ISME

Fuzzy Variable-Length Sliding Window Blockwise Least Square Algorithm with Application to Vehicle Heading Determination

J. Keighobadi [*]
Ph.D Candidate

M.J. Yazdanpanah [†]
Associate Professor

M. Kabganian [‡]
Associate Professor

In ground vehicles, three-axis magnetometers may be corrupted by both soft- and hard-iron disturbances. Therefore, it may not be possible to achieve qualified headings without online calibration of this magnetic system. First contribution of this paper is focused on improving the order of persistent excitation of the squared signal matrix through incorporation of a direction cosine matrix in estimation model. As the main contribution, a fuzzy change detection scheme for adjusting the length of data sliding window of blockwise least square (BLS) algorithms is presented in the framework of on-line estimation of system parameters under both abrupt and gradual changes. This is called fuzzy variable-length sliding window (FVLSW) BLS. Two change detection indices including generalized likelihood ratio and averaged parameter estimation errors together with their changes are considered as inputs of the fuzzy system. The defuzzified outputs consists a forgetting factor in order to place more emphasis on the recent data, and two adjusted lengths of data history windows.Simulations and real experiments revealed that the proposed approach has superior performance with respect to the latest variable-length sliding window (VLSW) BLS estimation algorithm. The superiority is more significant when the measurement noise power is substantial.

Keywords: fuzzy change detection, persistent excitation, vehicle navigation, least square, online calibration

1 Introduction

Using a three-axis magnetometers system together with a low-cost micro electro-mechanical attitude system, this paper develops a technique to determine heading of a vehicle in vicinity of the earth. Three-axis magnetometers system is commonly used to determine the attitude-heding of a vehicle. However, it should be coupled with gyros or three-axis strapdown accelerometers as an attitude heading reference system [1]. An important issue to the attitude accuracy obtained using a three-axis magnetometers system is the precision of its calibration parameters including biases, scale-factors and non-orthogonal misalignments, which may be determined using attitude-dependent or -independent methods [2-4]. On the other hand, unlike aerospace applications, in ground vehicles, magnetometers are commonly affected by considerable soft- and hard-iron

[*] Mechanical Engineering Department, Amirkabir University of Technology
[†] University of Tehran, Campus #2, Control & Intelligent Processing Centre of Excellence, School of Electrical and Computer Engineering, Karegare Shomali st., Tehran, Iran, P.O. Box 1439-515
[‡]Corresponding Author, Mechanical Engineering Department, Amirkabir University of Technology,
E-mail: Kabgan@aut.ac.ir

magnetic disturbances. These disturbances are generated due to steel made parts of the vehicle, its electrical devices, and other magnetic anomalies which may come from the environmental effects. However, these time-varying disturbance parameters should be considered and online calibration procedures (in addition to off-line calibrations) are required before correct vehicle heading can be determined to satisfy requirements of the vehicle navigation application [5]. It should be noted that during off-line calibration, there is no restrictions on rotating movements of three-axis magnetometers, which should be done to achieve input and output data for calibration. Therefore, input signals with a suitable order of persistent excitation, which are required for the purpose of convergence of the estimation/calibration algorithms, may be obtained. Since in online estimation/ calibration problem the input signals of regression model of the magnetometers system depend on the vehicle maneuvers, therefore, there is no guarantee that the input signals may generate suitable order of persistent excitation for squared regression (signal) matrix.

In the line of the other contribution of this paper, it is shown that using a direction cosine matrix to transform measurement vector of a strapdown system leads to a squared regression matrix together with high order of persistent excitation, in which many sine and cosine terms that act on the new inputs, are contributed in the appearance of new estimation models. Therefore, a new regression model of the vehicle heading reference system is generated to improve the convergence of parameter estimate in real-time applications.

Least square (LS) family of estimation algorithms, which may be formulated in blockwise or recursive forms, have shown a good tracking property because of the linear optimal features resulting from minimizing sum of the squared prediction errors [6]. For linear time-invariant systems, it has been demonstrated that the performance of blockwise least square (BLS) is always superior to that of the recursive LS [7]. In online estimation of time-varying system parameters, standard BLS may no longer be suitable due to its inefficiency in discarding old data. To deal with this limitation, sliding window BLS approach is proposed [7].

Two possible solutions for detecting a change of an unknown parameter are the weighted cumulative sum and generalized likelihood ratio [8]. The first is essentially a moving average method and the second scheme uses widely accepted generalized likelihood ratio instead of maximum likelihood. As the main contribution of this paper, a fuzzy change detection mechanism is designed for considering changes of disturbance parameters in adjusting data history length of adaptive BLS estimation algorithm. This fuzzy decision making system results in the enhancement of the estimation accuracy of BLS algorithm due to simultaneously using two independent change detection indices including averaged parameter estimation error and generalized likelihood ratio. When the measurement noise power is substantial, the performance of this new fuzzy change detection system is superior to that of non-fuzzy scheme. By extending recently developed variable-length BLS algorithms [9, 10], a new intelligent fuzzy variable-length sliding window (FVLSW) BLS estimator is designed to online identification of time-varying systems.

Performances of the proposed FVLSW BLS and recently developed variable-length sliding window (VLSW) BLS estimation algorithms are evaluated using both simulation and real test data of a low-cost heading reference system, which is strapped on a ride vehicle. The real tests data are collected during a wide range of the vehicle maneuvers in mountain roads, highways, and level streets of the city. Online estimation of six calibration/disturbance parameters may be possible only by using new estimation model of the heading reference system in BLS algorithms. Computer simulations and real experiments revealed that the tracking capability of proposed FVLSW BLS estimator on accurate estimation of time-varying magnetic parameters is

significantly better than that of the recent VLSW BLS algorithm [7]. Rest of the paper is organized follows: Section 2 describes the heading reference system modeling technique. Section 3 presents online parameter estimation algorithms and properties of the proposed regression model. In Section 4, the idea of fuzzy change detection scheme is presented. In section 5, in addition to simulations, performance of the proposed techniques is evaluated on a real heading reference system; and finally, Section 6 is devoted to concluding remarks.

2 Heading reference system

In general, the aim of attitude heading reference system is to obtain sufficient information for vector transformation from vehicle body coordinates to a pre-determined reference frame [11]. In this paper, heading reference system is considered to determine the magnetic heading of a vehicle using measurements made by three-axis magnetometers. Correct outputs of the three-axis magnetometers system should be projection of the earth's magnetic field vector along vehicle body axes. It has been assumed that the attitude including roll and pitch angles is determined with an acceptable accuracy (about 1°) using an aided low-cost inertial navigation system.

