

	Thermomechanical Buckling of Simply Supported Shallow FGM Spherical
	Shells with Temperature-dependent
<b>A. Hafezalkotob</b> <sup>*</sup> Postgraduate Student	Material The thermomechanical buckling of simply supported thin
<b>M. R. Eslami</b> <sup>†</sup> Professor	The thermomechanical buckling of simply supported thin shallow spherical shells made of functionally graded material is presented in this paper. A metal-ceramic functionally graded shell with a power law distribution for volume fraction is considered, where its properties vary gradually through the shell thickness direction from pure metal on the inner surface to pure ceramic on the outer surface. The mechanical properties of the metal and ceramic are assumed to be temperature dependent. The governing equations are derived using the first-order shell theory of Love and Kirchhoff, the Donnell-Mushtari-Vlasov kinematics equations, and the calculus of variations. The analytical results are obtained for various types of loadings. The detailed results are compared and validated with the known data in the literature.

Keywords: buckling, shallow spherical shell, FGM, temperature-dependent material

# 1 Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous commonly manufactured from a blend of metals and ceramics with continuous composition gradation from pure ceramic on one surface to pure metal on the other surface. Dissimilar to fiber-matrix composites, where cracking and debonding may take place at high temperatures due to the material properties mismatch at the interface of two discrete materials, the ceramic constituents of FGMs are capable of withstanding high temperature environments because of their better thermal resistance characteristics, while the metal constituents provide stronger mechanical performance and decrease the possibility of catastrophic fracture. FGMs are now developed for general use as structural components in high-temperature environments and high temperature environments while maintaining structural integrity, FGMs, whose material properties vary uniformly and continuously from one surface to the other by gradually varying the volume fraction of their constituent materials, have received substantial attention in various industries, particularly in high-temperature applications [2].

Shahsiah and Eslami [3] analyzed the thermal buckling of FGM cylindrical shell. The same authors [4] utilized the improved Donnell equations for the thermal instability analysis of

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FGM cylindrical shell. Sofiyev [5] analyzed the stability of FGM truncated conical shells subjected to aperiodic impulsive loading. Wu et al. [6] conducted studies on the thermoelastic stability of functionally graded cylindrical shells. Sofivev [7] considered the stability of compositionally graded ceramic-metal cylindrical shells under aperiodic axial impulsive loading. A thermoelastic finite element model to study the linear buckling behavior of FGM hemispherical shell with a cut-out at apex in thermal environment was developed by Bhangale and Ganesan [8]. Kadoli and Ganesan [9] studied the buckling and free vibration analysis of FGM cylindrical shells subjected to a temperature-specified boundary condition. Sofiyev [10] analyzed the thermoelastic stability of FGM truncated conical shells. Mirzavand et al. [11] studied the effect of imperfections on the thermal buckling of FGM cylindrical shells. Bhangale et al. [12] considered the linear thermoelastic buckling and free vibration behavior of FGM truncated conical shells. The study considers temperature-dependent material properties. The thermal buckling of a simply supported imperfect FGM cylindrical shell based on the Wan-Donnell geometrical imperfection model was studied by Mirzavand and Eslami [13]. Sofivev et al. [14] presented the vibration and stability analysis for generic three-layered conical shells containing an FGM layer subjected to axial compressive load.

Shahsiah and Eslami [15] developed a thermal instability analysis of FGM shallow spherical shell. Li and Batra [16] surveyed the buckling of axially compressed thin cylindrical shells with FGM middle layer. Matsunaga [17] presented a 2D higher-order theory for analyzing the natural frequencies and the buckling stresses of FGM shallow shells. The thermal and mechanical buckling of truncated conical shells made of FGM was considered by Naj et al. [18]. Sofiyev et al. [19] conducted a study on the stability analysis of thin threelayered, truncated, conical shells containing an FGM layer subjected to nonuniform lateral pressure. Mirzavand and Eslami [20] developed the thermal buckling of simply supported FGM cylindrical shells that are integrated with surface-bonded piezoelectric actuators. Sheng and Wang [21] investigated the thermal vibration, buckling and dynamic instability of FGM cylindrical shells embedded in an elastic medium, subjected to mechanical and thermal loads, considering rotary inertia and the transverse shear strains along the shell thickness. The buckling behavior of FGM stiffened cylindrical shells by rings and stringers under axial compression loading was studied by Najafizadeh et al. [22]. Naj et al. [23] presented a solution on the instability of FGM truncated conical shells with temperature-dependent material under combination of thermal and mechanical loads. Sofiyev [24] carried out the study of the vibration and buckling behaviors of freely supported FGM truncated and complete conical shells subjected to hydrostatic and lateral pressures.

Huang and Han [25] investigated the buckling behaviors of geometrically imperfect FGM cylindrical shells under axial compression by using the Donnell shell theory and the nonlinear strain-displacement relations of large deformation. Matsunaga [26] derived the free vibration and stability of FGM circular cylindrical shells according to a 2D higher-order deformation theory. Sofiyev et al. [27] have conducted a study on the buckling behavior of a thin FGM hybrid truncated conical shell subjected to hydrostatic pressure. The same workers [28] studied the stability of a three-layered composite conical shell containing an FGM layer subjected to external pressure. Huang and Han [29] extended the non-linear large deflection theory of cylindrical shells to discuss the non-linear buckling and postbuckling behaviors of FGM cylindrical shells which are synchronously subjected to axial and lateral loads. By taking the temperature-dependent material properties into account, various effects of external thermal environment were also investigated.

In this paper the equilibrium equations of an FGM shallow spherical shell are derived using the variational method, the first-order shell theory of Love and Kirchhoff, and the Donnell-Mushtari-Vlasov (DMV) kinematics equations [30]. The stability equations are then obtained by consideration of the second variation of the functional of total potential energy.

