

	Buckling and Static Analyses of Functionally
	Graded Saturated Porous Thick Beam Resting
	on Elastic Foundation Based on Higher Order
M.Babaei*	Beam Theory
M.Sc. Student	In this paper, static response and buckling analysis of
	functionally graded saturated porous beam resting on
	Winkler elastic foundation is investigated. The beam is
K Asemi†	modeled using higher-order shear deformation theory in
Assistant Professor	conjunction with Biot constitutive law which has not been
	surveyed so far. Inree different patterns are considered for porosity distribution along the thickness of the heam:
	1) poro/nonlinear non-symmetric distribution
	2) poro/nonlinear symmetric distribution and
D Safarnour [‡]	3) poro/monotonous distribution. To obtain the governing
Assistant Professor	equations, geometric stiffness matrix concept and finite
	element method is used. The effect of various parameters
	such as: 1) Stiffness of elastic foundation 2) Slender ratio
	3) Porosity coefficient 4) Skempton coefficient 5) Porosity
	distributions and 6) Different boundary conditions has
	been investigated to draw practical conclusions.

Keywords: Saturated porous beam, Functionally graded, Third-order beam theory, Buckling, Finite element method.

1 Introduction

Porous material is a media containing pores filled by a fluid. The skeletal part of the material is called matrix or frame and is usually a solid. Porosity is the ratio of holes to the overall space of porous material. Nowadays porous materials because of their permeability, high tensile strength and electrical conductivity is applied in an astonishingly large body of applications, including petroleum geophysics, civil engineering, geotechnical engineering, geology engineering, hydrology and biomechanics. In most of these applications, theory of poroelasticity is commonly exploited to examine the raised problems.

Biot [1] is the pioneer who has studied the poroelasticity. In this model, a porous material is composed of two phases namely solid and fluid.

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The linear poroelasticity theory of Biot has two characteristics:

- 1. An increase of pore pressure induces a dilation of pore.
- 2. 2- Compression of the pore causes a rise of pore pressure. There are several theories have also been developed for pore materials, but in practice they do not offer any advantage over the Biot theory. Theory of Biot poroelasticity in a drained condition is similar to the theory of elasticity in which the relation between strain and stress is based on Hook's law. During the last several years, porous material structures such as beams, plates, and shells have been used widely in structural design problems. Therefore, it is important to study the behavior of porous beams subjected to static and buckling loads.

Many researchers have studied static, buckling and vibration analyses of porous beams. But, in most of these studies, Hook's law (drained condition) and simple theories of beam are considered to model the problem in which some of them are referred here. Buckling of porous beams with varying properties was described by Magnucki and Stasiewicz [2]. They used a shear deformation theory to investigate the effect of porosity on the strength and buckling load of the beam. Magnucka-Blandzi [3] investigated the axi-symmetric deflection and buckling analysis of circular porous-cellular plate with the geometric model of nonlinear hypothesis. Jasion and Magnucka-Blandzi [4] presented the analytical, numerical and experimental buckling analysis of three-layered sandwich beams and circular plates with metal foam. Mojahedin and jabbari [5] investigated buckling of functionally graded circular plate made of saturated porous material based on higher order shear deformation theory. The effects of different parameters such as porosity coefficient, Skempton coefficient and porosity distribution on the critical buckling load were studied. Mojahedin et al. [6] employed the higher order shear deformation theory to examine the buckling of a fully clamped FG circular plate made of saturated porous materials subjected to an in-plane radial compressive load. Jabbari et al [7] studied the thermal buckling of solid circular plate made of porous material bounded with piezoelectric sensor-actuator patches.

The effects of thickness of porous plates, porosity and piezoelectric thickness on thermal stability of the plate were investigated. Buckling behavior of symmetric and antisymmetric FGM beams was investigated by khalid [8]. Tornabene and Fantuzzi et al. [9] presented a higher-order mathematical formulation for the free vibration analysis of arches and beams composed of the composite materials. The Euler- Bernoulli beam theory and Hamilton principle have been used to obtain the governing equations. Fouda [10] studied the bending, buckling and vibration of the functionally graded porous Euler- Bernoulli beam using finite element method. The governing equations are obtained using the Hamilton principle, and the finite element method is used to solve the equations.

Chen et al. [11, 12] presented elastic buckling, static bending, free and forced vibration analyses of shear deformable Timoshenko FG porous beams made of open-cell metal foams with two poro/nonlinear non-symmetric distribution and poro/nonlinear symmetric distribution. Galeban [13] studied free vibration of functionally graded thin beams made of saturated porous materials. The equations of motion were derived using Euler-Bernoulli theory and natural frequencies of porous beam have been obtained for different boundary conditions. The effects of poroelastic parameters and pores compressibility has been considered on the natural frequencies. Aghdam [14] studied nonlinear bending of functionally graded porous micro/nano-beams reinforced with graphene platelets based upon nonlocal strain gradient theory.

