A fiber-reinforced Transversely Isotropic Constitutive Model for Liver Tissue

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Biomechanical properties of soft tissue, such as liver, are important in modeling computer aided surgical procedures. Experimental evidences show that liver tissue is transversely isotropic. In this article, considering the liver tissue as an incompressible fiber-reinforced composite with one family of fibers, an exponential strain energy function (SEF) is proposed. The proposed SEF is based on a recently developed strain measure which is more consistent with the physics of deformation than the commonly used Green-Lagrange strain measure. To show the capabilities of the proposed SEF, comparison is done with the experimental data available in the literature. It is shown that the results of the proposed SEF is in a good agreement with the experimental data for both tensile and compression deformations.

Keywords: Liver tissue, transversely isotropic hyperelasticity, strain energy function, consistent strain measure

1 Introduction

Understanding the biomechanical properties of liver tissue is important for developing computer simulation programs that could assist in surgical planning, treatment and training, as well as for development and design of medical devices and procedures for treating liver disease [1]. In numerical simulation research, liver tissue is typically assumed to be isotropic. Nevertheless, a typical biological soft tissue is anisotropic [1].

In vitro uniaxial tension and compression experiments show that with the primary axis perpendicular to the cross sectional surface of samples, the liver tissue is stiffer with tensile or compressive force in the axial direction compared to that of the transverse direction. It was shown by Chui et al. [1] that at 20% strain, about twice as much force is required to elongate a longitudinal porcine liver tissue sample than that of a transverse sample. Results of their study suggest that liver tissue is transversely isotropic. However, liver, like many other soft biological tissues, is anisotropic and heterogeneous, hepatic surgical simulation programs were developed assuming isotropic and homogeneous tissue material properties [2-4]. Although much of the work in mathematical modeling of soft tissue was based on the assumption of the tissue having isotropic properties [5-7],

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it is important to consider the effects of liver tissue anisotropy when computer aided or integrated surgical models are developed.
Hu and Desai [8] reported that liver tissue is strain rate independent based on their indentation experiments at 6.096 mm/min and 0.69096 mm/min on 50x50 mm liver tissue sample with varying height [1].

Hyperelastic behavior of soft tissues is described by a strain energy function (SEF). Available SEFs are commonly based on right Cauchy-Green deformation tensor. Recently, Darijani and Naghdabadi [9] have reconsidered the strain measure definition to satisfy the consistency requirements: (1) strain should tend to +∞ when stretch approaches +∞[§]. (2) strain should approaches -∞ when stretch approaches to zero [**]. The well-known Seth-Hill strain measures do not satisfy the both requirements, simultaneously. For example, Green–Lagrange strain measure, $\mathbf{E}=\dfrac{\mathbf{U}^2-\mathbf{I}}{2}\rightarrow -\dfrac{1}{2}\mathbf{I}$ when $\mathbf{U}\rightarrow 0$ [10]. Based on this consistent strain measure, Fereidoonnezhad et al. [11] show that, using the strain measure proposed by Darijani and Naghdabadi [9] improves the constitutive modeling of fiber-reinforced rubbers.

In this paper, based on the strain measure proposed by Darijani and Naghdabadi [9], we introduce a transversely isotropic SEF for liver tissue. To this end, the liver tissue is considered as an incompressible fiber-reinforced composite with one family of fibers that form an angle ψ against the circumferential direction.

2 Preliminaries

A hyperelastic material can be described by a SEF $W=W_{\mathbf{F}}(\mathbf{F})$ per unit reference volume, where $\mathbf{F}$ denotes the deformation gradient tensor. The objectivity requirement can automatically be satisfied representing the SEF in terms of the right Cauchy–Green tensor $\mathbf{C}=\mathbf{F}^{\mathrm{T}}\mathbf{F}$ so that [12]:

$$W=W_{\mathbf{F}}(\mathbf{F})=W_{\mathbf{C}}(\mathbf{C}) \tag{1}$$

where the subscript $\mathbf{C}$ indicates that the SEF $W_{\mathbf{C}}(\mathbf{C})$ is a function of $\mathbf{C}$. A constitutive law can then be obtained for an unconstrained hyperelastic material by:

$$\mathbf{S}=2\frac{\partial W}{\partial \mathbf{C}} \tag{2}$$

Where $\mathbf{S}$ represents the second Piola–Kirchhoff stress tensor.