The aim of this work is to analyze the stability of simply supported shallow spherical shells made of an FGM with different power law index values under the combination of mechanical and thermal loads, where temperature-dependent material properties are considered. Approximate one-term solutions that satisfy the boundary conditions are assumed for the displacement components. The Galerkin method is used to minimize the errors due to this approximation. The critical buckling loads of the simply supported FGM shallow spherical shell under two different types of loading, hydrostatic pressure and uniform temperature rise, and their combinations are obtained. The results are confirmed with the known data in the literature. The novelty of the present work is to obtain analytical closed-form solutions for buckling loads of the FGM shallow spherical shells, which can be easily and safely used in engineering design applications. The improvements and differences of this paper comparing Shahsiah and Eslami [15] one which is the nearest paper on the subject are as follows: thermomechanical analysis, various power law index values for volume fraction of mechanical properties of the shell, and temperature-dependent material properties rather than thermal analysis, linear volume fraction of mechanical properties of the shell, and temperature-independent material properties, respectively.

## 2 Derivations

As shown in Fig. (1), an FGM thin shallow spherical shell of mean radius R and thickness h is considered. The shell is assumed to be a mixture of ceramic and metal, where the mechanical properties are assumed to be [18]

$$E(z,T) = E_m(T) + E_{cm}(T) \left(\frac{2z+h}{2h}\right)^k,$$
  

$$\alpha(z,T) = \alpha_m(T) + \alpha_{cm}(T) \left(\frac{2z+h}{2h}\right)^k,$$
(1)  

$$v(z,T) = v,$$

where

$$E_{cm}(T) = E_c(T) - E_m(T),$$

$$\alpha_{cm}(T) = \alpha_c(T) - \alpha_m(T).$$
(2)

In Eqs. (1), z is measured along the thickness direction, positive outward, and vary from -h/2 to h/2. Subscript *m* indicates metal and subscript *c* indicates ceramic. Poisson's ratio is assumed to be constant through the shell thickness. The mechanical properties change gradually from pure metal on the inner surface to pure ceramic on the outer surface. In the spherical shell,  $\phi$ ,  $\theta$ , and z are the meridional, circumferential, and radial directions, respectively. According to the first-order shell theory of Love and Kirchhoff, the normal and shear strains at a distance z from the middle surface of the shell are [31]

where  $\varepsilon_i$  and  $\gamma_{ij}$  are the normal and shear strains, respectively,  $k_{\theta}$  and  $k_{\phi}$  are the middle surface bending curvatures,  $k_{\phi\theta}$  is the middle surface twisting curvature, and the subscript *m* refers to the middle surface. According to the DMV strain-displacement relations, we have [31]

$$\begin{split} \varepsilon_{\phi m} &= \frac{u_{,\phi} + w}{R} + \frac{w_{,\phi}^2}{2R^2}, \\ \varepsilon_{\theta m} &= \frac{v_{,\theta} + u\cos\phi + w\sin\phi}{R\sin\phi} + \frac{w_{,\theta}^2}{2R^2\sin^2\phi}, \\ \gamma_{\phi \theta m} &= \frac{u_{,\theta} + v_{,\phi}\sin\phi - v\cos\phi}{R\sin\phi} + \frac{w_{,\phi}w_{,\theta}}{R^2\sin^2\phi}, \\ k_{\phi} &= -\frac{w_{,\phi\phi}}{R^2}, \\ k_{\theta} &= -\frac{w_{,\theta\theta}}{R^2\sin^2\phi} - \frac{w_{,\phi}\cot\phi}{R^2}, \\ k_{\phi\theta} &= \frac{w_{,\theta}\cot\phi - w_{,\phi\theta}}{R^2\sin\phi}, \end{split}$$
(4)

where u, v, and w are the meridional, circumferential, and radial displacements of the shell middle surface, respectively, and the subscript (,) indicates partial derivative. According to Hooke's law, the stress-strain relations for the spherical shell are [15]

$$\sigma_{\varphi} = \frac{E(z,T)}{1-v^{2}} \Big( \varepsilon_{\varphi m} + v \varepsilon_{\theta m} + z k_{\varphi} + v z k_{\theta} \Big) - \frac{E(z,T)\alpha(z,T)\Delta T}{1-v},$$

$$\sigma_{\theta} = \frac{E(z,T)}{1-v^{2}} \Big( \varepsilon_{\theta m} + v \varepsilon_{\varphi m} + z k_{\theta} + v z k_{\varphi} \Big) - \frac{E(z,T)\alpha(z,T)\Delta T}{1-v},$$

$$\tau_{\varphi \theta} = G(z,T) \Big( \gamma_{\varphi \theta m} + 2z k_{\varphi \theta} \Big).$$
(5)

The force and moment resultants according to the first-order shell theory are [30]

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} dz,$$

$$M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z dz.$$
(6)

Substituting Eqs. (5) in Eqs. (6) yields

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$$N_{\phi} = \beta_{1}\varepsilon_{\phi m} + \beta_{1}v\varepsilon_{\theta m} + \beta_{2}k_{\phi} + \beta_{2}vk_{\theta} - \frac{\beta_{3}}{1-v},$$

$$N_{\theta} = \beta_{1}\varepsilon_{\theta m} + \beta_{1}v\varepsilon_{\phi m} + \beta_{2}k_{\theta} + \beta_{2}vk_{\phi} - \frac{\beta_{3}}{1-v},$$

$$N_{\phi\theta} = \beta_{4}\gamma_{\phi\theta m} + 2\beta_{5}k_{\phi\theta},$$

$$M_{\phi} = \beta_{2}\varepsilon_{\phi m} + \beta_{2}v\varepsilon_{\theta m} + \beta_{6}k_{\phi} + \beta_{6}vk_{\theta} - \frac{\beta_{7}}{1-v},$$

$$M_{\theta} = \beta_{2}\varepsilon_{\theta m} + \beta_{2}v\varepsilon_{\phi m} + \beta_{6}k_{\theta} + \beta_{6}vk_{\phi} - \frac{\beta_{7}}{1-v},$$

$$M_{\phi\theta} = \beta_{5}\gamma_{\phi\theta m} + 2\beta_{8}k_{\phi\theta},$$
(7)

where

$$\beta_{1} = \int_{-h/2}^{h/2} \frac{E(z,T)}{1-v^{2}} dz,$$

$$\beta_{2} = \int_{-h/2}^{h/2} \frac{zE(z,T)}{1-v^{2}} dz,$$

$$\beta_{3} = \int_{-h/2}^{h/2} \Delta T E(z,T) \alpha(z,T) dz,$$

$$\beta_{4} = \int_{-h/2}^{h/2} G(z,T) dz,$$

$$\beta_{5} = \int_{-h/2}^{h/2} zG(z,T) dz,$$

$$\beta_{6} = \int_{-h/2}^{h/2} \frac{z^{2}E(z,T)}{1-v^{2}} dz,$$

$$\beta_{7} = \int_{-h/2}^{h/2} \Delta T E(z,T) \alpha(z,T) z dz,$$

$$\beta_{8} = \int_{-h/2}^{h/2} z^{2} G(z,T) dz.$$
(8)