Gorbanpour [15] studied the free vibration of functionally graded porous plate resting on Winkler foundation based on the third-order shear deformation theory.

Buckling analysis of two-directionally porous beam was investigated by Haishan Tang [16] based on Euler–Bernoulli beam theory, minimum total potential energy principle and generalized differential quadrature method. The above literature review shows that the analysis of porous beams has mainly been performed based on the simple beam theories (Euler and Timoshenko), and Hooke's law or drained condition is considered to model the porous behavior of beam.

In this paper, buckling and static bending analyses of thick saturated porous functionally graded beam resting on a Winkler elastic foundation has been investigated based on the third order shear deformation theory and Biot constitutive law which has not been surveyed so far. Distribution of porosity along the thickness is considered in three different patterns, which are uniform, symmetric nonlinear and nonlinear asymmetric distributions. Geometric stiffness matrix concept is used to express the stability equations and the finite element method is used to solve the governing equations .The effect of different boundary conditions and various parameters such as Biot, porosity and Skempton coefficients, slenderness ratio and stiffness of elastic foundation on buckling and static bending responses of porous beam have been studied.

2 Governing equations

Consider a beam made of saturated porous materials with rectangular cross section resting on Winkler elastic foundation. It is assumed that the length of the beam is *L* and cross section is $b \times h$. Cartesian coordinates is used such that the *x* axis is at the left side of the beam on its middle surface Figure 1).

As shown in Figure (1), the porosity distribution along the thickness is considered as: 1) nonlinear asymmetric, 2) nonlinear symmetric, and 3) uniform distributions. The relations of modulus of elasticity and the shear modulus for all three distributions are as following, respectively:

$$G(z) = G_0 \left[1 - e_0 cos\left(\left(\frac{\pi}{2 * h}\right)\left(z + \frac{h}{2}\right)\right) \right]$$

$$E(z) = E_0 \left[1 - e_0 cos\left(\left(\frac{\pi}{2 * h}\right)\left(z + \frac{h}{2}\right)\right) \right]$$

$$G(z) = G_0 \left[1 - e_0 cos\left(\frac{\pi z}{2 * h}\right) \right]$$

$$E(z) = E_0 \left[1 - e_0 cos\left(\frac{\pi z}{2 * h}\right) \right]$$

$$G(z) = G_0 [1 - e_0]$$

$$E(z) = E_0 [1 - e_0]$$

$$(3)$$

where e_0 is the coefficient of beam porosity (0< e_0 <1). For distribution 1, E_0 and E_1 are Young's modulus of elasticity at z=h/2 and z=-h/2, respectively. Also, G_0 and G_1 are the shear modulus at z=h/2 and z=-h/2, respectively.

The relationship between the modulus of elasticity and shear modulus is $E_j=2G_j$ (1+v) (j=0, 1) and v is Poisson's ratio, which is assumed to be constant across the beam thickness.

2.1 Constitutive equations

Constitutive equations of porous beam are based on Biot theory instead of Hook's law. Biot theory deals with the displacements of the skeleton and the pore fluid movement as well as their interactions due to the applied loads [17]. The linear poroelasticity theory of Biot has two characteristics [2]

- 1) An increase of pore pressure induces a dilation of pore.
- 2) Compression of the pore causes a rise of pore pressure. particularly when the fluid cannot move freely within the network of pores. These coupled mechanisms display the time dependent character of the mechanical behavior of the porous structures. Such interactional mechanics were firstly modeled by Biot.

The stress-strain law for the Biot poroelasticity is given by [18].

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} - p\alpha\delta_{ij}$$

$$p = \overline{M}(\zeta - \alpha\varepsilon_{kk})$$

$$\overline{M} = \frac{2G(v_u - v)}{\alpha^2(1 - 2v_u)(1 - 2v)}$$

$$v_u = \frac{v + \alpha\beta(1 - 2v)/3}{1 - \alpha\beta(1 - 2v)/3}$$
(4)

Here p is pore fluid pressure, \overline{M} is Biot's modulus, G is shear modulus, v_u is undrained Poisson's ratio ($v < v_u < 0/5$), α is the Biot coefficient of effective stress ($0 < \alpha < 1$), ε_{kk} is the volumetric strain, ζ is variation of fluid volume content, β is Skempton coefficient. For p=0, the Biot law reduces to conventional Hook's law or drained condition.

