a four bar mechanism

Equation (2) shows the position vector of the point p of the coupler in the {O- XY} coordinate system, in which the variables X and Y are considered as relations (2a) and (2b).

$$b_1 + b_2 I + l_2 e^{i(\theta+\xi)} + l_5 e^{i(\gamma+\alpha)} = X + YI \tag{2}$$

$$b_1 + l_2 \cos(\theta + \xi) + l_5 \cos(\gamma + \alpha) = X \tag{2a}$$

$$b_2 + l_2 \sin(\theta + \xi) + l_5 \sin(\gamma + \alpha) = Y \tag{2b}$$

Considering the following parameters:

$$\begin{aligned} A_1 &= l_2 \cos \xi \\ A_2 &= l_2 \sin \xi \\ A_3 &= l_5 \sin \gamma \\ A_4 &= l_5 \cos \gamma \end{aligned} \tag{3}$$

The relations (1a) – (2b) would be expressed as:

$$A_1 \sin \theta + A_2 \cos \theta + l_3 \sin \alpha = l_4 \sin \psi + l_1 \sin \xi \tag{4}$$

$$A_1 \cos \theta - A_2 \sin \theta + l_3 \cos \alpha = l_4 \cos \psi + l_1 \cos \xi \tag{5}$$

$$b_2 + A_1 \sin \theta + A_2 \cos \theta + A_3 \cos \alpha + A_4 \sin \alpha = Y \tag{6}$$

$$b_1 + A_1 \cos \theta - A_2 \sin \theta + A_4 \cos \alpha - A_3 \sin \alpha = X \tag{7}$$

We can easily show Eqs. (6) and (7) in matrix form as below:

$$\begin{bmatrix} A_4 & A_3 \\ -A_3 & A_4 \end{bmatrix} \begin{Bmatrix} \sin\alpha \\ \cos\alpha \end{Bmatrix} = \begin{Bmatrix} Y - A_1 \sin\theta - A_2 \cos\theta - b_2 \\ X - A_1 \cos\theta + A_2 \sin\theta - b_1 \end{Bmatrix} = \begin{Bmatrix} f \\ g \end{Bmatrix} \tag{8}$$

By Solving Eq. (8) we have:

$$\sin\alpha = \frac{1}{{A_3}^2 + {A_4}^2}[A_4 f - A_3 g]$$
$$\cos\alpha = \frac{1}{{A_3}^2 + {A_4}^2}[A_3 f + A_4 g] \tag{9a}$$

Where,

$$(\sin\alpha)^2 + (\cos\alpha)^2 = 1 \tag{9b}$$

In this way, one can find an equation as a function of X, Y and θ as below:

$$C_{11} \sin\theta + C_{12} \cos\theta = F_1 \tag{10a}$$

Where,

$$C_{11} = -A_1 Y + A_2 X + A_1 b_2 - A_2 b_1$$
$$C_{12} = -A_1 X - A_2 Y + A_1 b_1 + A_2 b_2$$
$$F_1 = \frac{1}{2}\left[-{A_1}^2 - {A_2}^2 + {A_3}^2 + {A_4}^2 - X^2 - Y^2 - {b_1}^2 - {b_2}^2\right] + b_1 x + b_2 y \tag{10b}$$

Substituting $(\sin\alpha)$ and $(\cos\alpha)$ in Eqs. (4) and (5) and using the following trivial relation:

$$(\sin\psi)^2 + (\cos\psi)^2 = 1 \tag{11}$$

Another equation is derived as a function of X, Y and θ as below:

$$C_{21} \sin\theta + C_{22} \cos\theta = F_2 \tag{12a}$$

Where,

$$\begin{aligned} C_{21} = {} & X\left[-2A_1A_3l_3 - 2A_2A_4l_3 + 2A_2{l_3}^2\right] + Y\left[2A_1A_4l_3 - 2A_2A_3l_3 - 2A_1{l_3}^2\right] \\ & + 2l_1A_1A_3[l_3\cos\xi - A_3\sin\xi] + 2l_1A_2A_3[l_3\sin\xi + A_3\cos\xi] \\ & + 2{l_3}^2(A_1b_2 - A_2b_1) + 2A_2l_3(b_2A_3 + b_1A_4) + 2A_1l_3(b_1A_3 - b_2A_4) \\ C_{22} = {} & X\left[2A_1A_4l_3 - 2A_2A_3l_3 - 2A_1{l_3}^2\right] + Y\left[2A_1A_3l_3 + 2A_2A_4l_3 - 2A_2{l_3}^2\right] \\ & + 2l_1l_2(l_3 - l_4)A_4 + 2l_1A_1A_3[-l_3\sin\xi - A_3\cos\xi] \\ & + 2l_1A_2A_3[l_3\cos\xi - A_3\sin\xi] + 2{l_3}^2(A_1b_1 + A_2b_2) \\ & + 2A_2l_3(b_1A_3 - b_2A_4) - 2A_1l_3(b_2A_3 + b_1A_4) \end{aligned} \tag{12b}$$

$$F_2 = \left(A_1{}^2 + A_2{}^2\right)\left[-l_3{}^2 - (A_4 - A_3)^2\right] + \left(A_3{}^2 + A_4{}^2\right)\left[l_4{}^2 - l_1{}^2\right] - l_3{}^2 X^2 + 2l_1 l_3 [A_4 \cos\xi - A_3 \sin\xi]X - l_3{}^2 Y^2 + 2l_1 l_3 [A_4 \sin\xi + A_3 \cos\xi]Y + 2A_3 l_1 l_3 [b_1 sin\xi - b_2 cos\xi] - 2A_4 l_1 l_3 [b_2 sin\xi + b_1 cos\xi] - (b_1^2 + b_2^2) l_3{}^2 + 2l_3{}^2 (b_1 x + b_2 y)$$