Even after calibration of the three-axis magnetometers system, remarkable uncertainties in its parameters including biases and scale factors may commonly exist. Therefore, norm of the three-axis magnetometers outputs that should be approximately a fixed value in a specified geographical region is together with serious gradual or abrupt changes. In this paper, online estimation of three-axis magnetometers system biases and scale factors is triggered for considering local hard- and soft-iron magnetic effects on the heading reference system algorithms. The following measurement model has been frequently examined for three-axis magnetometers calibration in literatures [2-4],

$$\hat{m}_k^b = (I_{3*3} + D_k) C_n^b m^n + B_k + v_k \,, \tag{1}$$

where $\hat{m}_k^b$ includes $\hat{m}_x$, $\hat{m}_y$ and $\hat{m}_z$ that stand for magnetic fields effects measured by the three-axis magnetometers along vehicle body axes at time t_k. m^n is the corresponding value of the geomagnetic field with respect to north-east-down coordinate system, C_n^b stands for the attitude and heading representation in the form of direction cosine matrix, D_k is an unknown matrix of scale factors (the diagonal elements are corresponding to soft-iron magnetic disturbances), B_k is the bias vector, and v_k is the measurement noise vector that is assumed to be a zero-mean Gaussian process. Now, two following nonlinear models may be considered for online calibration of the three-axis magnetometers system. The first is an attitude-independent observation model, which is inspired from the fact that the undistorted measurement vector by the three-axis magnetometers should trace a spherical surface during travel on the earth [4]. This sphere corresponds to the loci of the earth's magnetic field vector in the space. This sphere is shifted from the origin by applying biases, deformed by applying scale factors and finally corrupted by wide band sensor noises. The goal of the online calibration problem is to estimate *D* and *B.* An attitude-independent observation may be obtained as follows:

$$-\hat{m}_x^{b2} = [-2\hat{m}_x^b \quad \hat{m}_y^{b2} - 2\hat{m}_y^b \quad \hat{m}_z^{b2} - 2\hat{m}_z^b \quad 1]\begin{bmatrix} B_x \\ K_2 \\ K_2 B_y \\ K_3 \\ K_3 B_z \\ K_4 \end{bmatrix}, \tag{2}$$

where the auxiliary variables K_1 through K_4 are nonlinear functions of hard iron bias vector B_k (B_x, B_y, B_z) and soft iron scale factors (γ_x, γ_y and γ_z) which are diagonal elements of matrix *D*. In regression (signal) matrix of the aforementioned model (2), only measurements of the three-axis magnetometers exist. Therefore, due to slow variation of the vehicle attitude in required short-time intervals for online calibration updating, measurements by the three-axis magnetometers system may not be sufficiently rich to obtain a successful estimate of unknown parameters. On the other hand, during online calibration of a real three-axis magnetometers system, using test data and regression model of (2), expected divergence of estimated parameters is resulted. However, weak capability of the squared signal (regression) matrix of model (2) to have a high order of persistent excitation reveals necessity of new estimation model whose squared signal matrix should possess a high order of persistent excitation. This kind of regression model can be appropriate for online calibration of the three-axis magnetometers system. The second and new model of the three-axis magnetometers system is based on the fact that the horizontal and vertical components of the earth's magnetic field vector are not affected by heading angle ψ. Therefore, transformation of the measurements vector from body coordinate system to a local level frame is the base of new modeling technique, i.e.,

$$\begin{bmatrix} C\vartheta & S\phi S\vartheta & C\phi S\vartheta \\ 0 & C\phi & -S\phi \\ -S\vartheta & S\phi C\vartheta & C\phi C\vartheta \end{bmatrix} \begin{bmatrix} (m_x + B_x)\gamma_x \\ (m_y + B_y)\gamma_y \\ (m_z + B_z)\gamma_z \end{bmatrix} = \begin{bmatrix} M_{x1} \\ M_{y1} \\ M_D \end{bmatrix}, \tag{3}$$

where *S* and *C* stand for sine and cosine, respectively. ϕ and ϑ are roll and pitch angles, respectively. M_{x_1}, M_{y_1} and M_D are projected components of the earth's magnetic field vector in new local level frame (X_1Y_1D) which should be coincide to north-east-down frame trough rotation by the heading angle ψ along local normal axis. Therefore, vertical component of the earth's magnetic field vector (M_D) in both of north-east-down and X_1Y_1D frames is the same and is independent of ψ. In the mean time, the projection of the earth's magnetic field vector in local horizon can be found from world magnetic distribution models [3]. Therefore, two following regression models could be considered for online estimation of the calibration parameters,

$$M_D = \begin{bmatrix} -S\vartheta m_x & -S\vartheta & S\phi C\vartheta m_y & S\phi C\vartheta & C\phi C\vartheta m_z & C\phi C\vartheta \end{bmatrix} \begin{bmatrix} \gamma_x \\ \gamma_x B_x \\ \gamma_y \\ \gamma_y B_y \\ \gamma_z \\ \gamma_z B_z \end{bmatrix}, \tag{4}$$

$$\begin{aligned} \|M_H\|_2^2 = & \begin{bmatrix} (m_x + B_x)\gamma_x & (m_y + B_y)\gamma_y & (m_z + B_z)\gamma_z \end{bmatrix} \begin{bmatrix} C\vartheta & S\phi S\vartheta & C\phi S\vartheta \\ 0 & C\phi & -S\phi \end{bmatrix} * \\ & \begin{bmatrix} C\vartheta & S\phi S\vartheta & C\phi S\vartheta \\ 0 & C\phi & -S\phi \end{bmatrix}^T \begin{bmatrix} (m_x + B_x)\gamma_x & (m_y + B_y)\gamma_y & (m_z + B_z)\gamma_z \end{bmatrix}^T \end{aligned} \tag{5}$$

where M_H consists horizontal components of the earth's magnetic field vector (M_{x1} and M_{y1}).

In practical applications of heading reference system, providing signal/regression matrices using aforementioned modeling techniques leads to satisfaction of Lemma 1, which will be stated in the next section for achieving guaranteed convergence of parameter estimation algorithm. However, the components of magnetic field vector in local horizon (M_{x1} and M_{y1}) could not be known in the system under consideration. Therefore, we only focused on using the squared norm of M_H and M_D for modeling the three-axis magnetometers calibration problem.

3 Estimation of time-varying parameters

The following discrete time regression model with a white Gaussian noise v_k is considered to formulate the parameter estimation problem:

$$y_k = H_k \theta_k + v_k + \gamma(k), \tag{6}$$

$$v_k \sim N(0, R), \tag{7}$$

where R is a diagonal covariance matrix of noise vector v_k, θ is an *n*-dimensional vector of parameters and y_k is an *r*-dimensional measurement vector (*r*>*n*). H_k is the regression matrix of rank *n* and γ is a probability change in parameter or in measurement noise vector. In this paper, the developed fuzzy algorithm does not rely on any particular form of parameter changes. However, a change at unknown time k_a is regarded as [7]

$$\theta_k = \theta_0, \qquad k < k_a, \tag{8}$$

$$\theta_k = \theta_0 + \Delta\theta \quad k \geq k_a \quad , \|\Delta\theta\| \neq 0. \tag{9}$$

Equations (8) and (9) show the most general form of changes in vector θ, such that they may count for most of the change profiles which are described in refereed literatures [8].