The pore fluid property is introduced by the Skempton coefficient. The Biot's coefficient (α) describes the porosity effect on the behavior of the porous material without fluid, and states that due to porosity, the resistance of the body varies a few percent and is defined as follows:

$$\alpha = 1 - \frac{K}{K_S} \tag{5}$$



Figure 1 Distribution of porosity along the thickness

Where K_s is the bulk modulus of a homogeneous material. The relationship between the bulk modulus and the shear modulus is as follows:

$$K = \frac{2(1+v)}{3(1-2v)}G$$
(6)

The Skempton coefficient is an important dimensionless parameter for describing the effect of the fluid inside the cavities on the behavior of the porous material in the undrained state ($\zeta = 0$), and is the ratio of the cavity pressure to the total body stress.

$$\beta = \frac{dp}{d\sigma}|_{\zeta=0} = \frac{1}{1 + e_0} \frac{C_P}{C_c} = \frac{K_u - K}{\alpha K_u}$$
(7)

where K_u is the bulk modulus in the undrained state, K is the bulk modulus in the drained state, C_p is the fluid Compressibility in the pores and C_s is solid Compressibility. The Skempton coefficient also shows the effect of fluid Compressibility on the elastic modulus and the compressibility of the entire porous material [19].

2.2. Displacement field and strain

Different theories express the behaviors of the beam. In the third-order shear deformation theory, the displacement field is assumed to be of the third order of z diraction and as a result, the transverse shear stresses are second-order, and the problem of using the shear correction coefficient is eliminated. The displacement field in this theory are in x and z directions as follows [20]:

$$u(x,z) = u_0(x) + z\Phi_x(x) - 4\frac{z^3}{3h^2} \left[\Phi_x(x) + \frac{\partial w_0(x)}{\partial x}\right]$$
(8)

$$w(x,z) = w_0 \tag{9}$$

where u and w are the displacement components in the x and z directions, respectively. u_0 and w_0 are the midplane displacements and Φx is the bending rotation of x-axis. h is the total thickness of the beam. In this paper, beam is supported by a Winkler elastic foundation. Therefore we have [21]:

$$P(x) = k_w w(x) \tag{10}$$

where k_w is the elastic coefficient of the foundation.

The matrix form of the displacement field is as follows:

$$\overline{u} = \begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4\frac{z^3}{3h^2} & (z - 4\frac{z^3}{3h^2}) \end{bmatrix} \begin{bmatrix} u_0 \\ w_0 \\ \frac{\partial w_0}{\partial x} \\ \Phi_x \end{bmatrix} = \begin{bmatrix} Z_c \end{bmatrix} \begin{bmatrix} \overline{U} \end{bmatrix}$$
(11)

The strain-displacement relationship in the matrix form is given below:

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{bmatrix} = \begin{bmatrix} 1 & (z - 4\frac{z^3}{3h^2}) & -4\frac{z^3}{3h^2} & 0 \\ 0 & 0 & 0 & (1 - 4\frac{z^2}{h^2}) \end{bmatrix} \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial \Phi_x}{\partial x} \\ \frac{\partial^2 w_0}{\partial x^2} \\ \Phi_x + \frac{\partial w_0}{\partial x} \end{bmatrix} = [Z] [\overline{\varepsilon}]$$
(12)

In which $[\bar{\varepsilon}]$ is expressed as:

$$\left[\bar{\varepsilon}\right] = \left[d\right] \left[\bar{U}\right] \tag{13}$$

The matrix [d] is presented in the appendix. Substituting 13 in 12, we have:

$$[\varepsilon] = [Z] [d] [U] \tag{14}$$

The matrix form of stress-strain relations is as follows:

$$[\sigma] = [D] [\varepsilon] = [D] [Z] [\overline{\varepsilon}]$$
(15)

In which $[\sigma]$, $[\varepsilon]$ and [D] are:

$$\begin{bmatrix} \sigma \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xz} \end{bmatrix}^T \tag{16}$$

$$\begin{bmatrix} c \end{bmatrix} = \begin{bmatrix} c_{xx} & r_{xz} \end{bmatrix}$$
(17)
$$\begin{bmatrix} D \end{bmatrix} = \begin{bmatrix} Q_{11}(z) & 0 \\ 0 & Q_{rr}(z) \end{bmatrix}$$
(18)

$$Q_{11}(z) = \bar{M}\alpha^2 + \frac{E(z)}{1 - \vartheta^2}$$
(19)

$$Q_{55}(z) = G(z)$$
 (20)

2.3 Finite element model of governing equations

To solve the problem, finite element method is applied. The beam is divided to a number of element. It is assumed that each node of beam element has 4 degrees of freedom. Q(e) is considered as the vector of degrees of freedom for the beam element, and N(x) is the matrix of the shape functions. Therefore, the displacement approximation in each element of the beam can be considered as:

$$\left[\overline{U}^{(e)}(x)\right] = \left[N(x)\right] \left[Q^{(e)}\right] \tag{21}$$