For anisotropic materials, the symmetry group $\mathcal{G}$ can be defined with the aid of the so-called structural tensors $\mathbf{L}_i$ as [13]:

$$\mathcal{G}=\left\{\mathbf{Q}\in \text{orth} \ : \ \mathbf{Q}\mathbf{L}_i\mathbf{Q}^T=\mathbf{L}_i \ , \text{i=1,2,...,n}\right\} \tag{3}$$

The structural tensors are characterized by the following important properties [13].

$$\sum_{i=1}^{n}\mathbf{L}_i=\mathbf{I},\quad \mathbf{L}_i\mathbf{L}_j=0,\quad \operatorname{tr}\mathbf{L}_i=1, \tag{4}$$
$$i\neq j;\quad i,j=1,2,...,n$$

Where $\mathbf{I}$ represents the second-order identity tensor. The condition of material symmetry is written in terms of the SEF (1) and the symmetry group (3) by:

$$W_{\mathbf{C}}\left(\mathbf{Q}\mathbf{C}\mathbf{Q}^T\right)=W_{\mathbf{C}}(\mathbf{C}) \quad , \ \forall\mathbf{Q}\in\mathcal{G} \tag{5}$$

Condition (5) is satisfied if and only if the SEF can be represented as an isotropic tensor function of arguments containing the structural tensors [14]. In view of (3) one can thus write

[1] This means that under a very large stretching condition, strain should take a very large positive value
[2] This means that under a high compression condition, strain should take a very large negative value

$$W_{\mathbf{CL}}\left(\mathbf{QCQ}^{T},\mathbf{QL}_{i}\mathbf{Q}^{T}\right)=W_{\mathbf{CL}}\left(\mathbf{C},\mathbf{L}_{i}\right),\quad i=1,...,n\ ,\ \forall\mathbf{Q}\in\text{orth} \tag{6}$$

Transverse isotropy represents a material symmetry with respect to only one selected (principal material) direction. The structural tensors can be then expressed as [14]:

$$\mathbf{L}_1=\mathbf{l}_1\otimes\mathbf{l}_1,\ \ \mathbf{L}_2=\mathbf{L}_3=\frac{1}{2}\left(\mathbf{I}-\mathbf{l}_1\otimes\mathbf{l}_1\right) \tag{7}$$

We remark that the principal material direction is denoted by the index 1.
To satisfy (6) for transversely isotropic materials, $W_{\mathbf{CL}}$ should be a function of five invariants as follows [12]:

$$W_{\mathbf{CL}}=W_{\mathbf{CL}}\left(i_1,i_2,i_3,i_4,i_5\right) \tag{8}$$

where

$$\begin{aligned}&i_1=\text{tr}(\mathbf{C}),\ \ i_2=\text{tr}(\mathbf{C}^2),\ \ i_3=\det(\mathbf{C})\\&i_4=\mathbf{L}_1:\mathbf{C}\ ,\ \ i_5=\mathbf{L}_1:\mathbf{C}^2\end{aligned} \tag{9}$$