Maximum likelihood estimate is coincide with the LS estimation under linear and Gaussian assumptions, and they result in the following estimate using all observations up to current instant, k :

$$\hat{\theta}_k = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \left(Y_k - H\hat{\theta}\right). \tag{10}$$

3.1 Online blockwise least square

Estimation of time-varying parameters was investigated in many variants of LS algorithms in preceding literatures [9, 10]. The aim is to develop a strategy for adjusting the length of sliding window to achieve the best performance of the estimation algorithm in both transient and steady state intervals, which in turn require fast and reliable change detection mechanisms.
In this paper, the following cost function involving the noise covariance matrix is used for online estimation of parameters of the regression model (11) with additive changes,

$$y(k) = H_k\left(\theta_{k-1} + \Delta\theta_k\right) + v_k\ , \tag{11}$$

$$J_{N_k}(\theta, k) = \sum_{j=k-N_k+1}^{k} \lambda^{k-j} r_{j,j}^{-1} (y_j - H_k\theta_k)^2 \tag{12}$$

where N_k , 0< λ <1 and $r_{j,j}$ s respectively stand for the length of data history sliding window, a forgetting factor in order to place more emphasize on recent data and elements of noise covariance matrix R . Minimizing this cost function for known $\Delta\theta$ leads to the following estimate of parameters,

$$\hat{\theta}_{N_k}(k, \Delta\theta) = \left[\sum_{j=k-N_k+1}^{k} \lambda^{k-j} r_{j,j}^{-1} H_k^T H_k\right]^{-1} \left[\sum_{j=k-i+1}^{k} \lambda^{k-j} r_{j,j}^{-1} H_k\left(y_j - H_k\Delta\theta\right)\right] \tag{13}$$

and for unknown $\Delta\theta$,

$$\hat{\theta}_{N_k}(k, \Delta\theta) = \left[\sum_{j=k-N_k+1}^{k} \lambda^{k-j} r_{j,j}^{-1} H_k^T H_k\right]^{-1} \left[\sum_{j=k-i+1}^{k} \lambda^{k-j} r_{j,j}^{-1} H_k y_j\right] \tag{14}$$

Convergence of the estimations in equations (10), (13) and (14) is strictly dependent on order of persistent excitation of the squared regression matrix $H_k^T H_k$. Since generated input signals in online estimation are dependent on the vehicle trajectories and maneuvers, structure of the regression matrix that can be determined by designer is a most effective factor on increasing the order of persistent excitation. In this paper, a new strategy is proposed to improve the order of persistent excitation of the squared regression matrix when the input signals are not sufficiently rich. This significant idea is based on the fact that the direction cosine matrix for transforming a measurement vector from body to a reference coordinate system possesses many sinusoidal terms of attitude angles as new inputs. The following lemma plays an important role on the feasibility of online estimation of time-varying parameters in this paper.

Lemma 1. Transforming the measurement vector of a system from body coordinates to a reference frame generates new regression model that can improve the convergence of LS family estimators.

Proof, Consider the matrix form of the weighted least square (14) as,

$$\hat{\theta}(k) = \left[H^T R^{-1} \Lambda H \right]^{-1} \left[H^T R^{-1} \Lambda y(k) \right] \tag{15}$$

where, $R^{-1}\Lambda$ is a nonsingular weighting matrix because it consists two nonsingular matrices including noise covariance and user defined forgetting factors. Therefore, unique solution of (15) requires that $H^T H$ be a nonsingular matrix. This is the well-known persistent excitation condition of the squared form of the regression matrix H [12]. Before vector transformation, the elements of H are composed of only the measurements in body coordinates as, $H = f(u)$.

Now, consider the following direction cosine matrix to transform vectors from body- to navigation- coordinates system,

$$DCM = \begin{bmatrix} C\vartheta C\psi & -C\phi S\psi + S\phi S\vartheta C\psi & S\phi S\psi + C\phi S\vartheta C\psi \\ C\vartheta S\psi & C\phi C\psi + S\phi S\vartheta S\psi & -S\phi C\psi + C\phi S\vartheta S\psi \\ -S\vartheta & S\phi C\vartheta & C\phi C\vartheta \end{bmatrix}$$

By applying this direction cosine matrix for transforming the input vector u, which is measured in body coordinates system to a reference frame, the new regression matrix can be considered as, $H(k) = f(u, \sin\phi, \sin\vartheta, \sin\psi, \cos\phi, \cos\vartheta, \cos\psi)$.

Since elements of H after transformation consist many sine and cosine terms, which act on the new inputs in addition to principal input vector *u,* the capability of new regression system to being a high order of persistent excitation is evidently superior to that of the system $H = f(u)$. Therefore, regarding time-varying ϕ, ϑ and ψ inputs, the following persistent excitation matrix C_n could have full rank of order 2^r where *r* is the number of sinusoidal functions of rotation angles in H [12],

$$C_n = \begin{bmatrix} c(0) & c(1) & \ldots c(n-1) \\ c(1) & c(0) & \ldots c(n-2) \\ \vdots & & \\ c(n-1) & c(n-2) & \ldots c(0) \end{bmatrix}$$

Therefore, when the components of vector *u* are not persistently excited inputs; the new regression system leads to a squared signal/regression matrix with higher order of persistent excitation, which in turn results in accurate estimation of parameters. This completes the proof.

The above lemma shows that how the order of persistent excitation of squared signal matrix may be improved by transforming system measurement vector to a reference frame in addition to rich input signals. Therefore, using new regression model can increase the accuracy of estimated parameters and causes to fast convergence of the estimation algorithms.

For example, according to the aforementioned C_n matrix, appearance of each sinusoid of the pitch angle in the form of $\vartheta = at$ leads to a squared matrix with 2 order of persistent excitation where a is a fixed value. This means that for $h(t) = \sin at$, $C_2 = \begin{bmatrix} 1 & \cos a \\ \cos a & 1 \end{bmatrix}$.

Remark 1. Small forgetting factor may decline the order of persistent excitation of ill-conditioned regression matrices.

Remark 2. Because of small noise covariance matrix, using more reliable measurement sensors could prevent divergence of the estimation algorithm especially when the original regression is ill conditioned.

In the recent paper [13], dynamical model ($\dot{x} = f(x,u)$) of a wheeled mobile robot is changed from body coordinates to a global one using both linear and non-holonomic constraints. Nonlinear controller implementation based on new dynamics leads to accurate trajectory tracking and perfect compensation for initial off-track conditions.

4 Change detection

In adaptive BLS estimation, the length of sliding window is adjusted by considering detected changes of parameters. Many change detection schemes are developed to distinguish both abrupt and gradual changes [8, 14]. However, combined change detection schemes using output prediction error and averaged parameter estimation error are possible [8]. By considering the following output prediction error with the sliding window of length $L(k)$, occurrence of a parameter change will be considered as follows

$$d_e(k) = \frac{1}{M_e} \sum_{i=k-M_e+1}^{k} e^T(i) e(i) \begin{cases} > \bar{\rho}_e \\ \leq \underline{\rho}_e \end{cases}, \tag{16}$$

$$e(k) = y_{k-L(k)+1}^{k} - H_{k-L(k)+1}^{k} \hat{\theta}(k), \tag{17}$$

where M_e is the length of recent data to update change detection index d_e, and $\hat{\theta}$ is the vector of estimated parameters. Once this index exceeds an upper pre-set threshold $\bar{\rho}_e$, change process will start. On the other hand, if this index goes less than the lower threshold $\underline{\rho}_e$, change process will stop. Similarly, the following averaged parameter estimate in a sliding window of length M_θ may be used to detect change of parameters:

$$\bar{\theta}_{M\theta}(k) = \frac{1}{M_\theta} \sum_{j=k-M_\theta}^{k} \hat{\theta}_L(j). \tag{18}$$

A decision for parameter change (or its leakage) can also be made if the detection index (18) exceeds or goes less than the pre-set thresholds [7].