[N (x)] is given in the appendix. $[Q^{(e)}]$ contains ui, wi, $\partial wi / \partial x$ and Φi , or the components of the axial displacement, transverse displacement, gradient and rotation of the expected node i = 1, 2. For the approximation of Φi and ui, the linear bar element, and for the approximation of wi and $\partial wi / \partial x$, the Hermitian element of the Euler- Bernoulli beam is used. By using equation (9), (17) and (18), we have:

$$\left[\vec{\varepsilon}\right] = \left[B\right] \left[Q^{(e)}\right] \tag{22}$$

Where [B] is the matrix of derivative the shape functions and is presented in the appendix. Since, the displacements are large at the onset of buckling, the nonlinear terms of the straindisplacement relationship must be considered in the potential energy of system. The total potential energy of the system is due to the sum of the strain energy of linear and nonlinear terms of strain, and also the potential energy resulting from the elastic property of the foundation. Therefore, we have:

$$U = U_{1} + U_{2} + U_{3} = \frac{1}{2} \iiint \varepsilon^{T} \sigma \, dV + \frac{1}{2} \iint k_{w} w^{2} \, dx dy + \frac{1}{2} \iint Pw'^{2} \, dx$$

$$= \frac{1}{2} \left[Q^{(e)} \right]^{T} \left(b \int_{0}^{l^{(e)}} [B]^{T} [\overline{D}] [B] \, dx \right) \left[Q^{(e)} \right]$$

$$+ \frac{1}{2} \left[Q^{(e)} \right]^{T} \left(b \int_{0}^{l^{(e)}} [\overline{N}]^{T} \, k_{w} [\overline{N}] \, dx \right) \left[Q^{(e)} \right]$$

$$+ \frac{1}{2} \left[Q^{(e)} \right]^{T} \left(\int_{0}^{l^{(e)}} [\overline{Ng}]^{T} \, P [\overline{Ng}] \, dx \right) \left[Q^{(e)} \right]$$
(23)

In the above equation, P = 1 for buckling analysis.

Also, $[\overline{D}]$ and W are expressed by the following relations:

$$[\overline{D}] = \int_{-h/2}^{h/2} [Z]^{T} [D] [Z] dz$$
(24)

$$[w] = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overline{N} \\ \overline{N} \end{bmatrix} \begin{bmatrix} Q^{(e)} \end{bmatrix}$$
(25)

$$[w'] = [\overline{Ng}] [Q^{(e)}]$$
⁽²⁶⁾

$$[\overline{Ng}] = \begin{bmatrix} 0 \ \frac{\partial}{\partial x} (N_{4i-2}) \ \frac{\partial}{\partial x} (N_{4i-1}) \ 0 \ 0 \ \frac{\partial}{\partial x} (N_{4j-2}) \ 0 \ \frac{\partial}{\partial x} (N_{4j-1}) \ 0 \end{bmatrix}$$
(27)

 $[\overline{N}]$ is described in the appendix. In the static bending analysis, Pz is the transverse load applied on the beam. The work done by the transverse load is defined as follows:

$$W_f^{(e)} = \frac{1}{2} \iint \{f_2\} w \, dx dy = \frac{1}{2} \left[Q^{(e)}\right]^T \tag{28}$$

In the next, stiffness matrix for each element of the beam $[K_{\varepsilon}^{(e)}]$, the stiffness matrix due to the elastic property of the foundation $[K_{kw}^{(e)}]$, the geometric stiffness matrix $[K_g^{(e)}]$, the external load vector for each element $\{F^{(e)}\}$ are introduced:

$$\left[K_{\varepsilon}^{(e)}\right] = b \int_{0}^{l^{(e)}} [B]^{T}[\overline{D}] [B] dx$$
⁽²⁹⁾

$$[K_{kw}^{(e)}] = b \int_{0}^{l^{(e)}} [\bar{N}]^{T} k_{w} [\bar{N}] dx$$
(30)

$$\left[K_g^{(e)}\right] = P \int_0^{l^{(e)}} [\overline{Ng}]^T [\overline{Ng}]$$
(31)

$$\left\{F^{(e)}\right\} = b \int_{0}^{l^{(e)}} [\overline{N}]^{T} \begin{bmatrix} 0\\p_{z}\\0\\0 \end{bmatrix} dx$$
(32)

3 Numerical results

In this section, numerical results have been obtained for static bending and buckling of porous beam in undrained condition. The effects of different boundary conditions, porosity distribution, porosity parameters and slenderness ratio have been investigated. Critical buckling loads are non-dimensionalized according to the following equation:

$$P_{dimentionless} = P_{critial/(E_0*h)}$$

3.1 Verification

3.1.1 Buckling of isotropic homogenous beam

To validate results of present study, critical buckling load of isotropic homogenous beam for different boundary conditions and slender ratio (L/h) are obtained and compared with analytical results of [22] in Table (1). To derive results of Ref. [22] in the present study, Skempton coefficient is considered to be zero. This assumption gives $v_u = v$, Biot's modulus $\overline{M} = 0$ and pore fluid pressure p=0.