The equilibrium equations are derived using the functional of total potential energy equation and employing the Euler equations. Generally, the total potential energy of a loaded shell is the sum of the strain energy and the potential energy of the applied external loads [30]. For the shallow spherical shell of the present study, there exists a thermal term in the strain energy and the potential energy of the applied loads consists of the energy of the applied external hydrostatic pressure. Thus, the equilibrium equations using the variational principle are derived as [15]

$$\cos \phi N_{\theta} - (\sin \phi N_{\phi})_{,\phi} - N_{\phi\theta,\theta} = 0,$$

$$N_{\theta,\theta} + (\sin \phi N_{\phi\theta})_{,\phi} + \cos \phi N_{\phi\theta} = 0,$$

$$(\sin \phi M_{\phi})_{,\phi\phi} + \frac{1}{\sin \phi} M_{\theta,\theta\theta} - [R \sin \phi (N_{\phi}\beta_{\phi} + N_{\phi\theta}\beta_{\theta}) + \cos \phi M_{\theta}]_{,\phi} + 2[M_{\phi\theta,\phi\theta} + \cot \phi M_{\phi\theta,\theta}] - R \sin \phi (N_{\phi} + N_{\theta}) - R(N_{\theta}\beta_{\theta} + N_{\phi\theta}\beta_{\phi})_{,\theta} = PR^{2} \sin \phi,$$
(9)

where

$$\beta_{\phi} = -\frac{w_{,\phi}}{R}, \tag{10}$$
$$\beta_{\theta} = -\frac{w_{,\theta}}{R\sin\phi}.$$

In Eqs. (10),  $\beta_{\theta}$  and  $\beta_{\phi}$  are the rotations of the normal vector to the middle surface about the  $\phi$  and  $\theta$  axes, respectively.

The stability equations are obtained by consideration of the second variation of the functional of total potential energy. The displacement components are related to the terms representing the stable equilibrium and the terms of the neighboring state.

Accordingly, the force resultants  $N_{ij}$  and the moment resultants  $M_{ij}$  are divided into two terms representing the stable equilibrium and the neighboring state. Through the linear straindisplacement relations, the expression for the total potential energy is obtained. This expression, via the Taylor expansion, results in the sum of first and second variations in the total potential energy. Applying the Euler equations to the second variation of the total potential energy function results in the stability equations as [15]

$$\cos \phi N_{\theta_{1}} - (\sin \phi N_{\phi_{1}})_{,\phi} - N_{\phi\theta_{1},\theta} = 0,$$

$$N_{\theta_{1},\theta} + (\sin \phi N_{\phi\theta_{1}})_{,\phi} + \cos \phi N_{\phi\theta_{1}} = 0,$$

$$(\sin \phi M_{\phi_{1}})_{,\phi\phi} + \frac{1}{\sin \phi} M_{\theta_{1},\theta\theta} - \left[R \sin \phi \left(N_{\phi_{0}}\beta_{\phi_{1}} + N_{\phi\theta_{0}}\beta_{\theta_{1}}\right) + \cos \phi M_{\theta_{1}}\right]_{,\phi} + 2\left[M_{\phi\theta_{1},\phi\theta} + \cot \phi M_{\phi\theta_{1},\theta}\right] - R \sin \phi \left(N_{\phi_{1}} + N_{\theta_{1}}\right) - R\left(N_{\theta_{0}}\beta_{\theta_{1}} + N_{\phi\theta_{0}}\beta_{\phi_{1}}\right)_{,\theta} = 0.$$

$$(11)$$

Equations (11) are the stability equations of a shallow spherical shell. In Eqs. (11) the subscript 0 refers to the equilibrium state and the subscript 1 refers to the stability state. The terms with the subscript 0 are the solution of the equilibrium equations for the given load.

For simplicity, the membrane solution of the equilibrium equations is considered. By solving the membrane form of the equilibrium equations, it is found that

$$N_{\varphi_0} = N_{\theta_0} = -\frac{PR}{2} - \frac{1}{1-\nu} \int_{-h/2}^{h/2} \Delta T E(z,T) \alpha(z,T) dz = -\frac{PR}{2} - \frac{\Delta T h}{1-\nu} (E_m(T)\alpha_m(T) + \frac{1}{k+1} E_m(T)\alpha_{cm}(T) + \frac{1}{k+1} E_{cm}(T)\alpha_m(T) + \frac{1}{2k+1} E_{cm}(T)\alpha_{cm}(T)), \quad (12)$$

$$N_{\varphi_0} = 0,$$

where *P* is the applied hydrostatic pressure and  $\Delta T$  is the temperature change. The linear form of the strains and curvatures in terms of the displacement components are

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$$\varepsilon_{\phi m_{1}} = \frac{u_{1,\phi} + w_{1}}{R},$$

$$\varepsilon_{\phi m_{1}} = \frac{v_{1,\theta} + u_{1}\cos\phi + w_{1}\sin\phi}{R\sin\phi},$$

$$\gamma_{\phi \theta m_{1}} = \frac{u_{1,\theta} + v_{1,\phi}\sin\phi - v_{1}\cos\phi}{R\sin\phi},$$

$$k_{\phi_{1}} = -\frac{w_{1,\phi\phi}}{R^{2}},$$

$$k_{\theta_{1}} = -\frac{w_{1,\theta\theta}}{R^{2}\sin^{2}\phi} - \frac{w_{1,\phi}\cot\phi}{R^{2}},$$

$$k_{\phi \theta_{1}} = \frac{w_{1,\theta}\cot\phi - w_{1,\phi\theta}}{R^{2}\sin\phi}.$$
(13)