In this paper, a fuzzy decision making system is designed for simultaneously considering two efficient change detection indices including generalized likelihood ratio and averaged parameter estimation error. In the mean time, two independent functions of change rate of these indices are considered in the fuzzy change detection system. The importance of innovation vector as a tool for detecting change of parameters comes from the computation of the likelihood ratio. The widely accepted generalized likelihood ratio for the regression model can now be written as [8]:

$$g_k = \max_{1 \le j \le k} \ ln \ \frac{sup_\theta \prod_{i=j}^{k} P_{\Delta\theta}\left(y_i \,|\, \theta\right)}{\prod_{i=j}^{k} P_{\theta_\circ}\left(y_i \,|\, \theta\right)}, \tag{19}$$

where g_k is decision function of the generalized likelihood ratio, $P_{\Delta\theta}(y_i)$ and $P_{\theta_\circ}(y_i)$ are the probability distribution functions of y_i after and before change occurrence, respectively. By considering the factorized covariance matrix, $R = A^T A$, the simplified form of g_k is obtained after some mathematical manipulations as

$$g_k = \max_{1 \le j \le k} \frac{k-j+1}{2}\left(y_j^k - H\theta_0\right)\Lambda R^{-1}\left(y_j^k - H\theta_0\right) \quad = \max \frac{k-j+1}{2}\left(\chi_j^k\right)^2, \tag{20}$$

$$\left(\chi_{k-N_k+1}^k\right)^2 = e_k^T R^{-1} \Lambda e_k = y_k^T A^{-T} (I_r - A^{-1} H (H^T R^{-1} \Lambda H)^{-1} H^T A^{-T}) A^{-1} y_k, \tag{21}$$

where I_r is an identity matrix of order r.

4.1 Fuzzy change detection

The main contribution of this paper is devoted to developing a fuzzy change detection system for combination of generalized likelihood ratio and averaged parameter estimation error indices. The fuzzy change detection scheme may achieves more tracking capability of parameter estimate under both abrupt and gradual changes. The basic idea behind this intelligent change detection scheme, which is merely based on fuzzy *if-then* rules, is to cumulate advantages of both the innovation based and parameter estimate based methods conveniently. In addition, the fuzzy combination is more reliable because it turns away the probability drawbacks of each individual change detection scheme by extending them over.

Although it seems that in fuzzy scheme, more computing cost must be paid due to more input variables of this change detection system compared to that of the recent non-fuzzy algorithms, it should be noted that: (1) the fuzzy *if-then* rules are very simple, evident and easy to be adjusted by considering particular application issues; (2) the fuzzy change detection system has continuous outputs including the length of sliding windows for updating BLS estimations and the input functions of the fuzzy inference engine; (3) this new scheme is more robust against noises because of noise filtering capability of determined fuzzy membership functions; and finally, (4) in the fuzzy system, systematically tuned *if-then* rules are used instead of pre-set thresholds for detecting change points.

Fuzzy change detection system incorporates generalized likelihood ratio, averaged parameter estimation error and two other defined functions of their change rates to obtain superior performance in adjusting the length of sliding windows and the forgetting factor. Intelligently adjusted sliding windows and forgetting matrix resulted in an efficient BLS algorithm for estimation of time-varying parameters. Two first inputs of this fuzzy change-detecting system are: (1) decision function of the generalized likelihood ratio which was defined before in (20); (2) the following decision index of averaged parameter estimation error, which is the difference of averaged parameter estimates in two consecutive intervals of length N_t:

$$g_{\bar{\theta}k} = \frac{1}{N_t}\left[\sum_{i=0}^{N_t}\theta(k-i) - \sum_{i=N_t}^{2N_t-1}\theta(k-i)\right] \tag{22}$$

Third input to the fuzzy change detection system is difference of the averaged decision function of generalized likelihood ratios, which is defined below:

$$\nabla g_k = \sum_{i=0}^{N_t-1} g_{k-i} - \sum_{i=N_t}^{2N_t-1} g_{k-i} \tag{23}$$

Finally, change rate of the averaged parameter estimation error (22) is the fourth input to the fuzzy system as

$$\nabla g_{\bar{\theta}k} = g_{\bar{\theta}k} - g_{\bar{\theta}k-1} \tag{24}$$

In fuzzy system, the reciprocal condition number estimate of matrix $H^T R^{-1}\Lambda H$ may be considered as an optional input to enlarge the length of sliding windows when the squared signal matrix has a weak order of persistent excitation. Although this optional input helps to decrease the singularity probability of the BLS estimation algorithm, it in turn may result in a less estimation accuracy in addition to more computational cost. In this paper, the trade-off between accuracy and convergence of estimation algorithm may be removed due to generating a persistently excited regression model. However, profound effects of this new modeling technique on performance of the BLS parameter estimation algorithm will be shown through simulations and real tests of a vehicle.

Fuzzy change detection system uses a combination of several change detection indices to reveal both abrupt and gradual changes of parameters in a good quality. This fuzzy system, because of its continuous outputs is significantly superior to the recent non-fuzzy change detection systems developed to specify start- and stop-points of a change. In the meantime, the fuzzy approach is comprehensive and is designed in such a way to use all of the four input functions in parallel.

The fuzzy change detection system has a clear and simple structure as follows:

$$N = f\left(g_k, \nabla g_k, g_{\bar{\theta}k}, \nabla g_{\bar{\theta}k}\right), \tag{25}$$

where N, which is the defuzzified output vector of fuzzy system, includes: N_k, the length of sliding window of BLS estimator; N_t, the length of innovation data history to update both g_k and $g_{\bar{\theta}k}$; and λ, the forgetting factor parameter for weighting the recent data.

The rules of fuzzy change detector are obtained after some deep investigation on the system dynamics and trial-and-error tunings using both simulated and experimentally tested heading reference system of a vehicle. The fuzzy rules are simple and have a systematic tuning approach in comparison to rules of non-fuzzy mixed change detectors. The rules of fuzzy system are in the following generic form:

If g_k is A_1^l and ∇g_k is A_2^l and $g_{\bar{\theta}k}$ is A_3^l and $\nabla g_{\bar{\theta}k}$ is A_4^l, then N_k is B_1^l, N_t is B_2^l, λ is B_3^l; where, A_i^l $(i = 1, 2, 3, 4)$ and B_j^l $(j = 1, 2, 3)$ are fuzzy sets for linguistic input and output variables, respectively.

Here, 16 fuzzy *if-then* rules ($M = 16$) are constructed to deal with simple membership functions which are defined for inputs and outputs of fuzzy system. The response of fuzzy system corresponding to inputs A_1', A_2', A_3', A_4' is B' with the following membership function,

$$\mu_{B'}(N) = \max_{l=1}^{16}\left[\sup\begin{pmatrix}\min\{\mu_{A_1'}(g_k)\,\mu_{A_2'}(\nabla g_k)\,\mu_{A_3'}(g_{\bar{\theta}k})\,\mu_{A_4'}(\nabla g_{\bar{\theta}k}), \\ \mu_{A_1^l}(g_k)\,\mu_{A_2^l}(\nabla g_k)\,\mu_{A_3^l}(g_{\bar{\theta}k})\,\mu_{A_4^l}(\nabla g_{\bar{\theta}k}),\ \mu_{B^l}(N)\}\end{pmatrix}\right], \tag{26}$$

Defuzzified output vector, which includes the length of variable sliding windows and forgetting factor, is given below:

$$N^* = \frac{\sum_{j=1}^{16} N^j\ \mu_{B'}(N^j)}{\sum_{j=1}^{16} \mu_{B'}(N^j)}. \tag{27}$$

The fuzzy inference engine consists of the following operations,
- Individual-rule based inference with union combination,
- Mamdani's minimum implication,
- Algebraic product for all the t-norm operators,
- Max for all the s-norm operators.

Final tuning of fuzzy membership functions, which are plotted in Figures 1, are done by considering variations of the fuzzy system outputs against variations of its inputs (g_k, ∇g_k, $g_{\overline{\theta_k}}$ and $\nabla g_{\overline{\theta_k}}$) during tests of the heading reference system on a vehicle. The fuzzy rules, after final tuning, are gathered in the search Table 1, where S, M and L stand for small, medium and large, respectively.