If the position of point P is defined in the OXY coordinate system, then it can easily be expressed in the Oxy coordinate system by a rotation matrix and using the axes transfer rule as below:

$$\begin{bmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{bmatrix} \begin{Bmatrix} X - b_1 \\ Y - b_2 \end{Bmatrix} = \begin{Bmatrix} x \\ y \end{Bmatrix} \quad (13a)$$

As a result, we have:

$$\begin{aligned} x &= (X - b_1)\cos\xi + (Y - b_2) sin\xi \\ y &= -(X - b_1)\sin\xi + (Y - b_2) cos\xi \end{aligned} \quad (13b)$$

Where *x* and *y* are the components of position vector of point P in {D-xy} coordinate system.

3 Path-curve equation for point p of the coupler

Considering the trivial Eq. (15) and eliminating θ from Eq. (14) the path-curve equation (PCE) of the point P is derived as Eq. (16).

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \sin\theta \\ \cos\theta \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (14)$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1 \quad (15)$$

$$PCE = (x^2 + y^2)^3 + (k_1 x + k_2 y)(x^2 + y^2)^2 + (k_3 x^2 + k_4 xy + k_5 y^2)(x^2 + y^2) + k_6 x^3 + k_7 x^2 y + k_8 xy^2 + k_9 y^3 + k_{10} x^2 + k_{11} xy + k_{12} y^2 + k_{13} x + k_{14} y + k_{15} = 0 \quad (16)$$

Due to the creation of complex and long sentences during calculations, first we assume ξ, b_1 and b_2 are zero, so in this case, the coefficients of Eq. (16) are obtained as follows:

$$k_1 = -2l_1(1 + A_4/l_3)$$

$$k_2 = -2l_1 A_3/l_3$$

$$k_3 = -2l_2{}^2(-A_4/l_3 + 1) + l_1{}^2(1 + 4A_4/l_3) + \left(A_3{}^2 + A_4{}^2\right)\left(l_1{}^2/l_3{}^2 - 2\right) - l_4{}^2(2A_4/l_3) + 2A_4 l_3$$

$$k_4 = 4l_1{}^2 A_3/l_3$$

$$k_5 = k_3 - 4A_4 l_1{}^2/l_3$$

$$k_6 = (2A_4 l_1/l_3)\left(2A_4^2 - l_1^2 + l_4^2 - A_4 l_1^2/l_3 - l_2^2 + 2A_3^2\right) + 2l_1\left(A_3^2 + 2l_2^2\right) + 2A_3^2 l_1/l_3^2 \left(l_4^2 - l_1^2\right) - 2A_4^2 l_1/l_3^2 \left(l_2^2 - l_4^2\right) - 2A_4 l_1 (l_3 + A_4) - 2l_2^2 A_3^2 l_1/l_3^2$$

$$k_7 = 2A_3 l_1/l_3 \left(l_4^2 - l_1^2 - l_3^2 + l_2^2 - 2A_4 l_3 + 2A_3^2 + 2A_4^2\right)$$

$$k_8 = k_6, \quad k_9 = k_7$$

$$k_{10} = \left(-1/l_3^2\right)\Big(\left(A_3^4 + 2A_3^2 A_4^2\right)\left(2l_1^2 - l_3^2\right) + A_4\left(2A_3^2 + 2A_4^2\right)\left(l_1^2 + l_2^2 + l_3^2 - l_4^2\right)l_3 - A_3^2 l_1^2\left(l_1^2 + 4l_2^2 - 2l_4^2\right) - A_3^2 l_2^2\left(l_2^2 - 2l_4^2\right) - A_3^2\left(l_3^2\left(l_3^2 - 2l_4^2\right) + l_4^4\right) + A_4^4\left(2l_1^2 - l_3^2\right) - A_4^2\left(l_1^4 + l_2^4 + l_3^4 + l_4^4 + 4l_1^2 l_2^2 + 4l_1^2 l_3^2 - 2l_1^2 l_4^2 - 2l_3^2 l_4^2 + 4l_2^2 l_3^2 - 2l_2^2 l_4^2\right) + 2A_4 l_2 l_3\left(l_1^2 l_2 + l_2^3 + l_3^2 l_2 - l_4^2 l_2\right) + l_2^2 l_3^2\left(2l_1^2 - l_2^2\right)\Big)$$

$$k_{11} = \left(-4A_3 l_1^2/l_3\right)\left(l_2^2 + A_3^2\right) + 4A_3 A_4 l_1^2 (2 - A_4/A_3)$$

$$k_{12} = k_{10} + \left(4l_1^2/l_3^2\right)\left(A_3^2 A_4 l_3 - A_3^2 l_2^2 + A_3^2 l_3^2 + A_4^3 l_3 - A_4^2 l_2^2 - A_4^2 l_3^2 + A_4 l_2^2 l_3\right)$$