Also, the following material properties and geometrical parameters are used: E = 10 MPa, L = 1m, $e_0 = 0$, b = 1m, $\vartheta = 0.33$. Comparison of results in Table (1) shows excellent agreement between them. It should be noted that in [22], Hook's law (drained condition) and Timoshenko beam theory is used to model the beam.

3.1.2 Static bending of FGM porous beam in drained condition

Also, to validate static bending results of present study, non-dimensional transverse displacement (w/h) for different loading conditions (Distributed and concentrated load), slenderness ratio and different porosity distributions of a porous C-F beam in drained condition have been obtained and compared with results of [10]. Hence, the following parameters are considered: E = 200 GPa, h = 0.1m, b = 0.1m, e = 0.5, $\vartheta = 0.33$.

Also, to derive results of Ref. [11] in the present study, Skempton coefficient is considered to be zero (i.e. drained condition). Comparing results of the present study with reference [11] in Table (2) shows excellent agreement between them.

3.2 Buckling of FGM porous beam in undrained condition

An FGM saturated porous beam with the following parameters is considered: E = 200 GPa, h = 0.1m, b = 0.1m, $\vartheta = 0.33$. The effects of different boundary conditions, slenderness ratio, porosity coefficient and Skempton coefficient on the critical buckling load are investigated and shown in Tables (3), (4) and (5) for asymmetric, uniform and symmetric porosity distribution, respectively. As it is shown in these tables, by increasing porosity coefficient, the stiffness of the structure decreases and, as a result, the critical buckling load decreases. While by increasing the Skempton coefficient, critical buckling load increases.

Also, by increasing the slender ratio, the buckling load decreases. The results show that the maximum and minimum buckling loads associated with the symmetric and uniform porosity distributions, respectively. This is due to the fact that in the uniform distribution of pores, the stiffness of structure is lower than those of distributions. It should be noted that for asymmetric distribution of pores, an extra moment exerts to the beam, and C-C beam can handle this extra moment. Therefore, only beam with C-C boundary conditions shows bifurcation-type of buckling (Table (3)). Table (6) also shows that by increasing the stiffness of the elastic foundation, the buckling load increases.

Tuble T comparison of endean bucking foud in the present study with Ref. [22].				
Simple-Simple			Clamp-Clamp	n
	Exact solution [20]	present	Exact solution[20]	present
L/H=10	8013.8	8014.4	29766	29777
L/H=100	8.223	8.222	32.864	32.865
L/H=1000	0.0082	0.0082	0.0329	0.0329

Table 1 Comparison of critical buckling load in the present study with Ref. [22].

Table 2 Non-dimensional transverse displacement of C-F porous beam compared with [10] (e0=0.5, β =0).

Distributed load					
Porosity distribution 1			Porosity distri	Porosity distribution 2	
	Reference [10]	Present	Reference [10]	Present	
L/H=10	0.00083	0.000827	0.001	0.000998	
L/H=20	0.01307	0.013075	0.01582	0.001582	
L/H=50	0.5089	0.509027	0.61646	0.06164	
		Point load			
	Porosity distributi	ion 1	Porosity dist	ribution 2	
	Reference [10]	Present	Reference [10]	Present	
L/H=10	0.00219	0.002196	0.00265	0.00265	
L/H=20	0.01741	0.01741	0.02108	0.020763	
L/H=50	0.27142	0.27142	0.32874	0.32381	

Table 3 Critical buckling load of C-C beam for nonlinear asymmetric porosity distribution

		β=0	β=0.5	β=0.9
L/H=10	e0=0.3	0.0024	0.0026	0.0028
	e0=0.5	2.00E-03	2.30E-03	0.0026
	e0=0.7	0.0015	0.0019	0.0024
L/H=7	e0=0.3	0.0044	0.0047	0.005
	e0=0.5	0.0037	0.0041	0.0046
	e0=0.7	0.0028	0.0034	0.0041
L/H=5	e0=0.3	0.0073	0.0077	0.0081
	e0=0.5	0.006	0.0067	0.0073
	e0=0.7	0.0047	0.0054	0.0064