5 Simulations and experimental tests

In this section, the performance of the BLS estimation algorithm together with both proposed FVLSW and recently developed VLSW change detectors are evaluated using simulations and real tests of a vehicle. The magnetic heading reference system is simulated after reviewing test data that are obtained from a strapped Vitans-type attitude-heading reference system on a vehicle [15]. One hundred runs of magnetic heading system are executed to provide a Monte-Carlo type simulation.

The new regression model of heading reference system (4) is applied in both of FVLSW- and non-fuzzy VLSW- BLS algorithms to online estimation of disturbance/calibration parameters. The capability and performance of this new modeling scheme in parameter estimate tracking is very significant in comparison to that of the calibration model (2). In other words, using this new model results in expected convergence and accuracy of estimate parameters under both abrupt and gradual changes. On the other hand, implementing the BLS estimation algorithm using calibration model (2) caused to a diverged online estimate unlike its satisfactory performance in offline calibration of the three-axis magnetometers system. As seen in Figures 2 through 8 (subfigures a and b are concerned to FVLSW and VLSW respectively), tracking capability of new FVLSW BLS in online estimation of time-varying parameters is superior to that of the recent VLSW BLS algorithm. Although as shown in Figure 2, only estimate of B_x using the

VLSW BLS has a bit more deviation from its desired value with respect to that of the FVLSW algorithm, estimate of other five parameters using the FVLSW resulted in a remarkably better tracking precision, as can be observed easily in Figures 3 through 9. Specially, deviation of B_z and G_z from their desired values using the VLSW is not satisfactory. Due to devoting an individual index to detect the stop of changes in the VLSW [7], this algorithm resulted in a better performance for declaring the stop point of abrupt changes. However, in the particular application, the effect of this superiority of the VLSW BLS estimator is not considerable in overall performance of the heading reference system. The accuracy of heading angle, which is computed after application of estimated calibration parameters to raw measurements of the three-axis magnetometers, is evaluated as the overall performance of the estimation algorithms. Therefore, the overall performances of the BLS estimation algorithm with- and without- using change detection mechanisms are compared in Figure 8. It is shown that the overall performance of FVLSW BLS is evidently superior to that of the VLSW BLS.

Next, robustness of FVLSW and VLSW systems against noises is investigated after adding a white Gaussian noise on measurements of the three-axis magnetometers in addition to soft- and hard-iron magnetic disturbances. The covariance is taken to be isotropic with a standard deviation of 50 nano-tesla (NT), which is equal to noise of the common magnetometers. The measurements are sampled every 0.02 second over a 100-second span. Heading errors that are obtained from a calibrated heading reference system using FVLSW and VLSW change detectors together with BLS estimation algorithm have been shown in Figure 9. Although weakness of the VLSW BLS in attenuating noise effects on estimated parameters is evident, performance of the FVLSW BLS is excellent for parameter estimation from noisy measurements due to noise filtering capability of fuzzy membership functions. In Figures 10 and 11, estimation of B_y and G_z using the FVLSW BLS and VLSW BLS algorithms, shows the strong capability of the proposed fuzzy algorithm in noisy environments. Monte-Carlo simulation results in Table 2, which is executed for 100 runs, revealed a comprehensive evaluation of both fuzzy and non-fuzzy estimation algorithms in the sense of root-mean-square (RMS) errors.

Next, performance of the proposed estimation and modeling schemes is evaluated using experimental tests data. The system under test is a Vitans type magnetic heading reference system, which is strapped on a ride vehicle as shown in Figure 12. However, this system is affected by soft- and hard-iron disturbances due to non-geomagnetic local fields. Several tests are carried out in different maneuvering situations of the vehicle in mountain roads, highways, and city avenues to obtain as much as possible extensive data.

Figures 13 and 14 show that the FVLSW scheme resulted in a better tracking quality of the vehicle heading with respect to the VLSW scheme. Since the implementations are run using measurements of the three-axis magnetometers, the resulting heading angle of the vehicle should be considered with respect to magnetic reference north. Therefore, error of the estimated heading from that was found by integrated INS/GPS should accompany a bias. After performing the FVLSW- and VLSW- BLS algorithms for near one-hour tests data, mean of the heading biases were found to be 4.83º and 9.67º, respectively. On the other hand, declination angle around geographical location of the tests using international geomagnetic reference field model [16], was found to be 6.5º. Therefore, the FVLSW BLS worked well because of its less bias errors with respect to that of the VLSW BLS algorithm.

A final investigation of the aforementioned algorithms performance is carried out based on norm of the magnetometers output vector before and after removing disturbance effects. In Figure 15, plot of these norms for data of the vehicle test in mountain road revealed that online

calibration of the three-axis magnetometers system using estimated parameters led to obtain expected fixed norm of the measurement vector. Small deviations in the norm are because: new model of heading reference system is based on a predetermined downward component of the earth's magnetic field vector without considering other two local level components, the actual noises may are non-Gaussian, and the estimated parameters are together with small errors especially during abrupt changes. These errors are so small such that the expected results of estimation algorithms are completely obtained.

6 Conclusions

In the paper, a new real-time FVLSW BLS algorithm was proposed to estimate the parameters under both abrupt and gradual changes. Fuzzy combination of two important change detection indices including generalized likelihood ratio and averaged parameter estimation error together with their rates resulted in intelligently adjusted lengths of sliding windows for updating change detection indices and BLS estimation algorithm.

Incorporation of direction cosine matrix in regression models exposed a way for increasing the order of persistent excitation of the corresponding squared signal matrix. Using this modeling technique in the FVLSW BLS algorithm had profound effects on the accuracy and convergence speed of online parameter estimation. Performance of the proposed FVLSW was evaluated in comparison to that of the VLSW using both simulated and actual test data of a magnetic heading reference system. Monte-Carlo type simulation, executed for 100 runs, showed evidently better performance for FVLSW in the sense of RMS of estimation errors.

In addition to Table 2, more figured results showed that the performance of the FVLSW had significantly better quality compared to that of the VLSW. Especially, when the noise power of the Vitans sensors was considered in simulations, unlike VLSW, the FVLSW algorithm had excellent performance in detection of both abrupt and gradual change regimes. Therefore, it led to a satisfactory BLS parameter estimation.

In the meantime, experimental performances of the FVLSW and the VLSW algorithms together with proposed regression model of the three-axis magnetometers were examined using actual tests data of a strapped system on a ride vehicle. In comparison to heading angle by integrated INS/GPS as a reference, implementing both of the fuzzy and non-fuzzy change detectors in BLS estimation algorithm resulted in an acceptable tracking accuracy of the vehicle heading. However, the accuracy of heading angle obtained from the FVLSW is certainly superior to that obtained from the VLSW in which the mean of biases during one-hour tests were 1.67° and 3.17° using FVLSW and VLSW, respectively. Norm of the magnetic field vector before and after removing disturbance effects was investigated as another performance index. The relation between horizontal components of the earth's magnetic field vector (5), which was not used as a complementary regression model because of its strongly coupled and nonlinear structure, may be considered as a nonlinear constraint for regression model of (4) in the future. Therefore, small variations in norm of the magnetometers measurement vector may be decreased, which in turn can improve the accuracy of determined heading angle. From test results, it was shown that the FVLSW resulted in a better performance in the sense of vector norm than that of the VLSW. However, when fluctuations in norm of raw data were small, the resulted heading angle using both the FVLSW and VLSW algorithms was close to reference heading of the INS/GPS (errors were less than 1°). Once bigger fluctuations in norm of the magnetometers output appeared due to big disturbances, better performance of the FVLSW BLS estimator was evident with respect to

that of the VLSW BLS estimator in the sense of heading accuracy and of the norm of corrected outputs.