$$k_{13} = \left(-2l_1/l_3^2\right)\left(A_3^2 + A_4^2 - l_2^2\right)\left(A_3^2 A_4 l_3 - A_3^2 l_1^2 - A_3^2 l_2^2 + A_3^2 l_4^2 + A_4^3 l_3 - A_4^2 l_1^2 - A_4^2 l_2^2 - 2A_4^2 l_3^2 + A_4^2 l_4^2 + A_4 l_1^2 l_3 + 2A_4 l_2^2 l_3 + A_4 l_3^3 - A_4 l_3 l_4^2 - l_2^2 l_3^2\right)$$

$$k_{14} = (-2A_3 l_1/l_3)\left(A_3^2 + A_4^2 - l_2^2\right)\left(A_3^2 + A_4^2 - 2A_4 l_3 + l_1^2 + l_3^2 - l_4^2\right)$$

$$k_{15} = \left(l_1^2/l_3^2\right)\left(A_3^2 + (A_4 - l_3)^2\right)\left(A_3^2 + A_4^2 - l_2^2\right)^2$$

4 Optimization of the four bar mechanism parameters

In this section, the optimal parameters for a four-bar mechanism are obtained in such a way that point P of the coupler can best pass through the coupler-curve that comes from the curve fitting of N desired points as shown in Figure (2).