C-C					
	L/H=10		β=0	β=0.5	β=0.9
		e0=0.3	0.0021	0.0023	0.0026
		e0=0.5	0.0015	0.0018	0.0023
		e0=0.7	8.83E-04	0.0012	0.0019
	L/H=7				
		e0=0.3	0.0038	0.0042	0.0047
		e0=0.5	0.0027	0.0033	0.0041
		e0=0.7	0.0016	0.0022	0.0031
	L/H=5				
		e0=0.3	0.0063	0.0069	0.0075
		e0=0.5	0.0045	0.0053	0.0063
		e0=0.7	0.0027	0.0034	0.0069
S-S					
S-S	L/H=10				
S-S	L/H=10	e0=0.3	5.59E-04	6.43E-04	7.33E-04
S-S	L/H=10	e0=0.3 e0=0.5	5.59E-04 4.00E-04	6.43E-04 5.12E-04	7.33E-04 6.71E-04
S-S	L/H=10	e0=0.3 e0=0.5 e0=0.7	5.59E-04 4.00E-04 2.40E-04	6.43E-04 5.12E-04 3.48E-04	7.33E-04 6.71E-04 5.72E-04
S-S	L/H=10	e0=0.3 e0=0.5 e0=0.7	5.59E-04 4.00E-04 2.40E-04	6.43E-04 5.12E-04 3.48E-04	7.33E-04 6.71E-04 5.72E-04
S-S	L/H=10 L/H=7	e0=0.3 e0=0.5 e0=0.7	5.59E-04 4.00E-04 2.40E-04	6.43E-04 5.12E-04 3.48E-04	7.33E-04 6.71E-04 5.72E-04
S-S	L/H=10 L/H=7	e0=0.3 e0=0.5 e0=0.7 e0=0.3	5.59E-04 4.00E-04 2.40E-04 0.0011	6.43E-04 5.12E-04 3.48E-04 0.0013	7.33E-04 6.71E-04 5.72E-04 0.0014
S-S	L/H=10 L/H=7	e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5	5.59E-04 4.00E-04 2.40E-04 0.0011 7.92E-04	6.43E-04 5.12E-04 3.48E-04 0.0013 0.001	7.33E-04 6.71E-04 5.72E-04 0.0014 0.0013
S-S	L/H=10 L/H=7	e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5 e0=0.7	5.59E-04 4.00E-04 2.40E-04 0.0011 7.92E-04 4.75E-04	6.43E-04 5.12E-04 3.48E-04 0.0013 0.001 6.80E-04	7.33E-04 6.71E-04 5.72E-04 0.0014 0.0013 0.0011
S-S	L/H=10 L/H=7	e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5 e0=0.7	5.59E-04 4.00E-04 2.40E-04 0.0011 7.92E-04 4.75E-04	6.43E-04 5.12E-04 3.48E-04 0.0013 0.001 6.80E-04	7.33E-04 6.71E-04 5.72E-04 0.0014 0.0013 0.0011
<i>S-S</i>	L/H=10 L/H=7 L/H=5	e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5 e0=0.7	5.59E-04 4.00E-04 2.40E-04 0.0011 7.92E-04 4.75E-04	6.43E-04 5.12E-04 3.48E-04 0.0013 0.001 6.80E-04	7.33E-04 6.71E-04 5.72E-04 0.0014 0.0013 0.0011
S-S	L/H=10 L/H=7 L/H=5	e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5 e0=0.7 e0=0.3	5.59E-04 4.00E-04 2.40E-04 0.0011 7.92E-04 4.75E-04 0.0021	6.43E-04 5.12E-04 3.48E-04 0.0013 0.001 6.80E-04	7.33E-04 6.71E-04 5.72E-04 0.0014 0.0013 0.0011 0.0026
<i>S-S</i>	L/H=10 L/H=7 L/H=5	e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5 e0=0.7 e0=0.3 e0=0.5	5.59E-04 4.00E-04 2.40E-04 0.0011 7.92E-04 4.75E-04 0.0021 0.0015	6.43E-04 5.12E-04 3.48E-04 0.0013 0.001 6.80E-04 0.0023 0.0018	7.33E-04 6.71E-04 5.72E-04 0.0014 0.0013 0.0011 0.0026 0.0023

 Table 4 Critical buckling Load For uniform porosity distribution

Table 5 Critical buckling load for non-linear symmetric porosity distribution

C-C	L/H=10		$\beta = 0$	$\beta=0.5$	$\beta = 0.9$
		e0=0.3	0.0026	0.0027	0.0028
		e0=0.5	0.0023	0.0025	0.0026
		e0=0.7	0.002	0.0022	0.0024
	L/H=7				
		e0=0.3	0.0047	0.0048	0.005
		e0=0.5	0.0041	0.0044	0.0046
		e0=0.7	0.0036	0.0038	0.0041
	L/H=5				
		e0=0.3	0.0076	0.0078	0.008
		e0=0.5	0.0066		0.0071