7 References

[1] Deutschmann, J.K., and Izhtack, B., Evaluation of Attitude and Orbit Determination Using Actual Earth Magnetic Field Data, J. Guidance, Vol. 24, pp. 616-623, (2001).

[2] Alonso, R., and Shuster, M.D., A New Algorithm for Attitude-Independent Magnetometer Calibration, J. Astronautical Sciences, pp. 513-527, (2003).

[3] Ma, G.F., and Jiang, X.Y., Unscented Kalman Filter for Spacecraft Attitude Estimation and Calibration Using Magnetometer Measurements, Proc. of the Fourth Int. Conf. on Machine Learning and Cybernetics, Guangzhou, Korea, pp. 18-21, August (2005).

[4] Gebre-Egziabher, D., and Elkaim, G.H., Calibration of Strapdown Magnetometers in Magnetic Field Domain, ASCE Journal of Aerospace Engineering, Vol. 19, pp. 87-102, (2006).

[5] Kao, W.W., and Tsai, C.L., Adaptive and Learning Calibration of Magnetic Compass, IOP J. Measurement Science and Technology, Vol. 17, pp. 3073-3082, (2006).

[6] Ding, F., and Xiao, Y., A Finite-Data-Window Least Squares Algorithm with a Forgetting Factor for Dynamical Modeling, Applied Mathematics and Computation, Vol. 186, pp. 184–192, (2007).

[7] Jiang, J., and Zhang, Y., A Novel Variable-Length Sliding Window Blockwise Least-Squares Algorithm for On-Line Estimation of Time-Varying Parameters, Int. J. Adapt. Control Signal Process, Vol. 18, pp. 505–521, (2004).

[8] Basseville, M., and Nikiforov, I., Detection of Abrupt Changes: Theory and Application, Englewood Cliffs, New Jersey, Prentice Hall, 1993.

[9] Liu, H., and He, Z., A Sliding-Exponential Window RLS Adaptive Algorithm: Properties and Applications, Signal Processing, Vol. 45, pp. 357–368, (1995).

[10] Lee, Y.H., Kim, M., Chu, Y.H., and Han, C., Adaptive Multivariate Regression Modeling Based on Model Performance Assessment, Chemo Metrics and Intelligent Laboratory Systems, 78, pp. 63– 73, (2005).

[11] Rogers, R.M., Applied Mathematics in Integrated Navigation Systems, 2nd ed. Virginia, AIAA Series, 2003.

[12] Astrom, K.J., and Wittenmark, B., Adaptive Control, 1st ed., Addison-Wesley, 1989.

[13] Keighobadi, J., Menhaj, M.B., and Kabganian, M., Nonlinear Control of a Wheeled Mobile Robot: a Fuzzy Approach, Amirkabir J. Science and Technology Vol. 17, pp. 41-51(in Persian), (2007) .

[14] Malhotra, A., and Huang, B., Detection of Abrupt Change and Applications in Sensor Decalibration Monitoring, ISA Trans., Vol. 41, pp. 155–166, (2002).

[15] Http://www.vertexinertial.com/products/air/vitans, accessed March 2003.

[16] Http://www.ngdc.noaa.gov/seg/WMM/DoDWMM.shtml, accessed May 2006.

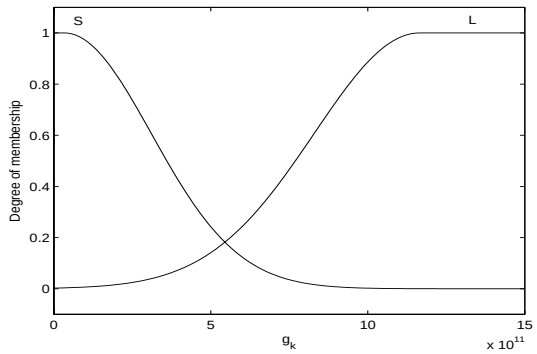
Tables

Table 1 Search table of fuzzy rules

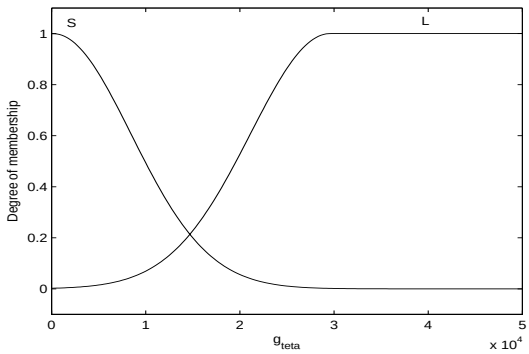
g_k	$g_{\bar{\theta}k}$	∇g_k	$\nabla g_{\bar{\theta}k}$	N_k	N_t	λ
S	*S*	*S*	*S*	*L*	*L*	*L*
S	*S*	*S*	*L*	*L*	*M*	*L*
S	*S*	*L*	*S*	*L*	*L*	*L*
S	*S*	*L*	*L*	*L*	*M*	*L*
S	*L*	*S*	*S*	*M*	*L*	*M*
S	*L*	*S*	*L*	*M*	*M*	*M*
S	*L*	*L*	*S*	*M*	*M*	*M*
S	*L*	*L*	*L*	*M*	*S*	*M*
L	*S*	*S*	*S*	*M*	*L*	*S*
L	*S*	*S*	*L*	*L*	*M*	*M*
L	*S*	*L*	*S*	*M*	*M*	*M*
L	*S*	*L*	*L*	*M*	*S*	*M*
L	*L*	*S*	*S*	*S*	*M*	*M*
L	*L*	*S*	*L*	*S*	*M*	*M*
L	*L*	*L*	*S*	*S*	*M*	*L*
L	*L*	*L*	*L*	*S*	*S*	*L*

Table 2 RMS errors of the BLS estimations

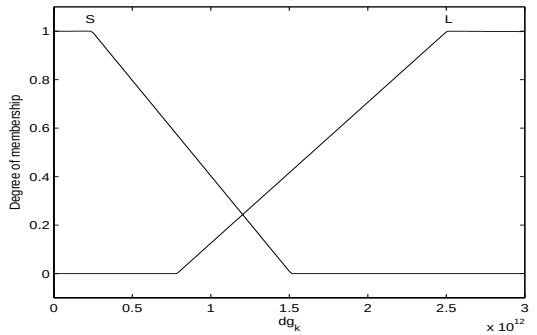
	Free of noise		**Noisy**	
	FVLSW	VLSW	FVLSW	VLSW
B_x	73.103	159.14	813.56	45220
B_y	104.062	255.84	177.93	80735
B_z	744.205	26900	24.222	16381
G_x	0.156	0.236	0.3578	0.593
G_y	0.181	0.259	0.3931	0.685
G_z	0.226	1.140	0.3464	2.759



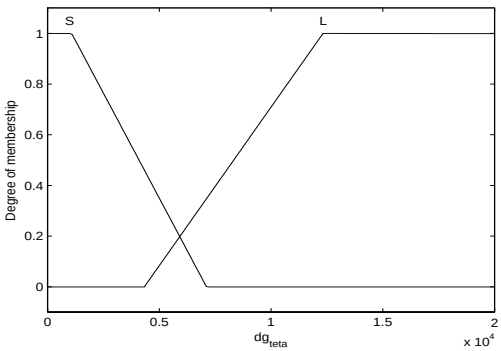
(a) Generalized likelihood ratio, g_k .