3 Strain energy function

Reconsidering the definition of the deformation measure, Darijani and Naghdabadi [9] proposed a new family of strain measures, their Lagrangian form is defined by:

$$\begin{cases}\boldsymbol{\Xi}^{(m,n)}=\dfrac{1}{m+n}\left(\mathbf{U}^{m}-\mathbf{U}^{-n}\right) & mn>0\\[2ex] \ln(\mathbf{U}) & m=n=0\end{cases} \tag{10}$$

where, m and n are integer numbers and $\mathbf{U}$ is the right stretch tensor. Let $m=n=1$, the strain measure is then takes the following form:

$$\boldsymbol{\Xi}^{(1,1)}=\frac{\mathbf{U}-\mathbf{U}^{-1}}{2} \tag{11}$$

We define the new deformation tensor $\mathbf{C}^*$ as:

$$\mathbf{C}^*=\left(\boldsymbol{\Xi}^{(1,1)}\right)^2=\left(\frac{\mathbf{U}-\mathbf{U}^{-1}}{2}\right)^2 \tag{12}$$

It should be noted that, $\mathbf{C}^*$ can also be represented as:

$$\mathbf{C}^*=\mathbf{H}^T\mathbf{H} \tag{13}$$

where $\mathbf{H}=\dfrac{\mathbf{F}-\mathbf{F}^{-T}}{2}$ is defined by Böck et al. [15] and Darijani and Naghdabadi [9] independently.

Now, we use the invariants of the proposed deformation tensor $\mathbf{C}^*$ instead of commonly-used right Cauchy-Green deformation tensor $\mathbf{C}$ to represent SEF of transversely isotropic materials as:

$$W=W\left(\mathrm{I}_1,\mathrm{I}_2,\mathrm{I}_3,\mathrm{I}_4,\mathrm{I}_5\right) \tag{14}$$

where I_1 to I_5 are defined similar to (9) except that $\mathbf{C}$ should be replaced by $\mathbf{C}^*$.

It is common to decompose the SEF of anisotropic materials into an isotropic part and a part which includes the effect of reinforcement (see e.g., [16-19]). Using this decomposition, the SEF of transversely isotropic materials can be represented as:

$$W=W_{iso}\left(\mathrm{I}_1,\mathrm{I}_2,\mathrm{I}_3\right)+W_{aniso}\left(\mathrm{I}_4,\mathrm{I}_5\right) \tag{15}$$

As experimental evidences indicate, soft tissues can be considered as incompressible materials with negligible error [20]. Since only two of $\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3$ are independent for incompressible materials, we represent W_{iso} as a function of $\mathrm{I}_1, \mathrm{I}_2$.
Considering (14), the constitutive equation is derived using the chain rule as:

$$\begin{aligned} \mathbf{\Sigma}^{(1)} &= \frac{\partial W}{\partial \mathbf{\Xi}^{(1,1)}} = \frac{\partial W}{\partial \mathbf{C}^*} \frac{\partial \mathbf{C}^*}{\partial \mathbf{\Xi}^{(1,1)}} \\ &= \left(\sum_{\substack{i=1 \\ i \neq 3}}^{5} \frac{\partial W}{\partial I_i} \frac{\partial I_i}{\partial \mathbf{C}^*} \right) \mathbf{\Xi}^{(1,1)} + \mathbf{\Xi}^{(1,1)} \left(\sum_{\substack{i=1 \\ i \neq 3}}^{5} \frac{\partial W}{\partial I_i} \frac{\partial I_i}{\partial \mathbf{C}^*} \right) \end{aligned} \tag{16}$$

where $\mathbf{\Sigma}^{(1)}$ is the stress tensor conjugate with $\mathbf{\Xi}^{(1,1)}$. The Cauchy stress tensor $\boldsymbol{\sigma}$ is obtained through the following relation [9]:

$$\mathbf{U}^{-1}\mathbf{R}^T\boldsymbol{\sigma}\mathbf{R} + \mathbf{R}^T\boldsymbol{\sigma}\mathbf{R}\mathbf{U}^{-1} = \mathbf{\Sigma}^{(1)} + \mathbf{U}^{-1}\mathbf{\Sigma}^{(1)}\mathbf{U}^{-1} \tag{17}$$