		0.0069			
		e0=0.7	0.0055	0. 0057	0.006
S-S	L/H=10		β=0	β=0.5	β=0.9
		e0=0.3	7.09E-04	7.42E-04	7.73E-04
		e0=0.5	6.48E-04	6.98E-04	7.50E-04
		e0=0.7	5.85E-04	6.46E-04	7.20E-04
	L/H=7				
		e0=0.3	1.40E-03	0.0015	0.0015
		e0=0.5	0.0013	0.0014	0.0015
		e0=0.7	0.0011	0.0012	0.0014
	L/H=5				
		e0=0.3	0.0026	0.0027	0.0028
		e0=0.5	0.0023	0.0025	0.0026
		e0=0.7	0.002	0.0022	0.0024

Table 6 Effect of elastic foundation on the critical buckling load for uniform porosity distribution $(I/H=5, \beta=0, c_0=0, 5)$

(L/H-3, p-0, e0-0.3)					
KW=0	KW=1E9	KW=5E9			
0.00054	9.12E-04	0.0018			

3.3 Static bending of FGM porous beam in undrained condition

In this section, static behavior of the beam for different porosity coefficients and boundary conditions and different slender ratio is investigated. For this purpose the following parameters have been considered: E = 200 GPa, e0 = 0.5, L = 1m, b = 0.1m, h = 0.2m. The normal and shear stresses have been non-dimensionalized by the following relations:

$$(\boldsymbol{\sigma}_{x}, \boldsymbol{\sigma}_{xy}) = ((\boldsymbol{\sigma}_{xx} * \mathbf{A})/\mathbf{Q} * \mathbf{L}, ((\boldsymbol{\sigma}_{xy} * \mathbf{A})/\mathbf{Q} * \mathbf{L})$$
(33)

Where Q is the load per unit length:

$$\mathbf{Q} = \mathbf{b} * \mathbf{p}_{\mathbf{z}} \tag{34}$$

Figure (2) shows the effect of the porosity coefficient on the deflection of S-S beam for L / H = 5 and asymmetric porosity distribution. As it can be seen, by increasing the porosity coefficient, the stiffness of the beam reduces and, as a result, deflection increases. Figure (3) shows the effect of the skempton coefficient on the deflection of S-S beam for L / H = 5 and asymmetric porosity distribution. This figure shows that by increasing the skempton coefficient, deflection of beam decreases. Figure (4) shows the effect of coefficient of elastic foundation on the transverse displacement of beam. This result denotes that by increasing the coefficient of elastic the foundation, transverse displacement of beam decreases significantly. Figure (5) investigates the effect of different porosity distribution on the deflection of the S-S beam for L / h = 5. The maximum and minimum magnitude of deflection belongs to the uniform and symmetric nonlinear distribution of porosity, respectively.

This is due to the fact that for uniform distribution of pores, the stiffness of the beam is lower than those of porosity distributions.

Figure (6) shows the effect of porosity coefficient on the normal stress along the thickness of beam for L / H = 5 and x=L/2. As it is shown in Figure (6), for e0 = 0 (beam without pores), the stress distribution along the thickness is linear. Also due to the asymmetric nonlinear distribution of pores, the neutral axis is not located at z = 0. Figure (7) shows the effect of the skempton coefficient on the normal stress along the thickness of beam for L / H = 5 and x=L/2. Figure (7) shows that by increasing the Skempton coefficient, normal stress decreases.



Figure 2 The effect of the porosity coefficient on the deflection of S-S beam



Figure 3 The effect of the skempton coefficient on the deflection of S-S beam



Figure 4 Effects of coefficient of elastic foundation on the deflection for S-S beam for nonlinear asymmetric porosity distribution and e0 = 0.25



Figure 5 The effects of porosity distribution on the deflection of S-S beam.



Figure 6 The effect of porosity coefficient on the normal stress of the S-S beam for asymmetric nonlinear distribution of porosity (z, x=L/2)



Figure 7 The effect of the skempton coefficient on the normal stress of the S-S beam for asymmetric nonlinear distribution of porosity (x=L/2)

Figure (8), shows the effect of porosity coefficient on the shear stress along the thickness of beam for L / H = 5 and x=L/4. Figure (8) denotes that the effect of the porosity coefficient on the shear stress in the lower surfaces is more distinct than the upper surface, and by increasing the porosity coefficient, the position of maximum shear stress is moved toward the upper surface of beam. Figure (9), shows the effect of the Skempton coefficient on the shear stress along the thickness of beam for L / H = 5 and x=L/4. Figure (9) shows that by increasing the Skempton coefficient, shear stress decreases significantly.