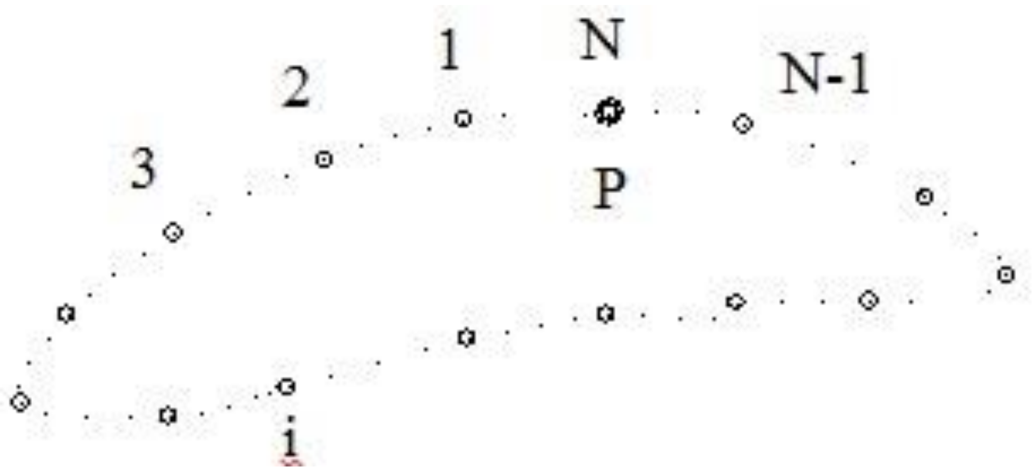


Figure 2 Discrete points for coupler-curve

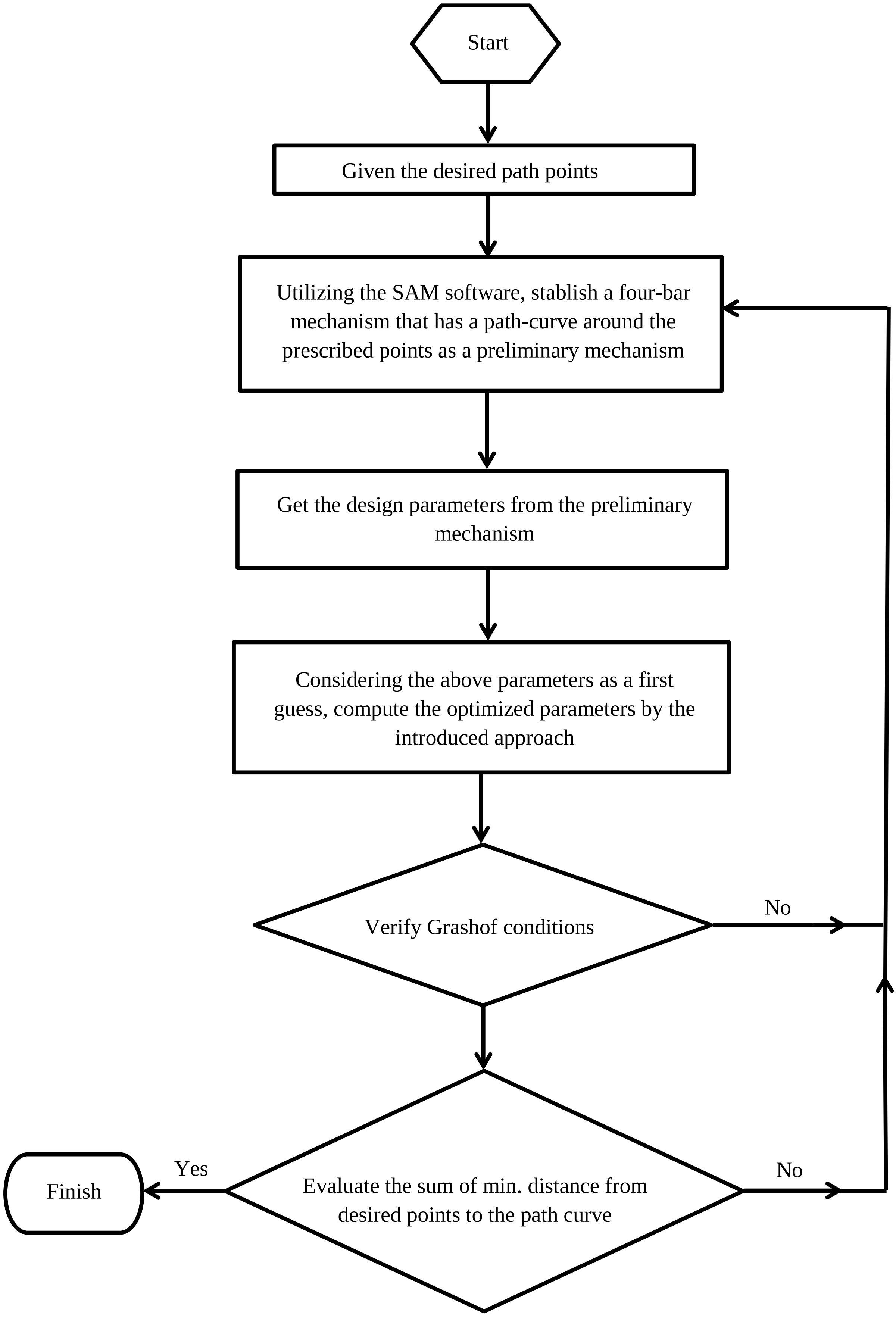


Figure 3 Flowchart of the proposed method

$$T(l_1, l_2, l_3, l_4, l_5, \gamma, b_1, b_2, \xi) = \sum_{i=1}^{N} [(x_i{}^2 + y_i{}^2)^3 + (k_1 x_i + k_2 y_i)(x_i{}^2 + y_i{}^2)^2 + (k_3 x_i{}^2 + k_4 x_i y_i + k_5 y_i{}^2)(x_i{}^2 + y_i{}^2) + k_6 x_i{}^3 + k_7 x_i{}^2 y_i + k_8 x_i y_i{}^2 + k_9 y_i{}^3 + k_{10} x_i{}^2 + k_{11} x_i y_i + k_{12} y_i{}^2 + k_{13} x_i + k_{14} y_i + k_{15}]^2 = 0 \quad (17)$$

For this purpose, we consider relation (17), which in general can be a function of nine design parameters, as the target function and it should be minimized considering the following relations:

$$\begin{gathered} \partial T/\partial l_j = 0\,, \quad j = 1,2,3,4,5 \\ \partial T/\partial b_j = 0\,, \qquad j = 1,2 \\ \partial T/\partial \xi = 0 \\ \partial T/\partial \gamma = 0 \end{gathered} \quad (18)$$

Obviously, to solve the nonlinear equations resulting from the optimal design, an initial guess for the unknown variables is needed. For this purpose, using the SAM software, we choose the configuration of the mechanism in such a way that the desired point of the coupler is placed close to the desired path. Now arbitrarily, some parameters can be considered as design parameters and the other parameters as constant. In general, all parameters can be optimally designed, in which case, complex nonlinear equations are created that require time-consuming algebraic calculations. In this case, the numerical values of all variables in the primary mechanism are considered as initial guesses in solving nonlinear equations. In order to check the performance of the proposed method, three case studies are presented. The evaluation of the obtained results shows that the optimal design of the mechanism with the aid of the proposed method was very effective and satisfactory in these three cases. The algorithm of the method proposed in this article is shown in Figure (3).

5 Case study

5.1 Case I

In this case, we assume that three parameters b_1, b_2 and ξ are zero. In addition, the values of parameters l_1, l_5 and γ are considered to be constant according to Table (1). It should be noted that the aforementioned parameters are selected according to the configuration of the mechanism in the conditions of initial guess. In the following three case studies, the numerical values of the geometric parameters of the coupler and the distance between the two supports connected to the ground are considered to be constant and the values of other geometric parameters are considered as initial guesses in solving nonlinear equations.

According to the explanations provided l_2 , l_3 and l_4 are the design parameters and the other parameters are considered constant, which can be seen in Table (1).

In order to obtain the optimal values of the design parameters and solve the nonlinear equations of relation (18), the initial guess for the numerical values of the parameters is determined by utilizing the SAM (Simulation and Analysis of Mechanisms) software and according to Table (2). As can be seen, after optimization, the values of the design parameters are obtained according to Table (2).

In Table (3), the information related to the position of the desired points on the path curve, the points on the path curve with the minimum distance from the desired points, in two cases, the estimated values of the parameters and the optimal parameters are provided.

Using Table (3), the error square values in these two cases are 0.15911 and 0.00018, respectively.

The significant reduction of the error square in the case where the parameters are optimized with the proposed method of this paper indicates the effectiveness and satisfactory performance of this method.

Figure (4) shows the trajectory curve in two cases of using guess values for parameters and using optimal values for parameters. The comparison of these two figures also confirms the information in Table (3) and shows the optimal performance of the proposed algorithm.