where $\mathbf{R}$ is the rotation tensor in the polar decomposition of $\mathbf{F}$. Also, the relation between nominal stress $\mathbf{P}$ and Cauchy stress $\boldsymbol{\sigma}$ can be written as:

$$\mathbf{P} = J\mathbf{F}^{-1}\boldsymbol{\sigma} \tag{18}$$

In view of (17) and (18), one can write:

$$\mathbf{F}^{-1}\mathbf{P}\mathbf{U} + \mathbf{R}^T\mathbf{P} = \mathbf{\Sigma}^{(1)} + \mathbf{U}^{-1}\mathbf{\Sigma}^{(1)}\mathbf{U}^{-1} \tag{19}$$

Now, based on the deformation measure $\mathbf{C}^*$, we introduce a transversely isotropic SEF for liver tissue. To this end, the liver tissue is considered as an incompressible fiber-reinforced composite with one family of fibers that form an angle ψ against the circumferential direction. Similar to [21], ψ is considered as a phenomenological variable.

The load-deformation response of soft biological tissues show an initial low-stiffness region, followed by an increasing stiffness at higher stretches [22]. This behavior motivated us to use an exponential function for anisotropic part of SEF. We also assume the isotropic part of SEF as a linear function of I_1 only, similar to well-known neo-Hookean SEF. Therefore,

$$\begin{aligned} W_{iso} &= c_1 \mathrm{I}_1 \\ W_{aniso} &= \frac{k_1}{k_2}\left(\exp\left(k_2 \mathrm{I}_4\right) - 1\right) \end{aligned} \tag{20}$$

where $k_1, ..., k_4$ and c_1 are material parameters. It should be noted that the fiber direction is $\mathrm{l}_1 = \left[0 \quad \cos\psi \quad \sin\psi\right]$.

4 Comparison with experimental results

It is generally believed that the mechanical properties of pig liver closely resemble those of human liver [1]. This motivates us to verify our model with the experimental data presented by Chui et al. [1] for porcine liver tissue. They have studied the transversely isotropic properties of porcine liver tissue experimentally. Therein a cylindrical sample is tested for tension and compression in longitudinal and circumferential directions and the nominal stress versus stretches in longitudinal and circumferential direction is presented. In this section, we examine the capability of the proposed SEF (20) to predict the experimental data of both tests, simultaneously.
To this end, substituting (20) in (16) and the results in (19) one can find the nominal stress versus stretches. It should be noted that the loading direction is not parallel or perpendicular to the fiber direction $l_1 = [0 \quad \cos\psi \quad \sin\psi]$.

Using the experimental data, the material parameters identification is achieved by means of least-squares and a Levenberg–Marquardt type algorithm [23, 24].

5 Results and Discussion

Table (1) shows the material parameters of the proposed SEF (20), adopted from the experimental data [1].

Table 1 material parameters of the proposed SEF (20), adopted from the experimental data [1].

c_1 (Pa)	k_1 (Pa)	k_2 (-)	ψ (deg)
335.28	864.84	26.54	57.3416

It should be noted that, since ψ is defined as the angle between fibers direction and circumferential direction, the absolute value greater than 45^0 shows that the fibers are closer to the longitudinal direction than circumferential direction. This indicates that the liver tissue should be stiffer in the longitudinal than circumferential direction which is consistent with experimental data [1]. Figure (1) and Figure (2), show the results of the proposed SEF as well as experimental data for longitudinal and circumferential direction, respectively. As can be seen from these figures, the results of the proposed model is in a good agreement with the experimental data in both tension and compression deformation for both tests. The material parameters in Table (1) are used in Figure (1) and Figure (2).