For the state of stability, the force and moment resultants are

$$N_{\phi_{1}} = \beta_{1}\varepsilon_{\phi m_{1}} + \beta_{1}v\varepsilon_{\theta m_{1}} + \beta_{2}k_{\phi_{1}} + \beta_{2}vk_{\theta_{1}},$$

$$N_{\theta_{1}} = \beta_{1}\varepsilon_{\theta m_{1}} + \beta_{1}v\varepsilon_{\phi m_{1}} + \beta_{2}k_{\theta_{1}} + \beta_{2}vk_{\phi_{1}},$$

$$N_{\phi\theta_{1}} = \beta_{4}\gamma_{\phi\theta m_{1}} + 2\beta_{5}k_{\phi\theta_{1}},$$

$$M_{\phi_{1}} = \beta_{2}\varepsilon_{\phi m_{1}} + \beta_{2}v\varepsilon_{\theta m_{1}} + \beta_{6}k_{\phi_{1}} + \beta_{6}vk_{\theta_{1}},$$

$$M_{\theta_{1}} = \beta_{2}\varepsilon_{\theta m_{1}} + \beta_{2}v\varepsilon_{\phi m_{1}} + \beta_{6}k_{\theta_{1}} + \beta_{6}vk_{\phi_{1}},$$

$$M_{\phi\theta_{1}} = \beta_{5}\gamma_{\phi\theta m_{1}} + 2\beta_{8}k_{\phi\theta_{1}}.$$
(14)

Substituting Eqs. (1) into Eqs. (8) yields

$$\begin{split} \beta_{1} &= \frac{h}{1 - \nu^{2}} \bigg( E_{m}(T) + \frac{E_{cm}(T)}{k + 1} \bigg), \\ \beta_{2} &= \frac{h^{2}}{1 - \nu^{2}} \frac{k}{2(k + 1)(k + 2)} E_{cm}(T), \\ \beta_{3} &= \Delta T h \bigg( E_{m}(T)\alpha_{m}(T) + \frac{E_{m}(T)\alpha_{cm}(T) + E_{cm}(T)\alpha_{m}(T)}{k + 1} + \frac{E_{cm}(T)\alpha_{cm}(T)}{2k + 1} \bigg), \\ \beta_{4} &= \frac{h}{2(1 + \nu)} \bigg( E_{m}(T) + \frac{E_{cm}(T)}{k + 1} \bigg), \\ \beta_{5} &= \frac{h^{2}}{1 + \nu} \frac{k}{4(k + 1)(k + 2)} E_{cm}(T), \\ \beta_{6} &= \frac{h^{3}}{1 - \nu^{2}} \bigg[ \frac{E_{m}(T)}{12} + \bigg( \frac{1}{k + 3} + \frac{1}{4(k + 1)} - \frac{1}{k + 2} \bigg) E_{cm}(T) \bigg], \\ \beta_{7} &= \Delta T h^{2} \frac{k}{2(k + 1)} \bigg( \frac{E_{m}(T)\alpha_{cm}(T) + E_{cm}(T)\alpha_{m}(T)}{k + 2} + \frac{E_{cm}(T)\alpha_{cm}(T)}{2k + 1} \bigg), \end{split}$$
(15)  
$$\beta_{8} &= \frac{h^{3}}{2(1 + \nu)} \bigg[ \frac{E_{m}(T)}{12} + \bigg( \frac{1}{k + 3} + \frac{1}{4(k + 1)} - \frac{1}{k + 2} \bigg) E_{cm}(T) \bigg]. \end{split}$$

Substituting Eqs. (13)-(15) into Eqs. (11), the stability equations in terms of the displacement components are derived. These equations, in terms of  $u_1$ ,  $v_1$ , and  $w_1$ , are a coupled set of three partial differential equations.

Consider a shallow spherical shell with the simply supported boundary edges. The boundary conditions are assumed as [15, 31]

$$u_{1,\phi} = v_1 = w_{1,\phi\phi} = w_1 = 0$$
 at  $\phi = \phi_L$ . (16)

The approximate one-term solutions for Eqs. (11), satisfying the boundary conditions given by Eqs. (16), may be assumed to be [15, 31]

$$\begin{cases} u_1 = A_1 \cos n\theta \cos \lambda\phi \\ v_1 = B_1 \sin n\theta \sin \lambda\phi \\ w_1 = C_1 \cos n\theta \sin \lambda\phi \end{cases}, \begin{cases} 0 \le \phi \le \phi_L \\ 0 \le \theta \le 2\pi \end{cases}$$
(17)

where  $\lambda = m\pi / \phi_L$ , m = 1, 2, 3, ... and n = 1, 2, 3, ... are the numbers of the meridional and circumferential waves, respectively, and  $A_1$ ,  $B_1$ , and  $C_1$  are constant coefficients. The approximate solutions (17) are substituted in Eqs. (11), utilizing the Galerkin minimization technique, to yield

$$a_{11}A_1 + a_{12}B_1 + a_{13}C_1 = 0,$$
  

$$a_{21}A_1 + a_{22}B_1 + a_{23}C_1 = 0,$$
  

$$a_{31}A_1 + a_{32}B_1 + a_{33}C_1 = 0.$$
(18)

The coefficients  $a_{ij}$  (i, j = 1, 2, 3) are calculated using Eqs. (11) and are given in Appendix. To derive the buckling load for the shallow FGM spherical shells, the determinant of the coefficients matrix of algebraic Eqs. (18) must be set equal to zero as

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0.$$
 (19)

The critical buckling load is the minimum load with respect to *m* and *n*.

### **3** Results and Discussion

Consider a metal-ceramic FGM shallow spherical shell. The geometry and loading of the shell are illustrated in Fig. (1). The combination of materials consists of steel and ceramic with temperature dependent Young's modulus and the thermal expansion coefficients as given in Table (1) [23, 32, 33].

Poisson's ratio is assumed to be 0.3 for the steel and ceramic. Simply supported boundary conditions are assumed.

Results are shown in the figures for a hydrostatic pressure and uniform temperature rise, and their combinations. The uniform temperature rise is considered with respect to the reference room temperature at  $25^{\circ}$ C.