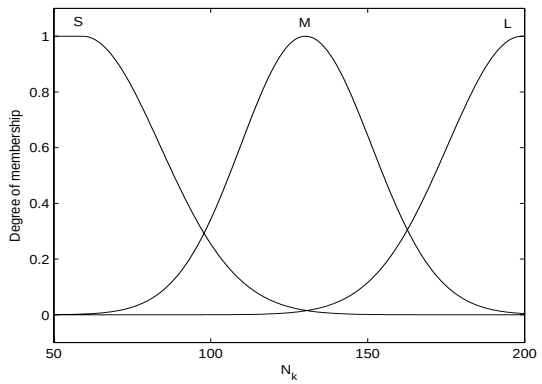
(b) Averaged parameter estimation error, $g_{\bar{\theta}k}$.



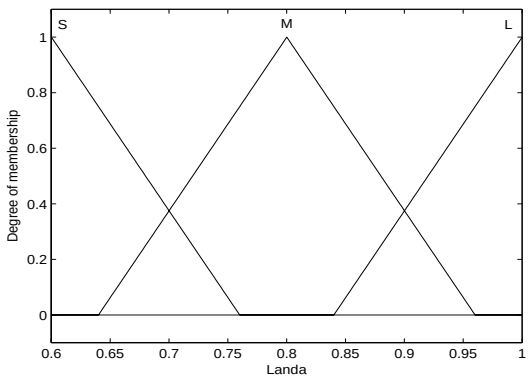
(c) Difference of averaged likelihood ratios, ∇g_k .



(d) Change rate of averaged estimation error.



(e) Length of BLS estimator sliding window, N_k .



(f) Forgetting factor λ .

Figure 1 Membership functions of fuzzy change detection system inputs and outputs

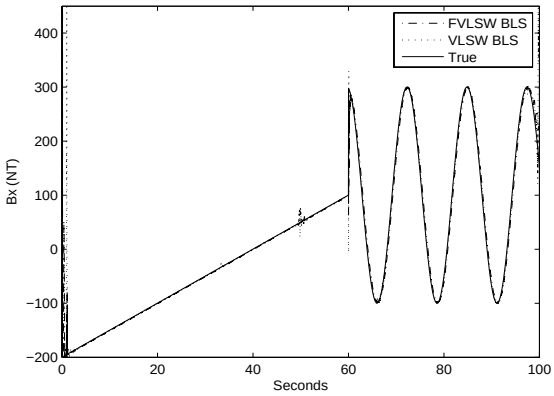


Figure 2 True and estimated hard-iron bias along *x*

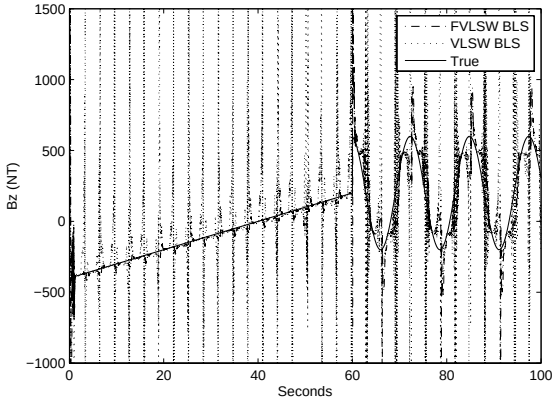


Figure 3 True and estimate hard-iron bias along *z*

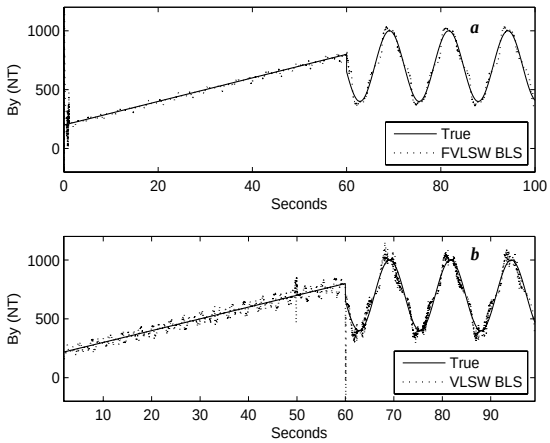


Figure 4 True and estimated second hard iron bias

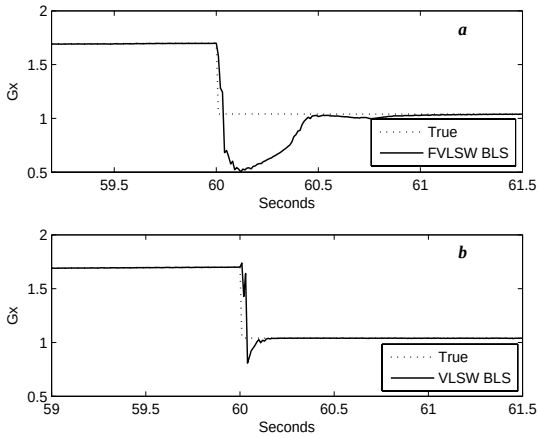


Figure 5 True and estimated first soft-iron gain

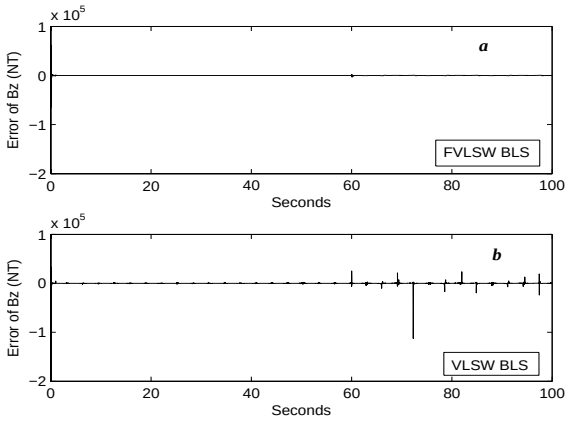


Figure 6 Estimation error of B_z.

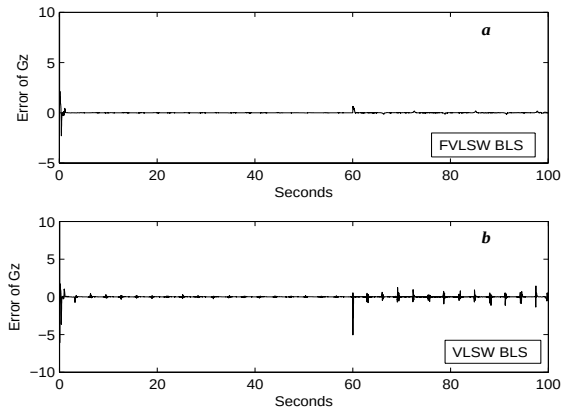


Figure 7:. Estimation error of G_z.