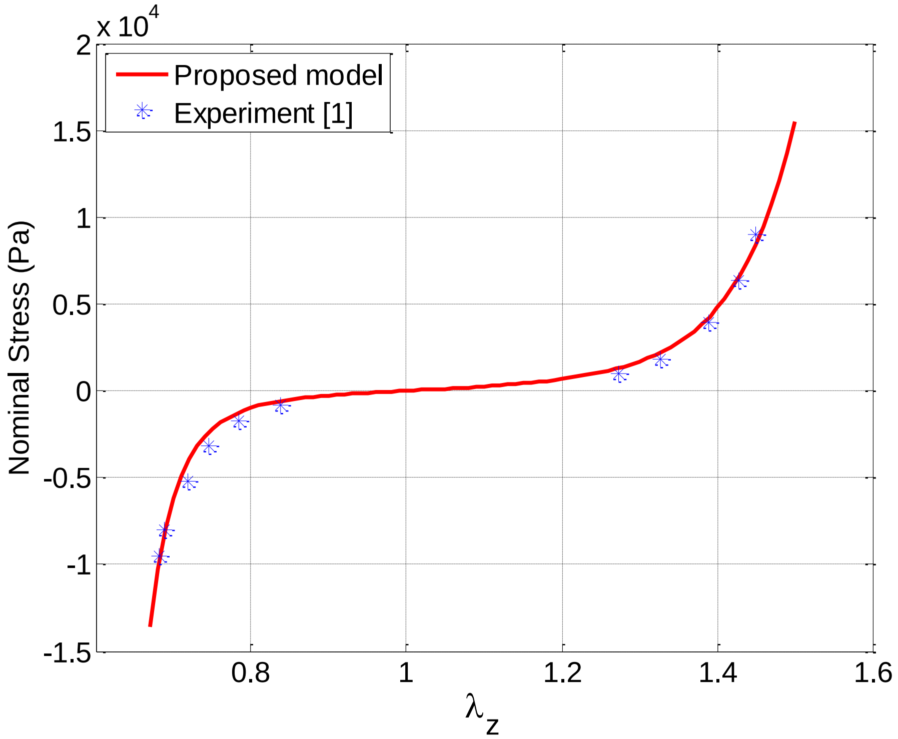

Figure 1 Nominal stress versus principal stretch for simple tension and compression in the longitudinal direction for porcine liver tissue.

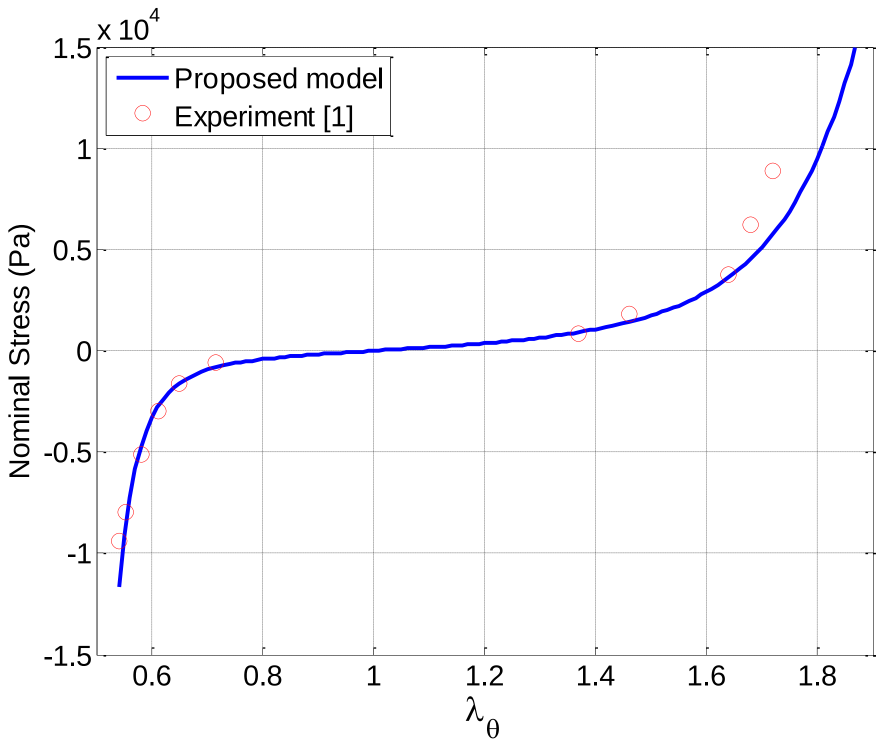

Figure 2 Nominal stress versus principal stretch for simple tension and compression in the circumferential direction for porcine liver tissue.