#### 3.1 Mechanical buckling

The classical hydrostatic buckling pressure for a shallow spherical shell is [34]

$$P_{cl} = \frac{2E}{\sqrt{3(1-v^2)}} \left(\frac{h}{R}\right)^2.$$
 (20)

Figure (2) and Table (2) compare the result of this paper for a pure isotropic (metal) shallow spherical shell, where  $E = E_m(T)$ , with those reported in reference [32]. Figure (2) and Table (2) show the dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for different values of  $\phi_L$ . The dimensionless critical hydrostatic pressure is the ratio of the critical hydrostatic pressure to the classical one given by Eq. (20), where the ratio 1 shows identical estimated buckling pressure with that of the classical load. Close agreements between the results of this paper and those reported in reference [32] are observed, as seen from Fig. (2) and Table (2). It is found that for the larger values of  $\phi_L$ , better agreement between the values of the critical hydrostatic pressure and the classical one are observed. In Table (2), the numbers in the parentheses are related to the circumferential and meridional buckling modes of the shells, respectively.

Figure (3) illustrates the critical hydrostatic pressure versus the dimensionless parameter h/R for the steel shallow spherical shells with  $\phi_L = 10^\circ$ ,  $\phi_L = 20^\circ$ , and  $\phi_L = 30^\circ$ . It is found that the classical buckling hydrostatic pressure expression has more reliability for larger values of  $\phi_L$ . The most precise expression is for a complete spherical shell.

Figure (4) demonstrates the dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with  $\phi_L = 10^\circ$  and different power law indices k. The curves show the critical hydrostatic pressure decreases as the power law index increases.

Figure (5) indicates the dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and various  $\phi_L$ . It is found that the higher the value of  $\phi_L$ , the lower the value of dimensionless critical hydrostatic pressure will be.

### 3.2 Thermal buckling

Figure (6) displays the dimensionless critical temperature difference versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with different power law indices k and  $\phi_L = 10^\circ$ . The dimensionless critical temperature difference is critical temperature difference times the steel thermal expansion coefficient. The material properties are assumed to be temperature independent. The temperature difference is compared with the reference temperature ( $T = 25^\circ$ C). The curves illustrate that under thermal loading, the stability of an FGM shallow spherical shell is larger than that of a pure metallic one and smaller than that of a pure ceramic one. Additionally, the stability of FGM shallow spherical shells increases as the dimensionless parameter h/R increases. The curves for metallic and FGM shallow spherical shells with k = 1, are compared with those given in reference [31] and [15], respectively. From this comparison, close agreement between the results of this paper and those reported in the related references are observed. The reason of the small differences between the results of the present study and the given references is the selection of different material properties, as follows: Eslami et al. [31]

(metallic shell): E = 200 GPa,  $\alpha = 11.7 \times 10^{-6} \,^{\circ}\text{C}^{-1}$  and v = 1/3; Shahsiah et al. [15] (FGM shell):  $E_{\rm m} = 200$  GPa,  $\alpha_{\rm m} = 11.7 \times 10^{-6} \,^{\circ}\text{C}^{-1}$ ,  $E_{\rm c} = 380$  GPa,  $\alpha_{\rm c} = 7.4 \times 10^{-6} \,^{\circ}\text{C}^{-1}$ , and v = 0.3.

Figure (7) shows the dimensionless critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and various  $\phi_L$ . The material properties are assumed to be temperature independent. It can simply be observed from the curves that the higher the value of h/R, the higher the value of the dimensionless critical temperature difference will be.

Figure (8) presents the critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and various  $\phi_L$ . The material properties are assumed to be temperature independent. Similar to Fig. (7), the higher the value of h/R, the higher the value of the critical temperature difference will be.

#### 3.3 Thermomechanical buckling

Figure (9) indicates the dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and  $\phi_L = 10^\circ$  at  $T = 25^\circ$ C,  $T = 50^\circ$ C, and  $T = 100^\circ$ C. The buckling load is obtained using both the temperature dependent and independent material properties, as given by Table (1). According to this figure, it is found that at higher temperatures the value of the buckling pressure becomes less. This is an obvious fact due to the increase of thermal stresses at larger temperatures. Furthermore, another phenomenon that can be realized is the less effect of the temperature change on the critical pressure value for larger values of h/R.

Figure (10) presents the critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and  $\phi_L = 10^\circ$  at  $T = 100^\circ$ C. The buckling load is obtained using both the temperature dependent and independent material properties. As the curves illustrate, at  $T = 100^{\circ}$ C, the value of the buckling pressure related to the temperature dependent material curve is less than the one related to the temperature independent material curve. According to Table (1), at higher temperatures, Young's modulus decreases and thermal expansion coefficient of steel and ceramic increases. Therefore, according to the relations between the material properties and the dependence of the mechanical and thermal strains on material properties ( $\varepsilon_{\text{mechanical}} \sim \sigma$ , 1/E;  $\varepsilon_{\text{thermal}} \sim \alpha$ ,  $\Delta T$ ), at the same temperature the hydrostatic buckling pressure for the case of temperature dependent condition is less than that of temperature independent condition. The reason is the increase of 1/E and thermal expansion coefficient for the temperature dependent case. However, in Fig. (9), it is observed that at  $T = 100^{\circ}$ C for larger h/R the value of dimensionless buckling pressure related to the temperature dependent material curve is more than the one related to the temperature independent material curve. At this temperature for smaller h/R, the value of the dimensionless buckling pressure related to the temperature dependent material curve is less than the one related to the temperature independent material curve. This happens because of the different values of the classical buckling hydrostatic pressure for the temperature independent and dependent cases.

Figure (11) gives the dimensionless critical hydrostatic pressure versus the power law index k for the FGM shallow spherical shells  $\phi_L = 10^\circ$  and h/R = 0.01 at  $T = 25^\circ$ C,  $T = 50^\circ$ C, and  $T = 100^\circ$ C. The buckling load is obtained using the temperature dependent material properties. It is found that as the temperature rises, the dimensionless critical hydrostatic pressure decreases. The higher the value of k, the lower the value of the dimensionless critical hydrostatic pressure will be.

Figure (12) shows the dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with k = 0.5, k = 2, and  $\phi_L = 10^\circ$  at  $T = 100^\circ$ C. The material properties are assumed to be temperature

dependent. According to this figure, under the thermomechanical loading the critical buckling pressure of an FGM shallow spherical shell is less than that of a pure ceramic shallow spherical shell and higher than that of a pure metallic shallow spherical shell.