Table 1 Constant parameters of the mechanism

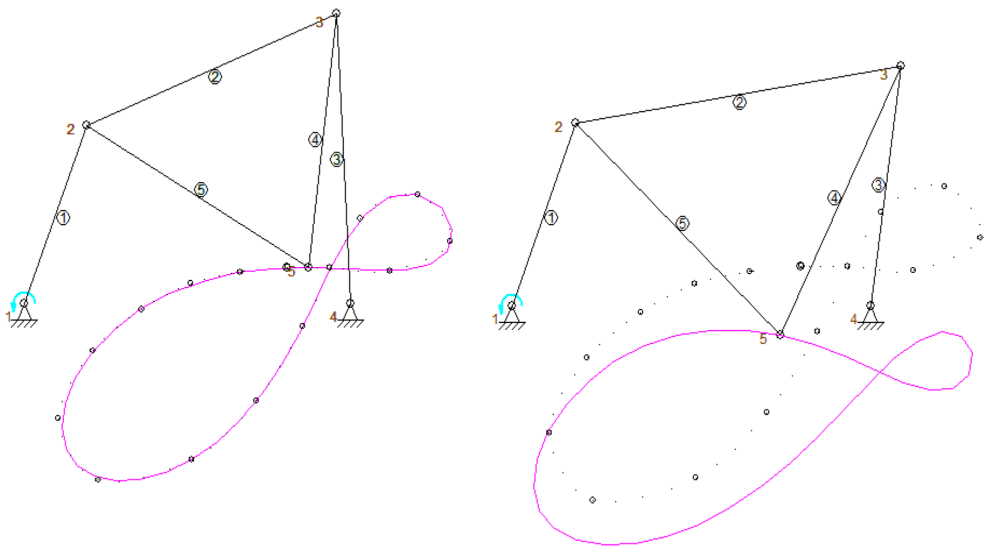
Parameters	l_1(mm)	l_5(mm)	γ(degree)	b1(mm)	b2(mm)	ξ(degree)
Constant Value	500	395	-55	0	0	0

Table 2 Design parameters for the first guess and after optimization

Parameters	l_2(mm)	l_3(mm)	l_4(mm)
First Guess	251	459	312.5
After Optimization	269.74	414.83	412.28

Table 3 Coordinates of points on the coupler curve

Index	*Points on desired path-curve*		*Points on preliminary path-curve with min. distance*		*Points on optimized path-curve with min. distance*	
i	$x_{desired}(i)$	$y_{desired}(i)$	$x_{pr}(i)$	$y_{pr}(i)$	$x_{opt}(i)$	$y_{opt}(i)$
1	0.40364	0.05143	0.37990	-0.05062	0.40358	0.04768
2	0.46843	0.05112	0.42857	-0.06473	0.46802	0.04783
3	0.56043	0.04627	0.58941	-0.04861	0.56043	0.04627
4	0.65404	0.08824	0.61768	-0.04708	0.65484	0.08836
5	0.60401	0.15441	0.60647	-0.04575	0.60401	0.15441
6	0.51524	0.12052	0.58289	-0.05090	0.51724	0.11867
7	0.42646	-0.03280	0.41741	-0.06102	0.42921	-0.03380
8	0.35545	-0.13771	0.40846	-0.19957	0.35834	-0.13999
9	0.25699	-0.22164	0.28614	-0.28184	0.25789	-0.22340
10	0.11334	-0.25070	0.08188	-0.28388	0.11444	-0.24702
11	0.05201	-0.16354	0.04814	-0.16107	0.05881	-0.16375
12	0.10527	-0.06670	0.12023	-0.08945	0.10744	-0.06843
13	0.17952	-0.00859	0.19393	-0.05480	0.18053	-0.01046
14	0.25538	0.02852	0.26158	-0.04152	0.25712	0.02331
15	0.33124	0.04466	0.32371	-0.04157	0.33193	0.04094



(**b**) (**a**)

Figure 4 Path curve of point 5, (**a**) Considering an initial guess for parameters (**b**)After optimization of parameters

5.2 Case II

In this case, we assume that b_1, b_2 and ξ are zero. In addition, we consider the values of parameters l_1, l_5 and γ to be constant according to Table (4).
In this way, l_2, l_3 and l_4 are the design parameters. In order to obtain the optimal values of the design parameters and solve the nonlinear equations (18), the initial guess for the numerical values of the design parameters is determined by considering the explanation of the first case and according to Table (5). As can be seen, after optimization, the values of the design parameters are obtained according to Table (5). According to Table (6), the error square values in these two cases are 0.01073 and 0.00012, respectively, and the significant reduction of the error square in the case where the parameters are optimized with the proposed method of the current study is an awesome achievement. Figure (5) shows the path curve in two cases of using guess and optimal values for the parameters, and the comparison of these couple of figures also confirms the information in Table (6) and shows the optimal performance of the proposed algorithm.

Table 4 Constant parameters of the mechanism

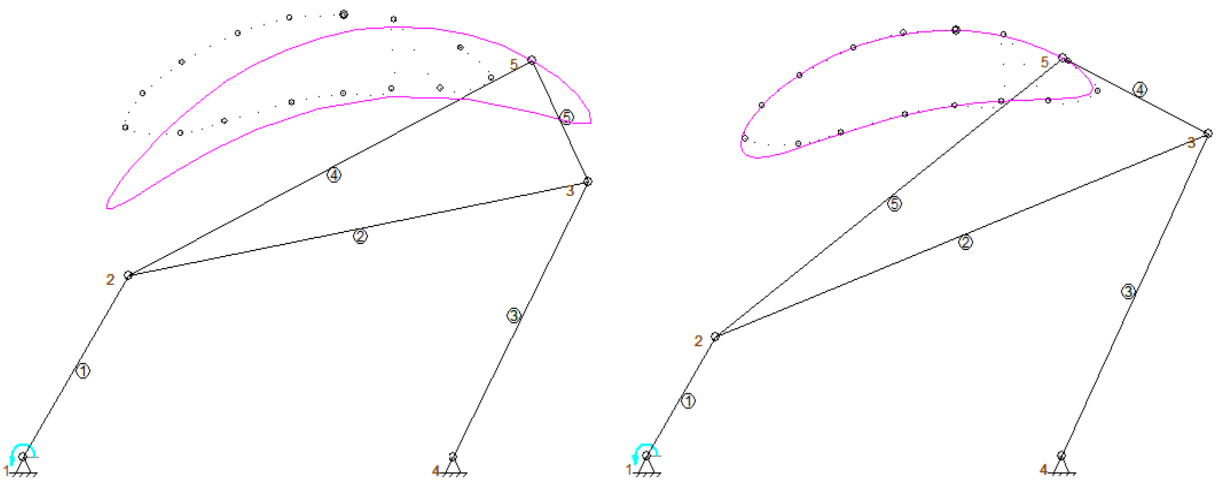
Parameters	l_1(mm)	l_5(mm)	γ(degree)	b1(mm)	b2(mm)	ξ(degree)
Constant Value	500	527	16	0	0	0