6 Summary and Conclusions

In this work, considering the liver tissue as an incompressible fiber-reinforced composite with one family of fibers, an exponential SEF is proposed. The proposed SEF is based on the strain measure introduced by Darijani and Naghdabadi [9], which is more consistent with physics of deformation than commonly-used Green-Lagrange strain measure.

The results of the proposed SEF are fitted to experimental data of tensile and compression tests, simultaneously. The results show that the proposed SEF is capable of capturing the experimental data for tensile and compression deformation in both longitudinal and circumferential directions, simultaneously.

The capability of the proposed SEF to model the experimental data suggests that anisotropy properties of liver tissue should be considered in surgical simulation.

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Nomenclature

$\mathbf{C}$: Right Cauchy-Green deformation tensor
c_1 : Material parameter
$\mathbf{C}^*$: New deformation tensor
$\mathbf{E}$: Green-Lagrange strain tensor
$\mathbf{F}$: Deformation gradient tensor
$\mathbf{G}$: Symmetry group
$\mathbf{H}$: The deformation tensor defined by Böck and Holzapfel [15]
$\mathbf{I}$: Identity tensor
i_1, i_2, i_3, i_4, i_5 : Invariants of $\mathbf{C}$
$\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3, \mathrm{I}_4, \mathrm{I}_5$: Invariants of $\mathbf{C}^*$
k_1, k_2, k_3, k_4 : Material parameters
$\mathbf{L}_i$: Structural tensors
$\mathbf{l}_1$: Fiber direction
m, n : Index of strain measure
orth : Orthogonal
$\mathbf{P}$: Nominal stress
$\mathbf{Q}$: Orthogonal mapping
$\mathbf{R}$: Rotation tensor
$\mathbf{S}$: Second Piola–Kirchhoff stress tensor
SEF: Strain energy function
tr : Trace of a tensor
$\mathbf{U}$: Right stretch tensor
W : Strain energy function per unit initial volume

Greek symbols

$\mathbf{\Xi}^{(m,n)}$: The strain measure proposed by Darijani and Naghdabadi [9].
$\mathbf{\Sigma}^{(1)}$: Stress conjugate to strain $\mathbf{\Xi}^{(1,1)}$
ψ : The angle of fibers with circumferential direction
λ_θ : Stretch in circumferential direction
λ_z : Stretch in axial direction

چکیده

خواص بیومکانیکی بافت‌های نرم بدن، از جمله بافت‌های کبد، در شبیه‌سازی کامپیوتری فرآیندهای جراحی از اهمیت ویژه‌ای برخوردار هستند. با وجود اینکه بسیاری از مقالات برای سادگی رفتار کبد را همسانگرد در نظر می‌گیرند اما مشاهدات تجربی بیانگر آن است که بافت‌های کبد همسانگرد جانبی هستند. در این مقاله یک مدل هایپرالاستیک همسانگرد جانبی برای پیش‌بینی رفتار مکانیکی بافت‌های کبد ارائه شده است. سینماتیک مساله بر اساس یک معیار کرنش سازگار با فیزیک تغییر شکل است [۹]. مقایسه نتایج حاصل از مدل پیشنهادی با نتایج تجربی نشان می‌دهد که مدل ارائه شده قادر به پیش‌بینی رفتار غیرهمسانگرد بافت‌های کبد هم در تغییرشکل‌های کششی و هم تغییرشکل‌های فشاری می‌باشد.