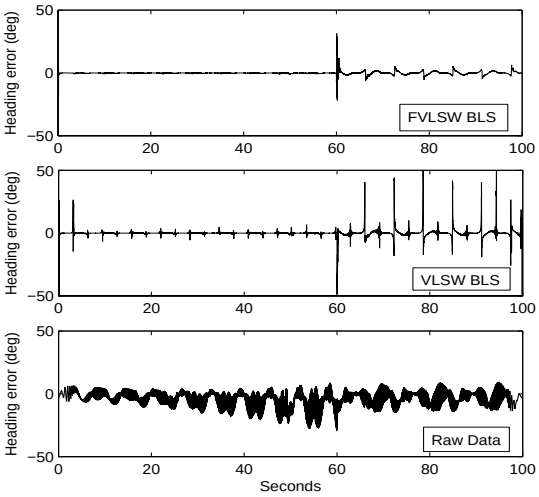


Figure 8 Heading angle error with- and with-out on-line calibration of three-axis magnetometers system.

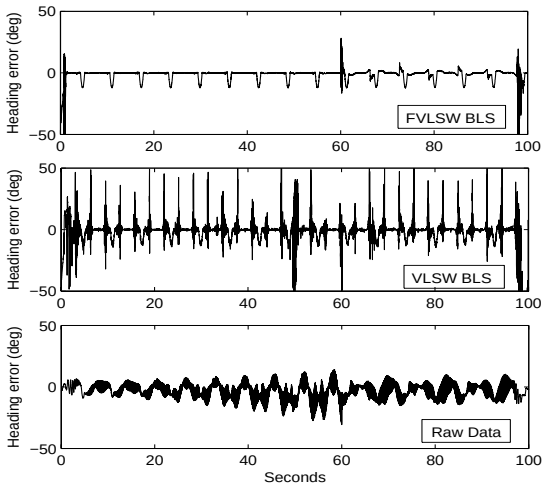


Figure 9 Noise effects on heading error with and with-out online calibration

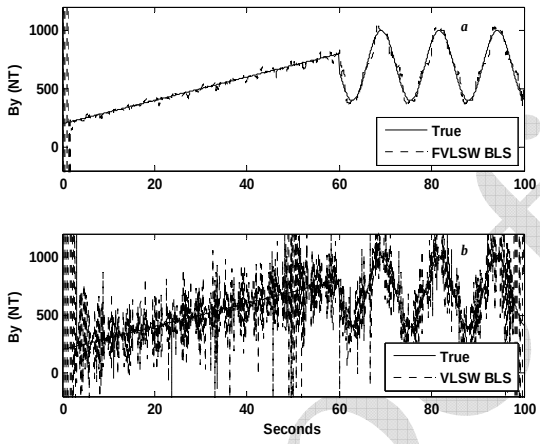


Figure 10 Estimated B_y using noisy measurements.

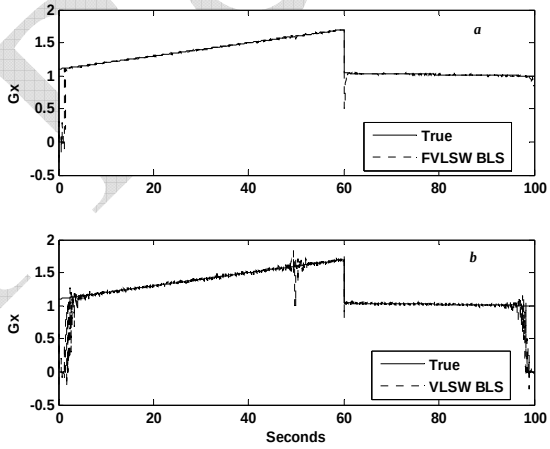


Figure 11 Estimated G_x using noisy measurements.

Figure 12 Test vehicle together with Vitans system.

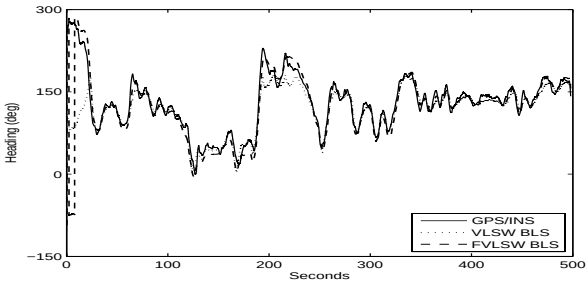


Figure 13 Estimated and INS/GPS headings.

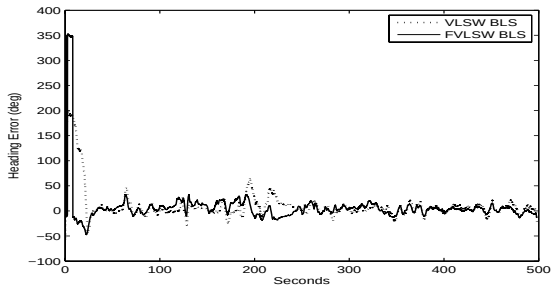


Figure 14 Error between the estimated and INS/GPS. headings.

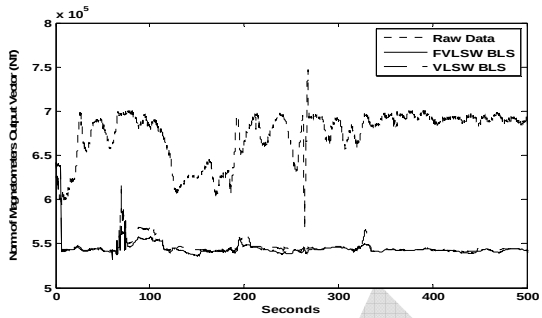


Figure 15 Norm of the magnetometers output vector after and before estimation algorithms.

چكيده

در خودروهاي زميني، سنسورهاي مغناطيس‌سنج متعامد سه‌محوره تحت تأثير اغتشاشات مغناطيسي نرم و سخت زيادي قرار مي‌گيرند. بنابراين، بدست آوردن زاويه سمت از شمال با دقت و كيفيت مورد نياز، بدون تخمين اغتشاشات كاليبراسيون و جبران آنها در بردار اندازه‌گيري امكان‌پذير نيست. براي تخمين برخط پارامترهاي متغير با زمان، ماتريس رگرسور (يا سيگنال) با حدأقل مرتبه غناي معادل تعداد پارامترها ضروري است. در اين مقاله، ابتدا ايده بديع افزايش مرتبه غناي سيستم از طريق درگير كردن ماتريس سمت كسينوسي در مدل رگرسور تخمين ارائه مي‌شود. سپس، تخمين‌زن بديع حدأقل مربعات بلوكي فازي همراه با طول پنجره لغزشي متغير به منظور تخمين برخط و بلادرنگ پارامترهائي از سيستم كه تحت تغييرات سريع و يا تدريجي هستند ارائه مي‌شود. ايده اصلي اين روش، تركيب فازي دو مكانيزم تشخيص تغييرات شامل نسبت حداكثر احتمال تعميم‌يافته و خطاي متوسط پارامترها و همچنين نرخ تغيير آنها براي توليد يك سيستم فازي تشخيص‌دهنده تغيير و يا خرابي در سيستم است. خروجي سيستم فازي طول دو پنجره لغزشي براي بروزرساني مكانيزم‌هاي آشكارساز تغييرات و بروزرساني تخمين پارامترها در الگوريتم حدأقل مربعات و همچنين يك ضريب فراموشي براي وزن‌دهي بيشتر به داده‌هاي جديدتر است. نتايج شبيه‌سازي‌ها و تست‌هاي خودرو نشان مي‌دهند كه عملكرد سيستم تعيين سمت با تخمين‌زن جديد بهتر از عملكرد آن با الگوريتم حدأقل مربعات دسته‌ايي اخير ارائه شده است. مزاياي اين روش در مواقعي كه قدرت نويزها قابل ملاحظه باشد بيشتر مشخص مي‌شود.