Table 5 Design parameters for the first guess and after optimization

Parameters	l_2(mm)	l_3(mm)	l_4(mm)
First Guess	235	545	343
After Optimization	160.920	638.847	411.392

Table 6 Coordinates of points on the coupler curve

Index	*Points on desired path-curve*		*Points on preliminary path-curve with min. distance*		*Points on optimized path-curve with min. distance*	
i	$x_{desired}(i)$	$y_{desired}(i)$	$x_{pr}(i)$	$y_{pr}(i)$	$x_{opt}(i)$	$y_{opt}(i)$
1	0.37300	0.49000	0.37623	0.47191	0.37299	0.48968
2	0.31000	0.48700	0.32023	0.45731	0.31021	0.48564
3	0.25000	0.47000	0.268032	0.43497	0.24980	0.47057
4	0.18500	0.43800	0.212172	0.40051	0.18419	0.43929
5	0.13900	0.40300	0.171233	0.36728	0.13882	0.40317
6	0.11900	0.36500	0.143957	0.34089	0.11419	0.36646
7	0.16400	0.35400	0.160676	0.35749	0.16531	0.34907
8	0.18300	0.35900	0.173673	0.36947	0.18436	0.35460
9	0.23400	0.37200	0.243387	0.35138	0.23443	0.37063
10	0.31200	0.39300	0.316230	0.37890	0.31218	0.39222
11	0.37200	0.40300	0.373841	0.39265	0.37196	0.40331
12	0.42800	0.40800	0.428556	0.39925	0.42794	0.40891
13	0.48500	0.40900	0.484505	0.39956	0.48496	0.41080
14	0.54400	0.42000	0.539652	0.39355	0.53841	0.42152
15	0.50900	0.45400	0.512459	0.46949	0.50890	0.45384



(**a**) (**b**)

Figure 5 Path curve of point 5, (**a**) Considering an initial guess for parameters (**b**)After optimization of parameters

5.3 Case III

In order to compare the performance of the proposed algorithm in the current research activity with the research of other researchers, the optimal design of the four-bar mechanism in this latter case is done by considering the proposed path that has been used in various references [7,10,11,12,25]. In this case, we assume that $\xi = 0$, in addition, we consider the values of parameters l_1, l_5, and γ as constant according to Table (7). Thus, in this case l_2, l_3, l_4, b_1 and b_2 are the design parameters, and the initial guess values for these parameters are considered according to Table (8).

Using Table (9), the error square values in these two cases are 1.0951 and 0.0058, respectively, and the significant reduction of the error square in the case where the parameters are optimized with the proposed method of the article indicates the effectiveness of this method.

Figure (6) shows the path curve in two cases of using guess values for parameters and using optimal values for parameters, and the comparison of these two figures confirms the information in Table (9) and shows the optimal performance of the proposed algorithm.

Figure (7) shows the comparison of performance results of different methods of designing the four-bar mechanism that researchers have proposed in past works. As it is clear, the optimal design method proposed in this article gives satisfactory results compared to the results of other methods, and even compared to the results of some algorithms, the results of the proposed method show a significant improvement. One can observe the Error Square Values (ESV) for different methods proposed in Table (10). As it is clear, the result related to the current study shows significant improvement as compared to the most of the previous approaches.

Table 7 Constant parameters of the mechanism

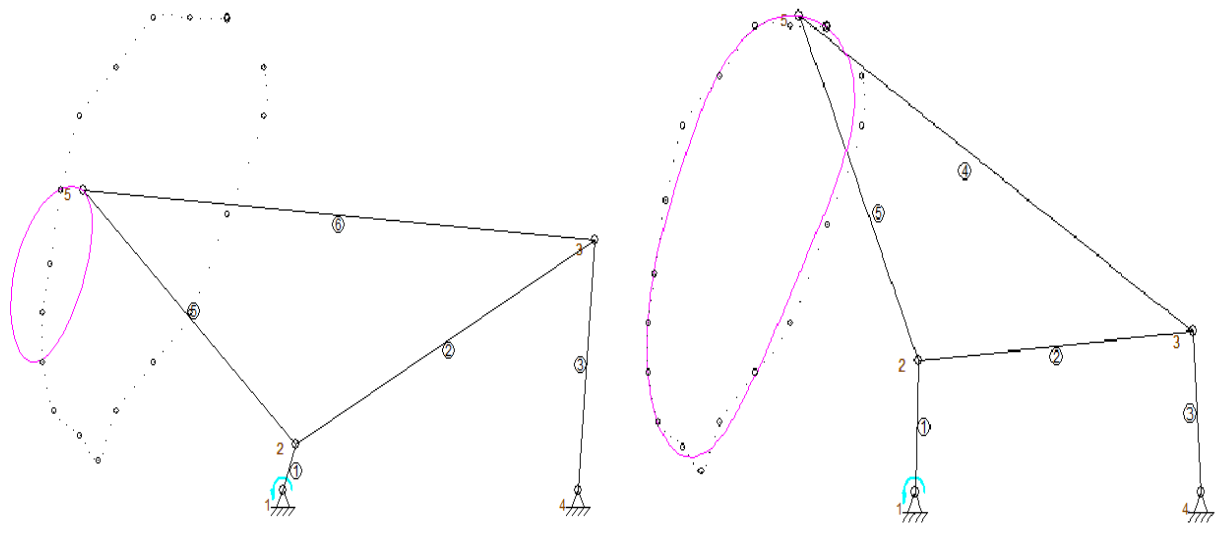
Parameters	l_1(mm)	l_5(mm)	γ(degree)	ξ(degree)
Constant Value	800	774	111	0