Figure (13) illustrates the dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2, various  $\phi_L$ , and the temperature dependent material at  $T = 100^{\circ}$ C. According to the curves at high temperatures, the higher the value of h/R, the higher the value of the dimensionless critical hydrostatic pressure will be.

Figure (14) demonstrates the dimensionless critical hydrostatic pressure versus the power law index k for the FGM shallow spherical shells with  $\phi_L = 10^\circ$ , various h/R, and the temperature dependent material at  $T = 100^\circ$ C. It is found that as the value of h/R falls, the dimensionless critical hydrostatic pressure decreases. Moreover, similar to Fig. (11), the higher the value of k, the lower the value of the dimensionless critical hydrostatic pressure will be.

Figure (15) presents the critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and  $\phi_L = 10^\circ$  under P = 0, P = 40 kPa, and P = 80 kPa. The material properties are assumed to be temperature independent. According to this figure, it is found that for higher hydrostatic pressures the value of the buckling temperature difference becomes less. This occurs because of the increase of mechanical stresses at larger values of the hydrostatic pressure. For higher values of h/R the effect of internal pressure on the critical buckling temperature becomes less.

Figure (16) displays the critical temperature difference versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with k = 0.5, k = 2, and  $\phi_L = 10^\circ$  under P = 80 kPa. The material properties are assumed to be temperature independent. The curves illustrate that under thermomechanical loading, the stability of an FGM shallow spherical shell is larger than that of a pure metallic one and smaller than that of a pure ceramic one. Additionally, the stability of FGM shallow spherical shell decreases as the power law index *k* increases.

Figure (17) gives the critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with k = 2 and various  $\phi_L$  under P = 80 kPa. The material properties are assumed to be temperature independent. It can easily be perceived from the curves that the higher the value of h/R, the higher the value of the critical temperature difference will be.

Figure (18) indicates the thermomechanical stability for the steel, the ceramic, and the FGM shallow spherical shells with different values of h/R, k, and  $\phi_L$  under a thermal load and hydrostatic pressure. The material properties are assumed to be temperature independent. The curve shows that for the steel, the ceramic, and the FGM shallow spherical shells with different values of h/R, k, and  $\phi_L$  under a thermal load and hydrostatic pressure, the steel, the ceramic, and the FGM shallow spherical shells with different values of h/R, k, and  $\phi_L$  under a thermal load and hydrostatic pressure, the stability boundary is a perfectly straight line. The area above the stability boundary can be considered as the instability region. Adversely, the area below the stability boundary is the stability region.

#### 4 Conclusion

In the present paper, the thermomechanical buckling of simply supported shallow spherical shells made of functionally graded materials is considered. The equilibrium and stability equations for the FGM shallow spherical shells are obtained. Derivations are based on the first-order shell theory and Sanders kinematics relations. The buckling analysis of FGM shallow spherical shells under two different types of loading (hydrostatic pressure and uniform temperature rise) and their combinations are investigated. To compute the critical

hydrostatic pressure in a given thermal environment, the mechanical properties are assumed to be temperature dependent. However, to calculate the critical temperature difference under a given hydrostatic pressure, the temperature independent material is assumed. That is, the mechanical properties at room temperature are utilized. The following conclusions are drawn:

- 1. At any given temperature, the critical mechanical buckling load of an FGM shallow spherical shell is less than that of a pure ceramic shallow spherical shell and higher than that of a pure metallic shallow spherical shell, where the material temperature dependency is considered. Additionally, under any given hydrostatic pressure, the critical thermal buckling load of an FGM shallow spherical shell is less than that of a pure ceramic shallow spherical shell and higher than that of a pure metallic shell we have a spherical shell is less than that of a pure ceramic shallow spherical shell and higher than that of a pure metallic shallow spherical shell, where the material temperature independency is applied.
- 2. The thermomechanical stability boundary for a steel, ceramic, or FGM shallow spherical shells with different values of h/R, k, and  $\phi_L$  under a combined thermal load and hydrostatic pressure is always a straight line, as shown in Fig. (18).
- 3. The stability of an FGM shallow spherical shell decreases with increasing temperature or hydrostatic pressure.
- 4. When the power law index *k* increases, the critical buckling hydrostatic pressure and temperature of an FGM shallow spherical shell decrease.
- 5. At any fixed temperature, the buckling hydrostatic pressure of a temperature dependent FGM shallow spherical shell is less than that of a temperature independent FGM shallow spherical shell.
- 6. The higher the value of h/R, the higher the stability of an FGM shallow spherical shell under a thermal, mechanical, or thermomechanical load.

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### Nomenclature

$E_m, E_c$ :	moduli of elasticity of the metal and the ceramic respectively
<i>G</i> :	modulus of rigidity
<i>h</i> :	thickness
<i>k</i> :	power law index
$k_{ij}$ :	curvature of the middle surface
$M_{ij}$ :	moment resultant
$M_{iji}$ :	moment resultant related to the stable state
$N_{ij}$ :	force resultant
$N_{ij0}$ :	force resultant related to the equilibrium state
$N_{ijl}$ :	force resultant related to the stable state
<i>P</i> :	hydrostatic pressure
$P_{cl}$ :	classical buckling hydrostatic pressure
$P_{cr}$ :	critical buckling hydrostatic pressure
<i>R</i> :	mean radius
$T, \Delta T$ :	temperature and temperature difference
$\Delta T_{cr}$ :	critical buckling temperature difference
<i>u, v, w</i> :	the middle surface meridional, circumferential, and radial displacements
	respectively
$u_1, v_1, w_1$ :	the middle surface meridional, circumferential, and radial displacements
	related to the stable state respectively
<i>Z</i> :	radial direction

### Greek symbols

$\alpha_m, \alpha_c$ :	thermal expansion coefficients of the metal and the ceramic respectively
$\beta_{\theta}$ :	rotation of the normal vector to the middle surface about the $\boldsymbol{\varphi}$ direction
$\beta_{ heta_l}$ :	rotation of the normal vector to the middle surface about the $\varphi$ direction related to the stable state
$\beta_{\phi}$ :	rotation of the normal vector to the middle surface about the $\theta$ direction
$p_{\phi_l}$ .	related to the stable state
γ <sub>ij</sub> :	shear strain