Figure 8 The effect of the porosity coefficient on the shear stress of the S-S beam for asymmetric nonlinear distribution of porosity (x=L/4)



Figure 9 The effect of the porosity coefficient on the shear stress of the S-S beam for asymmetric nonlinear distribution of porosity (x=L/4)

4 Conclusion

In this paper, static and buckling analysis of thick functionally graded saturated porous beam on an elastic foundation is investigated. Third-order beam theory in conjunction with Biot constitutive law and finite element method has been used to model the problem for the first time. The effect of various parameters such as boundary conditions, porosity coefficient, Skempton coefficient, porosity distribution and elastic coefficient of the foundation on the critical buckling load, displacements and stresses of beam have been investigated. The results show that by increasing the elastic coefficient of the foundation, the critical buckling load increases and the deflection decreases. Also, the maximum and minimum magnitude of critical buckling loads obtained for nonlinear symmetric and uniform porosity distribution, respectively. Results denote that the effect of porosity coefficient on the shear stress in the lower surfaces is more distinct than the upper surface, and by increasing the porosity coefficient, the position of maximum shear stress is moved toward the upper surface of beam. Also, by increasing the skempton coefficient, transverse displacement, normal and shear stresses decreases.

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Nomenclature

- L: Length of the beam
- *b*: Depth of the beam
- *h*: Height of the beam
- e₀: Coefficient of beam porosity
- \overline{M} : Biot's modulus
- v_u : Undrained Poisson's ratio
- v: Poisson's ratio

- Fluid pore pressure *p*:
- ζ: Variation of fluid volume content
- α: Biot coefficient
- Bulk modulus of a homogeneous material K_S :
- K_u : Bulk modulus in the undrained state
- C_p : Solid Compressibility

The midplane displacements and Φx is the bending rotation of x-axis u_0, w_0 :

- Elastic coefficient of the foundation. k_w :
- Matrix of the shape functions N(x):
- $K_{\varepsilon}^{(e)}$: Stiffness matrix for each element of the beam
- Elastic stiffness matrix for each element of the beam
- $K_{\varepsilon}^{(e)}$: $K_{g}^{(e)}$: Geometric stiffness matrix for each element of the beam
- $F^{(e)}$: Geometric stiffness matrix for each element of the beam

Appendix

The shape functions are as:

$$\begin{split} & N_{4i-3} = 1 - \frac{x}{l} \\ & N_{4j-3} = \frac{x}{l} \\ & N_{4j} = 1 - \frac{x}{l} \\ & N_{4j} = \frac{x}{l} \\ & N_{4i-2} = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \\ & N_{4i-1} = x - \frac{2x^2}{l} + \frac{x^3}{l^2} \\ & N_{4j-2} = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \\ & [N] = \begin{bmatrix} N_{4i-3} & 0 & 0 & 0 & N_{4j-3} & 0 & 0 & 0 \\ 0 & N_{4i-2} & N_{4i-1} & 0 & 0 & N_{4j-2} & N_{4j-1} & 0 \\ 0 & \frac{\partial N_{4i-2}}{\partial x} & \frac{\partial N_{4i-1}}{\partial x} & 0 & 0 & \frac{\partial N_{4j-2}}{\partial x} & \frac{\partial N_{4j-1}}{\partial x} & 0 \\ & 0 & 0 & 0 & N_{4i} & 0 & 0 & 0 & N_{4j} \end{bmatrix} \\ & [d] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial^2}{\partial x^2} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ & [B] = [d][N(x)] \end{split}$$

$$[B] = \begin{bmatrix} \frac{\partial N_{4i-3}}{\partial x} & 0 & 0 & 0 & \frac{\partial N_{4j-3}}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_{4i}}{\partial x} & 0 & 0 & 0 & \frac{\partial N_{4j}}{\partial x} \\ 0 & \frac{\partial^2 N_{4i-2}}{\partial x^2} & \frac{\partial^2 N_{4i-1}}{\partial x^2} & 0 & 0 & \frac{\partial^2 N_{4j-2}}{\partial x^2} & \frac{\partial^2 N_{4j-1}}{\partial x^2} & 0 \\ 0 & \frac{\partial N_{4i-2}}{\partial x} & \frac{\partial N_{4i-1}}{\partial x} & N_{4i} & 0 & \frac{\partial N_{4j-2}}{\partial x} & \frac{\partial N_{4j-1}}{\partial x} & N_{4j} \end{bmatrix}$$
$$[\overline{N}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{4i-2} & N_{4i-1} & 0 & 0 & N_{4j-2} & N_{4j-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$