Table 8 Design parameters for the first guess and after optimization

Parameters	b1	b2	l_2(mm)	l_3(mm)	l_4(mm)
First Guess	650	140	100	910	410
After Optimization	747.70	157.70	267.20	767.88	326.33

Table 9 Coordinates of points on the coupler curve

Index	*Points on desired path-curve* [6]		*Points on preliminary path-curve with min. distance*		*Points on optimized path-curve with min. distance*	
i	$x_{desired}(i)$	$y_{desired}(i)$	$x_{pr}(i)$	$y_{pr}(i)$	$x_{opt}(i)$	$y_{opt}(i)$
1	0.50000	1.10000	0.11723	0.73936	0.50000	1.10000
2	0.40000	1.10000	0.10834	0.74887	0.39779	1.12208
3	0.30000	1.10000	0.10114	0.75465	0.30105	1.09751
4	0.20000	1.00000	0.09999	0.75350	0.19557	1.00376
5	0.10000	0.90000	0.08734	0.75925	0.12711	0.88678
6	0.05000	0.75000	0.04904	0.75234	0.06280	0.74555
7	0.02000	0.60000	-0.06006	0.63794	0.01536	0.60120
8	0.00000	0.50000	0.05645	0.46393	-0.00140	0.50005
9	0.00000	0.40000	-0.00360	0.40563	-0.00047	0.39993
10	0.03000	0.30000	-0.01522	0.40351	0.02732	0.29852
11	0.10000	0.25000	-0.00657	0.40635	0.09829	0.23135
12	0.15000	0.20000	0.00322	0.40991	0.13984	0.23148
13	0.20000	0.30000	0.03795	0.43841	0.21734	0.28476
14	0.30000	0.40000	0.08603	0.51143	0.30099	0.39941
15	0.40000	0.50000	0.12138	0.60066	0.37384	0.51671
16	0.50000	0.70000	0.13273	0.69147	0.48619	0.70845
17	0.60000	0.90000	0.12045	0.73810	0.57167	0.90508
18	0.60000	1.00000	0.12309	0.73942	0.57497	0.99482



(**a**) (**b**)

Figure 6 Path curve of point 5, (**a**) Considering an initial guess for parameters (**b**)After optimization of parameters