Yijm:	shear strain of the middle surface
Yijm1:	shear strain of the middle surface related to the stable state
$\varepsilon_i$ :	normal strain
$\mathcal{E}_{im}$ :	normal strain of the middle surface
$\mathcal{E}_{im1}$ :	normal strain of the middle surface related to the stable state
θ.	circumferential direction
v:	Poisson's ratio
$\sigma_i$ :	normal stress
$\tau_{ii}$ :	shear stress
φ:	meridional direction
$\phi_L$ :	supportive angle

# Appendix

The arrays of the coefficients matrix of algebraic Eqs. (18) are as follows:

$$\begin{aligned} a_{11} &= \frac{\pi}{R} \int_{0}^{\phi_{1}} \left[ \left( \cos^{2} \lambda \phi \cos^{2} \phi + \frac{\lambda}{4} \sin 2\lambda \phi \sin 2\phi + (\lambda^{2} + \nu) \cos^{2} \lambda \phi \sin^{2} \phi \right) \beta_{1} + n^{2} \cos^{2} \lambda \phi \beta_{4} \right] d\phi, \\ a_{12} &= -\frac{\pi n}{R} \int_{0}^{\phi_{1}} \left[ \left( -\frac{1}{2} \sin 2\lambda \phi \cos \phi + \nu \lambda \cos^{2} \lambda \phi \sin \phi \right) \beta_{1} + \left( -\frac{1}{2} \sin 2\lambda \phi \cos \phi + \lambda \cos^{2} \lambda \phi \sin \phi \right) \beta_{4} \right] d\phi, \\ a_{13} &= \frac{\pi}{R^{2}} \int_{0}^{\phi_{1}} \left[ -\lambda (1 + \nu) R \cos^{2} \lambda \phi \sin^{2} \phi \beta_{1} + \left( -\cos^{2} \lambda \phi \cos^{2} \phi - (\lambda^{2} + \nu) \lambda \cos^{2} \lambda \phi \sin^{2} \phi \right) + \frac{(1 + \nu) n^{2}}{2} \sin 2\lambda \phi \cot \phi - \frac{\lambda^{2}}{4} \sin 2\lambda \phi \sin 2\phi - \lambda \nu n^{2} \cos^{2} \lambda \phi \right] \beta_{2} \\ &+ n^{2} (-2\lambda \cos^{2} \lambda \phi + \sin 2\lambda \phi \cot \phi) \beta_{5} \right] d\phi, \\ a_{21} &= -\frac{\pi n}{R} \int_{0}^{\phi_{1}} \left[ \left( \frac{1}{2} \sin 2\lambda \phi \cos \phi - \nu \lambda \sin^{2} \lambda \phi \sin \phi \right) \beta_{1} + \left( \frac{1}{2} \sin 2\lambda \phi \cos \phi - \lambda \sin^{2} \lambda \phi \sin \phi \right) \beta_{4} \right] d\phi, \\ a_{22} &= \frac{\pi}{R} \int_{0}^{\phi_{1}} \left[ -n^{2} \sin^{2} \lambda \phi \beta_{1} + \left( -2 \sin^{2} \lambda \phi \cos 2\phi + \frac{\lambda}{4} \sin 2\lambda \phi \sin 2\phi - \lambda^{2} \sin^{2} \lambda \phi \sin^{2} \phi \right) \beta_{4} \right] d\phi, \\ a_{23} &= \frac{\pi n}{R^{2}} \int_{0}^{\phi_{1}} \left[ -(1 + \nu) R \sin^{2} \lambda \phi \sin \phi \beta_{1} + \left( \frac{1}{2} \sin 2\lambda \phi \cos \phi - (\nu \lambda^{2} + n^{2} \csc^{2} \phi) \sin^{2} \lambda \phi \sin \phi \right) \beta_{2} \\ &+ 2(1 - \lambda^{2}) \sin^{2} \lambda \phi \sin \phi \beta_{5} \right] d\phi, \\ a_{31} &= \frac{\pi}{R} \int_{0}^{\phi_{1}} \left[ (1 + \nu) R (\lambda \sin^{2} \lambda \phi \sin^{2} \phi - \frac{1}{4} \sin 2\lambda \phi \sin 2\phi \right] \beta_{1} + \left( (1 + \lambda^{2} + \nu) \lambda \sin^{2} \lambda \phi \sin^{2} \phi \right) \beta_{2} \\ &+ 2\lambda n^{2} \sin^{2} \lambda \phi \beta_{5} \right] d\phi, \end{aligned}$$

$$a_{32} = -\frac{\pi n}{R} \int_0^{\phi_L} \left[ (1+\nu)R\sin^2 \lambda\phi \sin\phi \beta_1 + (\frac{\lambda}{2}\sin 2\lambda\phi\cos\phi - (1-\lambda^2\nu)\sin^2 \lambda\phi\sin\phi + (n^2 - \cos^2\phi)\sin^2 \lambda\phi\csc\phi )\beta_2 + 2(-1+\lambda^2)\sin^2 \lambda\phi\sin\phi \beta_5 \right] d\phi,$$

$$\begin{aligned} a_{33} &= -\frac{\pi}{R^2} \int_0^{\phi_L} [2(1+\nu)R^2 \sin^2 \lambda \phi \sin^2 \phi \beta_1 + (1+\nu)R(-\frac{\lambda}{2} \sin 2\lambda \phi \sin 2\phi + 2(n^2 + \lambda^2 \sin^2 \phi) \sin^2 \lambda \phi) \beta_2 + (\lambda^2 \sin^2 \lambda \phi \cos^2 \phi + \frac{\lambda}{4} (\cot^2 \phi (1-2\nu) + 2\lambda - 2\lambda^2 - \nu) \sin 2\lambda \phi \sin 2\phi + (n^2 + \cos^2 \phi) \lambda \nu \sin 2\lambda \phi \cot \phi - 2(1+\nu) n^2 \sin^2 \lambda \phi \cot^2 \phi + (1+\lambda^2+\nu) \lambda^2 \sin^2 \lambda \phi \sin^2 \phi + (-1-\nu+2\lambda^2\nu+n^2 \csc^2 \phi) n^2 \sin^2 \lambda \phi) \beta_6 + (4(-1+\lambda^2 - \cot^2 \phi) n^2 \sin^2 \lambda \phi + 2\lambda n^2 \sin 2\lambda \phi \cot \phi) \beta_8 + ((n^2 + \lambda^2 \sin^2 \phi) R^2 \sin^2 \lambda \phi - \frac{\lambda R^2}{4} \sin 2\lambda \phi \sin 2\phi) N_{\phi_0}] d\phi. \end{aligned}$$