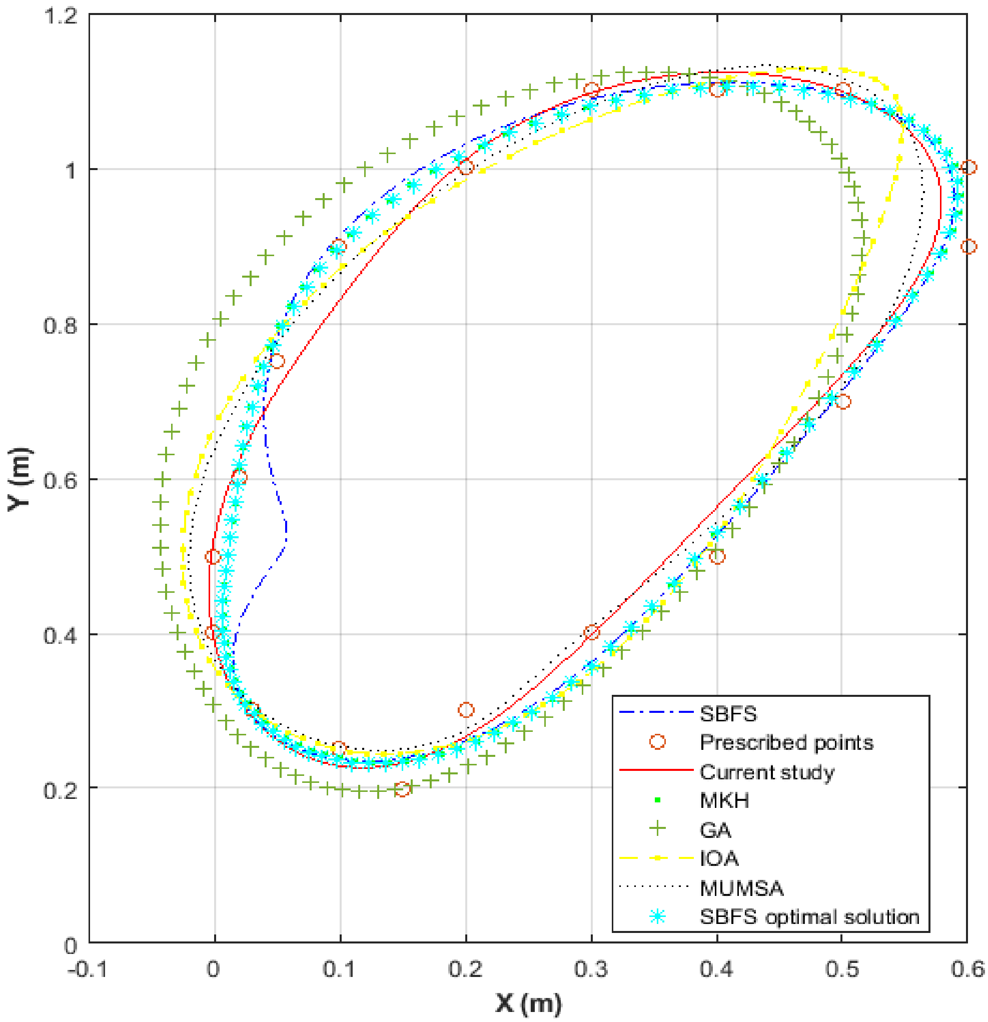


Figure 7 Prescribed points and the path curve obtained by the current study and the results obtained by other researchers

Table 10 Final results for case study II

Variables	SBFS [7]	Optimal SBFS [7]	Present study	GA [25]	IOA[11]	MUMSA [10]	MKH [12]
b_1	0.2409	0.2707	0.7477	1.1321	1.8918	-1.3092	0.2689
b_2	0.1429	0.1470	0.1577	0.6634	-0.7613	2.8070	0.1771
ξ	0.2878	0.2906	0.0000	4.3542	1.1878	2.7387	0.2929
l_1	1.1309	1.0610	0.8000	1.8797	4.0404	4.4538	1.0043
l_2	0.4326	0.4232	0.2672	0.2749	0.2452	0.2971	0.4218
l_3	0.9709	0.9239	0.7679	1.1803	6.3829	3.9131	0.8782
l_4	0.5934	0.5974	0.3263	2.1382	2.6205	0.8494	0.5801
l_5	0.5556	0.5522	0.7735	0.9156	2.1863	2.6520	0.5234
γ	0.8263	0.8270	1.9398	3.5681	1.0228	2.4647	0.8148
ESV	0.0153	0.0076	0.0115	0.0671	0.0318	0.0154	0.0078

- *All dimensions are in meter and angles are in radian*
- *ESV is in m^2*

Table 11 CPU-run-time and Number of Iterations for considered case studies

Case study	CPU-run-time(sec)	Number of Iterations
I	0.0469	17
II	0.0312	5
III	0.0715	31

6 Conclusions

In this article, the optimal dimensional synthesis of the four-bar path generating mechanism is performed utilizing the optimization method of multivariable functions. For this purpose, the equation of the coupler-curve is obtained according to the design parameters and with a completely analytical method. The results obtained from the considered case studies show that the method used to obtain the optimal value of the design parameters is very efficient and satisfactory and is clearly superior in many cases compared to the research results of other researchers. One can find the CPU-run-time and number of iterations for three case studies in Table (11) that can show the efficiency of the proposed method. One of the advantages of the proposed method is that it can be used for the optimal dimensional synthesis of other planar and three-dimensional mechanisms, which the author will address in future researches.

Declaration of competing interests

The author declares no conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Nomenclature

English symbols

b_1	Coordinate of the left revolute joint in the *X* direction in the reference coordinate system, *XYZ*
b_2	Coordinate of the left revolute joint in the *Y* direction in the reference coordinate system, *XYZ*
l_1	Length of the fixed link

l_2	Length of the link 2
l_3	Geometric parameter of the coupler
l_4	Geometric parameter of the coupler
l_5	Length of the link 5
x	Coordinate of the desired coupler point in the *x* direction in the coordinate system attached to the mechanism, *xyz*
X	Coordinate of the desired coupler point in the *X* direction in the reference coordinate system, *XYZ*
$x_{opt}(i)$	Optimum coordinate of the desired coupler-point in the *x* direction in the coordinate system attached to the mechanism, *xyz*
$x_{pr}(i)$	Preliminary coordinate of the desired coupler- point in the *x* direction in the coordinate system attached to the mechanism, *xyz*
$x_{desired}(i)$	Desired coordinate of the coupler-point in the *x* direction in the coordinate system attached to the mechanism, *xyz*
y	Coordinate of the desired coupler point in the *y* direction in the coordinate system attached to the mechanism, *xyz*
Y	Coordinate of the desired coupler point in the *Y* direction in the reference coordinate system, *XYZ*
$y_{opt}(i)$	Optimum coordinate of the desired coupler-point in the *y* direction in the coordinate system attached to the mechanism, *xyz*
$y_{pr}(i)$	Preliminary coordinate of the desired coupler- point in the *y* direction in the coordinate system attached to the mechanism, *xyz*
$y_{desired}(i)$	Desired coordinate of the coupler-point in the *y* direction in the coordinate system attached to the mechanism, *xyz*

Greek symbols

α	Angle of the link 3 relative to the horizontal surface
y	Angle of link 4 relative to the fixed element
x	Angle of the fixed link relative to the horizontal surface
γ	Angle of the link 3 relative to link 5
θ	Angle of the link 2 relative to fixed element.