# Tables

**Table 1** Temperature-dependent Young's moduli of elasticity and thermal expansion

 coefficients for steel and ceramic (from Refs. [23, 32, 33])

	Steel		Ceramic	
Temperature (°C)	Modulus of elasticity (GPa)	Thermal expansion coefficient $(10^{-6\circ}C^{-1})$	Modulus of elasticity (GPa)	Thermal expansion coefficient $(10^{-6\circ}C^{-1})$
25	202.97	11.7	361.3	5.59
50	201.44	11.76	359.6	5.66
100	198.12	12.05	356.8	5.89
150	195.01	12.37	352.3	6.17
200	191.29	12.7	350.0	6.42
400	171.13	13.83	338.7	7.15

**Table 2** Variation in the dimensionless critical hydrostatic pressure  $P_{cr}/P_{cl}(n,m)$  with the dimensionless parameter h/R for various  $\phi_L$ 

	$\phi_L$				
h/R	$10^{\circ}$	$20^{\circ}$	$30^{\circ}$	$45^{\circ}$	$60^{\circ}$
0.0005	1.0291 (1,4)	1.0021 (1,9)	1.0034 (1,14)	1.0011 (1,20)	1.0005 (1,27)
0.0006	1.0071 (1,4)	1.0039 (1,8)	1.0030 (1,12)	1.0019 (1,19)	1.0008 (1,25)
0.0007	1.0147 (1,4)	1.0078 (1,8)	1.0045 (1,11)	1.0011 (1,17)	1.0007 (1,23)
0.0008	1.0408 (1,4)	1.0036 (1,7)	1.0033 (1,11)	1.0011 (1,16)	1.0016 (1,21)
0.0009	1.0241 (1,3)	1.0072 (1,7)	1.0021 (1,10)	1.0013 (1,15)	1.0011 (1,20)
0.0010	1.0111 (1,3)	1.0103 (1,6)	1.0061 (1,10)	1.0026 (1,14)	1.0011 (1,19)
0.0011	1.0090 (1,3)	1.0043 (1,6)	1.0026 (1,9)	1.0024 (1,14)	1.0015 (1,18)
0.0012	1.0150 (1,3)	1.0068 (1,6)	1.0044 (1,9)	1.0017 (1,13)	1.0027 (1,17)
0.0013	1.0274 (1,3)	1.0158 (1,6)	1.0070 (1,8)	1.0036 (1,13)	1.0015 (1,17)
0.0014	1.0447 (1,3)	1.0144 (1,5)	1.0031 (1,8)	1.0020 (1,12)	1.0016 (1,16)
0.0015	1.0656 (2,2)	1.0076 (1,5)	1.0044 (1,8)	1.0030 (1,12)	1.0025 (1,16)



Figure 1 The geometry and loading of the FGM shallow spherical shell.



**Figure 2** The dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the homogenous isotropic (steel) shallow spherical shells with various  $\phi_L$ .



**Figure 3** The critical hydrostatic pressure versus the dimensionless parameter h/R for the homogenous isotropic (steel) shallow spherical shells with various  $\phi_L$ .



**Figure 4** The dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with different power law indices *k*.



**Figure 5** The dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with various  $\phi_L$ .



**Figure 6** The dimensionless critical temperature difference versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with different power law indices k and the temperature independent material.



**Figure 7** The dimensionless critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with various  $\phi_L$  and the temperature independent material.



**Figure 8** The critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with various  $\phi_L$  and the temperature independent material.



**Figure 9** The dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with the temperature dependent and independent material under various temperature rises.



**Figure 10** The critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with the temperature dependent and independent material in the thermal environment.



Figure 11 The dimensionless critical hydrostatic pressure versus the power law index k for the FGM shallow spherical shells with the temperature dependent material under various temperature rises.



**Figure 12** The dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with different power law indices k and the temperature dependent material in the thermal environment.



**Figure 13** The dimensionless critical hydrostatic pressure versus the dimensionless parameter h/R for the FGM shallow spherical shells with various  $\phi_L$  and the temperature dependent material in the thermal environment.



**Figure 14** The dimensionless critical hydrostatic pressure versus the power law index k for the FGM shallow spherical shells with various h/R and the temperature dependent material in the thermal environment.



**Figure 15** The critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with the temperature independent material under various hydrostatic pressures.



**Figure 16** The critical temperature difference versus the dimensionless parameter h/R for the steel, the ceramic, and the FGM shallow spherical shells with different power law indices *k* and the temperature independent material under the hydrostatic pressure.



**Figure 17** The critical temperature difference versus the dimensionless parameter h/R for the FGM shallow spherical shells with various  $\phi_L$  and the temperature independent material under the hydrostatic pressure.



**Figure 18** The thermomechanical stability for the steel, the ceramic, and the FGM shallow spherical shells with different values of h/R, k,  $\phi_L$ , and the temperature independent material under a thermal load and hydrostatic pressure.

# چکیدہ

در این مقاله، تحلیل کمانش ترمومکانیکی پوستههای کروی کمعمق نازک با تکیهگاه ساده از جنس مواد FGM ارائه گردیده است. پوسته FGM در نظر گرفته شده از نوع فلز – سرامیک با کسر حجمی توانی است به طوری که خواص آن در راستای ضخامت پوسته به صورت تدریجی از فلز خالص در سطح درونی به سرامیک خالص در سطح بیرونی تغییر مییابد. خواص مکانیکی فلز و سرامیک وابسته به دما فرض گردیدهاند. روابط بنیادی با استفاده از نظریه مرتبه اول پوستهها Love-Kirchhoff، روابط سینماتیک -Iove-Invell-Mushtari بنیادی با استفاده از نظریه مرتبه اول پوستهها Subjection روابط سینماتیک -Iove-Kirchhoff بنیادی با استفاده از نظریه مرتبه اول پوسته و مقایسه میگردد.